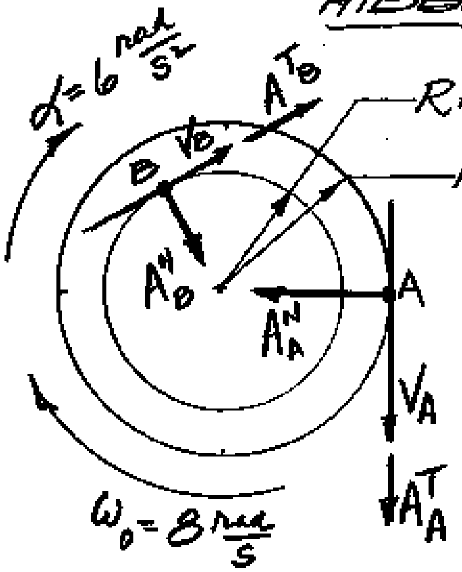


pt. A
$r = 2' = 24''$

ANGULAR VEL. INITIALLY, $\omega_0 = 8 \frac{rad}{s}$

ANGULAR ACC., $\alpha = 6 \frac{rad}{s^2}$

ACCELERATION ∴ SAME CW DIR.

FIND LINEAR VEL. + ACC. WHEN $t = 0.5s$

SIMILAR TO DERIVATION FOR LINEAR TERMS, USE DEFINITION:

$$\alpha = \frac{d\omega}{dt}$$

$$\int_0^{0.5s} \alpha \, dt = \int_{\omega_0 = 8\frac{rad}{s}}^{\omega_f} d\omega$$

$$\alpha t \Big]_0^{0.5} = \omega_f - \omega_0 \Rightarrow \omega_f = \omega_0 + \alpha t$$

$$\omega_f = 8\frac{rad}{s} + \left(6 \frac{rad}{s^2}\right)(0.5s)$$

$$\boxed{\omega_f = 11 \frac{rad}{s}}$$

ANG. ACC. CAUSED THE ANG. VEL. TO INCREASE FROM $8 \frac{rad}{s}$ TO $11 \frac{rad}{s}$ IN 0.5 SECONDS

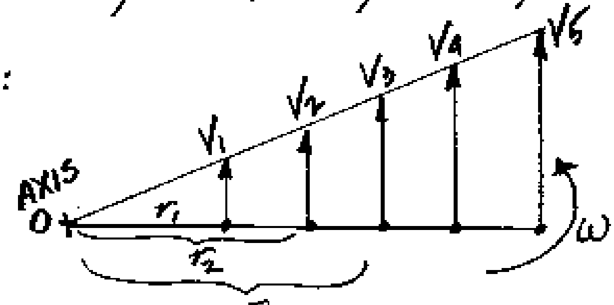
LINEAR VEL., $V_A = r\omega$ @ $t = 0.5s$ $= (2')(11 \frac{rad}{s}) = 22 \frac{'}{s} = V_A$

while $V_B = (1.5')(11 \frac{rad}{s}) = 16.5 \frac{'}{s} = V_B$

NOTE: For the same angular velocity, ω, the linear velocity increases as the pt. moves further from the center axis.

VELOCITY proportional to RADIUS:

$$V = r\omega$$



$$\therefore \omega = \frac{V_1}{r_1} = \frac{V_2}{r_2} = \frac{V_3}{r_3} = \frac{V_4}{r_4}$$

WE NEED THIS RELATIONSHIP FOR MESHING GEARS.

CENTRIPETAL or NORMAL ACC., $A^N = \frac{V^2}{r} = r\omega^2$

If $V = r\omega$, $V^2 = r^2\omega^2$ $\therefore \frac{V^2}{r} = \frac{r^2\omega^2}{r} = \boxed{r\omega^2 = A^N}$

$$\left.\begin{array}{l} \text{for pt. A,} \quad A^N = (2')(11\tfrac{rad}{s})^2 = 242\tfrac{1}{s^2} \\ \text{for pt. B,} \quad A^N = (1.5')(11\tfrac{rad}{s})^2 = 181.5\tfrac{1}{s^2} \end{array}\right\}$$ DIRECTED IN TOWARD CENTER AXIS.

TANGENTIAL ACCELERATION 'pulls' tangent to the curve as the motion speeds up.

$A^T = r\alpha$ exists ONLY if there is an angular acceleration.

for pt. A, $A^T = (2')(6\tfrac{rad}{s^2}) = 12\tfrac{1}{s^2}$

for pt. B, $A^T = (1.5')(6\tfrac{rad}{s^2}) = 9\tfrac{1}{s^2}$

TOTAL ACCEL. = VECTOR SUM of these $\perp$ COMPONENTS.

$\vec{A}_{TOT} = \vec{A^N} + \vec{A^T}$ or $|A_{TOT}| = \sqrt{(A^N)^2 + (A^T)^2}$

$$\left.\begin{array}{l} \text{for pt. A,} \quad A_{TOT} = \sqrt{(242)^2 + (12)^2} = 242.3\ rad/s^2 \\ \text{for pt. B,} \quad A_{TOT} = \sqrt{(181.5)^2 + (9)^2} = 181.7\ rad/s^2 \end{array}\right\}$$ ACCEL.

FOR PROB. 16.3, FIND VEL. + ACC. AFTER 2 REVOLUTIONS or 2 COMPLETE TURNS. THIS IS ANGULAR DISPLACEMENT.

1 REVOL. $= 360° = \theta = 2\pi$ radians

$\therefore$ 2 REVOL $= 2\,(2\pi\ rad) = 4\pi$ radians

ANGULAR DISPLACEMENT, $0 \longrightarrow 4\pi$ radians

ANGULAR VELOCITY, $\omega_0 = 8\frac{rad}{s} \longrightarrow \omega_f = ?$

CONSTANT ANG. ACC., $\alpha = 6\ rad/s^2$

NOTE: TIME IS $\underline{\underline{\text{NOT}}}$ A FACTOR

SIMILAR DERIVATION AS FOR LINEAR MOTION.

$\alpha = \frac{d\omega}{dt}$ where $\omega = \frac{d\theta}{dt}$ $\therefore dt = \frac{d\theta}{\omega}$

$= \frac{\omega\, d\omega}{d\theta}$ } OR USE CHAIN RULE $\frac{d\omega}{d\theta}\left(\frac{d\theta}{dt}\right) = \frac{d\omega}{d\theta}(\omega)$

SEPARATE VARIABLES & INTEGRATE

$$\int_0^{4\pi} \alpha\, d\theta = \int_8^{\omega_f} \omega\, d\omega$$

$$\alpha\theta \Big]_0^{4\pi} = \frac{\omega^2}{2}\Big]_{8\frac{rad}{s}}^{\omega_f} = \frac{1}{2}\left[\omega_f^2 - \left(8\tfrac{rad}{s}\right)^2\right]$$

or $2\alpha[\theta_f - \theta_0] = \omega_f^2 - \omega_0^2$

$$\omega_f^2 = \omega_0^2 + 2\alpha[\theta_f - \theta_0] = \left(8\tfrac{rad}{s}\right)^2 + 2\left(6\tfrac{rad}{s^2}\right)\left[4\pi \text{ radians}\right]$$

$\omega_f^2 = 214.8 \frac{rad}{s^2}$ $\therefore \boxed{\omega_f = 14.66 \tfrac{rad}{s}}$

The angular acceleration has increased the ang. velocity from $8 \frac{rad}{s}$ to $14.66 \frac{rad}{s}$ after 2 revolutions.

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for pt. A

$V_A = r\omega = (2')(14.66 \frac{rad}{s})$

$= 29.32 \frac{1}{s}$

$A^N = r\omega^2 = V\omega = 429.83 \,^1/_{s^2}$

$A^T = r\alpha = (2')(6 \frac{rad}{s^2})$

$= 12 \,^1/_{s^2}$

$A_{TOT} = \sqrt{} \doteq 430 \,^1/_{s^2}$

for pt. B

$V_B = r\omega = (1.5')(14.66 \frac{rad}{s})$

$= 21.99 \,^1/_s$

$A^N = r\omega^2 = V\omega = 322.37 \,^1/_{s^2}$

$A^T = r\alpha = (1.5')(6 \frac{rad}{s^2}) = 9 \,^1/_{s^2}$

$A_{TOT} = \sqrt{} = 322.5 \,^1/_{s^2}$

NOTE: TANGENTIAL ACC. for pt. A + B were unchanged.