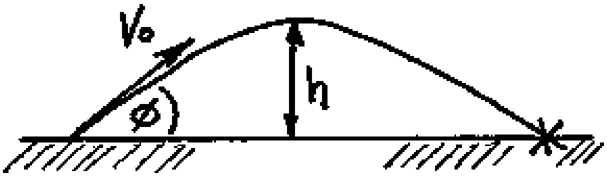
PROJECTILES



LEVEL TARGET

1. THE TRAJECTORY IS PARABOLIC
2. IMPACT VELOCITY = INITIAL VELOCITY, V_0
3. IMPACT ANGLE = INIT. LAUNCH ANGLE, ϕ
4. RANGE IS MAX. WHEN $\phi = 45°$
5. TIME TO REACH MAX. HT. = TIME FROM APEX TO IMPACT POINT.
6. TIME FROM APEX TO IMPACT = SAME TIME FOR FREE FALL A DIST., h

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FOR THESE PROBLEMS . . .

- NEGLECT AIR DRAG
- PROJECTILE IN MOTION HAS ONLY GRAVITATIONAL FORCE (ITS WEIGHT)

EXAMPLE A shot is fired at an angle of 45° with the horizontal and has an initial velocity of 300 ft./sec.

Find the height and range of the projectile.

SOLUTION: PROJECTILE HAS ZERO VELOCITY AT APEX. SEE #6 ABOVE.

$$V_y = V_{0_y} \pm gt \qquad \text{for max. ht., use APEX}$$

$$0 = 300\tfrac{ft}{sec} \sin 45° - 32.2\tfrac{ft}{s^2}\, t \qquad \text{SOLVE } t$$

$$\frac{212.1}{32.2} = t = 6.6 \text{ sec.}$$

USE t TO GET HEIGHT $= S = v_0 t \pm \frac{1}{2} g t^2$

$$\therefore\ h = \left(300\tfrac{1}{s} \sin 45°\right)(6.6 \text{ sec}) - \tfrac{1}{2}\left(32.2\tfrac{1}{s^2}\right)(6.6 \text{ sec.})^2$$

$$h \doteq 700 \text{ ft.} \uparrow \quad (698.5 \text{ ft.})$$

RANGE = HORIZ. COMPONENT

$= 2 \times V_{0_x} t = 2(300 \cos 45°)(6.6 \text{ sec})$

$= \underline{2800 \text{ ft.}}$ → NOTE: $\times 2$ BECAUSE t IS TIME TO REACH APEX, NEED TIME TO IMPACT

WHY ? ∴ because

HORIZ.

$$x(t) = V_0 \cos\phi \, t$$

$$V_x(t) = V_0 \cos\phi$$

VERT.

$$y(t) = (V_0 \sin\phi)t - \frac{1}{2}gt^2$$

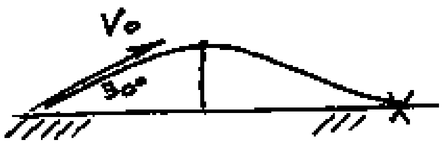
$$V_y(t) = V_0 \sin\phi - gt$$

SPECIAL FORMULAS:

$$\text{max. ht.} = \frac{V_0^2 \sin^2\phi}{2g}$$

$$\text{max. range} = \frac{V_0^2 \sin^2\phi}{g} = V_0 t \cos\phi$$

COMPARE FOR A 30° ANGLE, $V_0 = 300 \text{ ft/s.}$



AT APEX, $V = 0$

$$V_y = V_{0_y} - gt$$

$$0 = 300\frac{1}{s} \sin 30° - 32.2\, t$$

$$\frac{150}{32.2} = \underline{t = 4.66 \text{ sec.}}$$

$$h = v_0 t - \frac{1}{2}gt^2 = (300 \sin 30°)(4.66 \text{ sec.}) - \frac{1}{2}(32.2)(4.66 \text{ sec})$$

$h = 623.97 = \underline{624 \text{ ft.}}$ compared to 700 ft. ↑

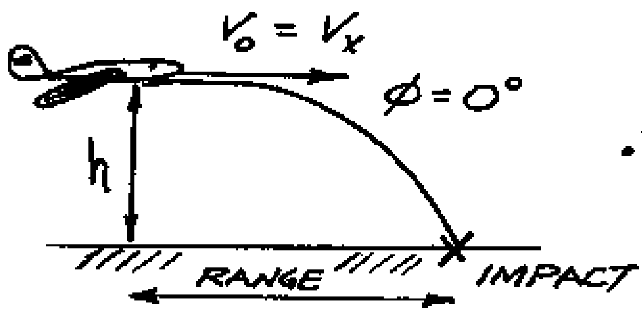
RANGE $= 2 \times V_{0_x} t = 2(300 \cos 30°)(4.66 \text{ sec.})$

$= \underline{2421 \text{ ft.}}$ compared to 2800 ft. →

PROJECTILES

FROM FLIGHT

∴ HORIZONTAL PROJECTION →



$x(t) = V_0 t$ $\qquad$ $y(t) = h - \frac{1}{2} g t^2$

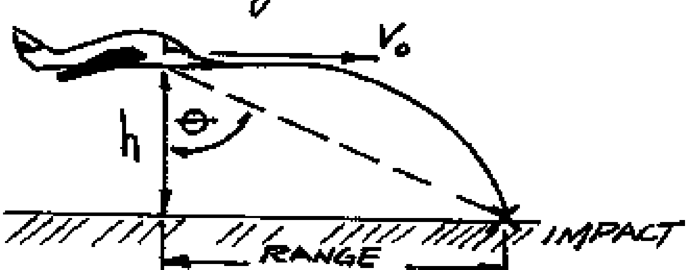
$V_x(t) = V_0$ $\qquad$ $V_y(t) = -gt$

max. ht. $= \frac{1}{2} g t^2$

max. range $= V_0 t$

EXAMPLE A different type of question!

A bomber flies horizontally at 275 mph at an altitude of 9000 ft. At what viewing angle, θ, from the bomber to the target should the bombs be dropped?



FALLING TIME DEPENDS ONLY ON ALTITUDE

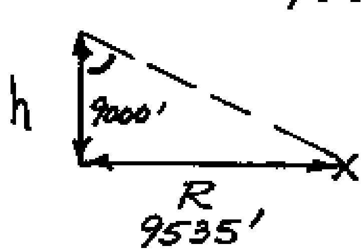
$$h = \frac{1}{2} g t^2 \quad \therefore \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(9000\,ft.)}{32.2\,'/sec^2}} = \underline{23.64\ sec.}$$

BOMB HAS SAME VELOCITY AS BOMBER!

∴ DIST. TRAVELED = RANGE = $V_0 t$

$$= \left[275 \frac{mi.}{hr.} \times \frac{5280\,ft.}{mi.}\right](23.64\,sec.) \times \frac{hr.}{3600\,sec.}$$

$$= 9534.8 \approx \underline{9535\,ft.}$$ HORIZONTAL DISTANCE



$$\theta = \tan^{-1} \frac{9535}{9000} = 46.65°$$

$$\boxed{\theta = 46.7°}$$