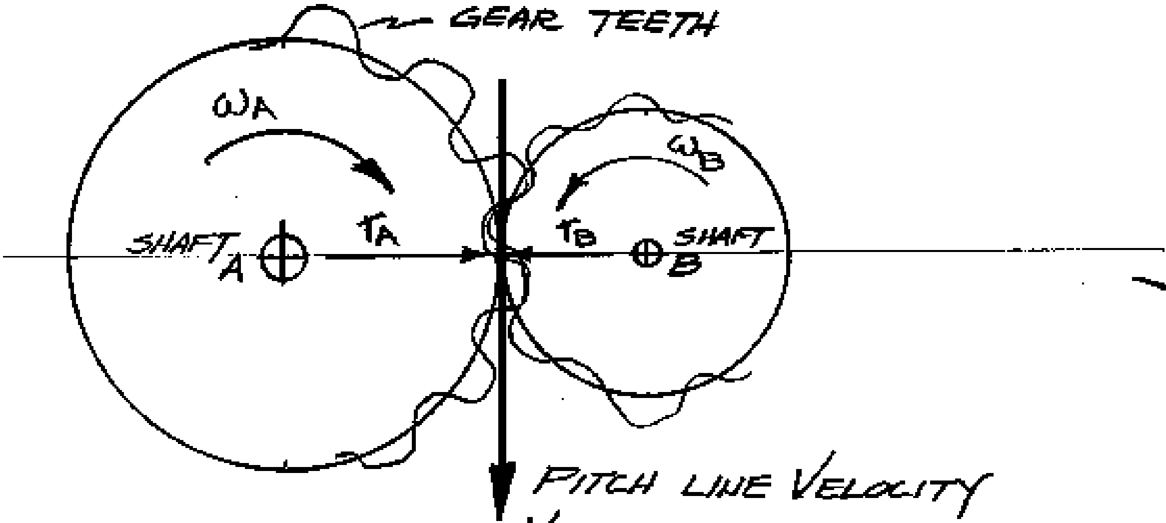
MESHING GEARS

GEARS MESH OR CONTACT ALONG A PITCH CIRCLE. FOR ANALYSIS, CONSIDER 2 CYLINDERS IN CONTACT. THE CIRCUMFERENCE OF THE CYLINDERS WOULD BE THE PITCH CIRCLES.



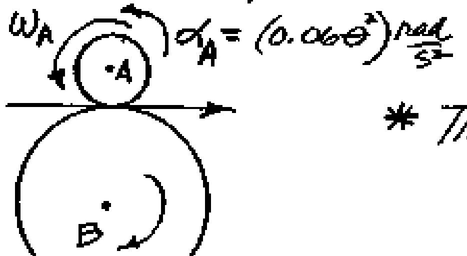
PITCH LINE VELOCITY

$$V_A = r_A \omega_A = V_B = r_B \omega_B$$

$$\therefore \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{d_B/2}{d_A/2} = \frac{\text{diameter}_B}{\text{diameter}_A} \Bigg\} \text{ INVERSELY PROPORTIONAL}$$

FOR THIS PROBLEM, A MOTOR TURNS (OR DRIVES) A SHAFT WITH TEETH CUT ON THAT SHAFT, A. THOSE TEETH MESH WITH A GEAR, B.

SHAFT DIAM = 24 mm $\therefore r_A = 12$ mm with $\omega_0 = 50 \frac{rad}{s} = \omega_A$



FIND: ω_B after $\theta_A = 10$ revolutions $= 20\pi$ radians

$r_B = 60$mm

* TIME IS NOT A FACTOR

$$\therefore \alpha\, d\theta = \omega\, d\omega$$

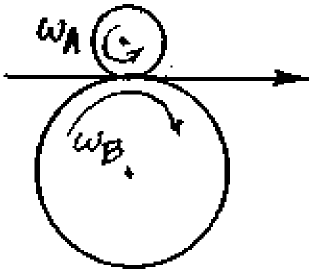
$$\int_0^{20\pi} (0.06\theta^2)\, d\theta = \int_{50}^{\omega_A} \omega\, d\omega$$

$$0.06 \frac{\theta^3}{3} \Big]_0^{20\pi} = \frac{\omega^2}{2} \Big]_{50}^{\omega_A}$$

$$0.02 \left[20\pi\right]^3 = \frac{1}{2}\left[\omega_A^2 - (50)^2\right]$$

$$9.922 \times 10^3 + 50^2 = \omega_A^2 \qquad \therefore \boxed{\omega_A = 111.45 \frac{rad}{s}}$$

AFTER 10 REVOLUTIONS



$V_A = r_A\omega_A = V_B = r_B\omega_B$

BECAUSE LINEAR VELOCITY IS TANGENT TO THE CURVE.

$\therefore V_A = V_B$ AT THE PITCH POINT

PITCH POINT IS THE POINT WHERE THE TWO CYLINDERS OR TWO CIRCLES COME IN CONTACT.

NOTE: AT THIS POINT, THE RADII ARE CO-LINEAR.

THE VELOCITY VECTOR IS $\perp$ RADII AT THIS POINT.

CONSIDER A = DRIVER or MOTOR INPUT.

$\therefore$ GEAR B = DRIVEN or OUTPUT

$$\frac{\text{OUTPUT}}{\text{INPUT}} = \frac{\omega_B}{\omega_A} = \frac{r_A}{r_B}$$

INVERSELY PROPORTIONAL

$r_A\omega_A = r_B\omega_B$

$$\therefore \omega_B = \frac{r_A}{r_B}\omega_A = \frac{(12\text{ mm})}{(60\text{ mm})}\left(111.45\tfrac{\text{rad}}{\text{s}}\right) = 22.29\tfrac{\text{rad}}{\text{s}}$$

SUMMARY:

INITIALLY $\omega_A = 50\frac{\text{rad}}{\text{s}}$

$$\omega_B = \frac{r_A\omega_A}{r_B} = \frac{12}{60}\left(50\tfrac{\text{rad}}{\text{s}}\right) = 10\tfrac{\text{rad}}{\text{s}}$$

and the PITCH LINE VEL. $V_A = r_A\omega_A = (12\text{mm})(50\frac{\text{rad}}{\text{s}}) = 600\frac{\text{mm}}{\text{s}}$

$V_B = r_B\omega_B = (60\text{mm})(10\frac{\text{rad}}{\text{s}}) = 600\frac{\text{mm}}{\text{s}}$

WITH AN ANGULAR ACC. OF $(0.06\times[20\pi]^2) = 236.87\frac{\text{rad}}{\text{s}^2}$

ANGULAR VELOCITIES INCREASED TO: $\omega_A = 111.45\text{ rad/s}$

$\omega_B = 22.3\text{ rad/s}$

$\therefore$ PITCH LINE VELOCITIES INCREASED:

$V_A = (12\text{ mm})(111.45\frac{\text{rad}}{\text{s}}) = V_B = (60\text{ mm})(22.3\frac{\text{rad}}{\text{s}}) = 1.33\frac{\text{m}}{\text{s}}$

ABOUT DOUBLED