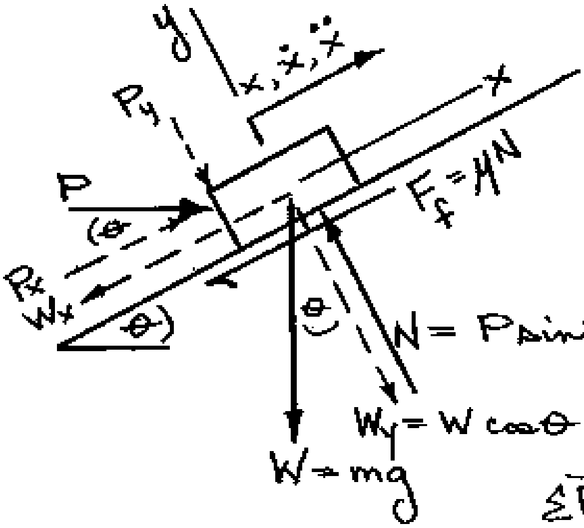
ENERGY METHOD

FORCE, DISTANCE + VELOCITY



To determine N,

$$\Sigma \vec{F}_y = m\cancel{\vec{a}_y}^{\,0}$$

$$N = P\sin\theta + W\cos\theta \quad *$$

$$\Sigma \vec{F}_x = m\vec{a}_x$$

$$P\cos\theta - W\sin\theta - F_f = m\vec{a}_x$$

$$\text{where} \quad a = \frac{dV}{dt} = \frac{dV}{dx}\left(\frac{dx}{dt}\right) = \frac{dV}{dx}V$$

SEPARATE VARIABLES + INTEGRATE:

$$\int_{x_i}^{x_f}\left(P\cos\theta - W\sin\theta - \mu N\right)dx = \int_{V_i}^{V_f} m\,V\,dV$$

$$\left(P\cos\theta - W\sin\theta - \mu P\sin\theta - \mu W\cos\theta\right)\left[x_f - x_i\right] = m\frac{V^2}{2}\Big]_{V_i}^{V_f}$$

$$\underbrace{\left(P\left[\cos\theta - \mu\sin\theta\right] - mg\left[\sin\theta + \mu\cos\theta\right]\right)}_{\text{FORCES ACTING ON MASS}}\Delta X = \frac{1}{2}m\left[V_f^2 - V_i^2\right]$$

ΔX = DISTANCE THE MASS MOVES

CONSTANT OF INTEGRATION: $V = V_i$ when $X = 0$

$$\underbrace{(\Sigma F)\,\Delta X}_{\text{WORK of a FORCE}} = \Delta K.E.$$

$$\text{WORK of a FORCE} = \Delta KE$$

$$\Delta PE = \Delta KE$$

TEXT NOTATION:

$$T = K.E._{1-2} = \int_1^2 = \tfrac{1}{2}mV^2$$

$$U = P.E._{1-2} = \int_1^2 = (mg)h = Wd$$

$$\underbrace{\text{WORK}_{PE}}_{} = \Delta KE$$

DETERMINE COMPONENTS IN SAME DIRECTION — $= \frac{1}{2} m \left[V_2^2 - V_1^2 \right]$

OR —

CONSIDER PRODUCTIVE WORK (+) & $\underbrace{\text{LOSS OF WORK}}_{\text{GRAVITY, FRICTION}}$ (−)

SAME MASS, CHANGING VELOCITY, ∴ ACCELERATION (+) or DECELERATION (−)

$$\Sigma \vec{F} = m\ddot{x} = m \frac{dV}{dt}$$

$$\int (\Sigma F)\, dt = \int m\, dV$$

$$(\Sigma F) \underbrace{\left[t_2 - t_1 \right]}_{\Delta t} = \underbrace{m \left[V_2 - V_1 \right]}_{\text{MOMENTUM}}$$

IF TIME IS AN ISSUE & YOU KNOW THE TIMES

ASSUME @ $t_1 = 0$, $V_1 = 0$ or V_0 for I.C.

DEFINE $(F)t = mV = m \frac{dx}{dt}$

$$F t\, dt = m\, dx$$

$$F \frac{t^2}{2} = m \left[x_2 - x_1 \right]$$

IF YOU KNOW DISPLACEMENT, FIND TIME, WORK BACK TO FIND VELOCITY

STORED ENERGY IN THE SPRING

$$-kx\, dx = m\, V dV$$

$$-k \frac{x^2}{2} \Big]_{x_1}^{x_2} = m \frac{V^2}{2} \Big]_{V_1}^{V_2}$$

$$-\frac{1}{2} k x_2^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 \quad \text{REARRANGE}$$

$$\frac{1}{2} m V_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m V_2^2 + \frac{1}{2} k x_2^2$$

$$\underbrace{KE_1 + PE_1}_{\text{INITIAL}} = \underbrace{KE_2 + PE_2}_{\text{FINAL}}$$

CONSERVATION of ENERGY

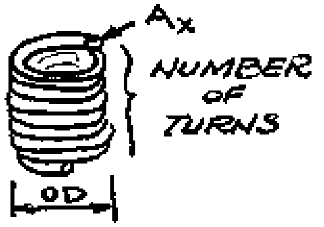
MECH 236

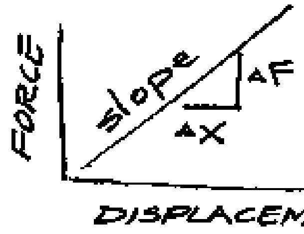
DERIVE EXPRESSION FOR $k_{EQUIVALENT} = \text{"}\Sigma k\text{"}$?

k = SPRING MODULUS or SPRING CONSTANT, f[geometry + physical properties]

k depends on wire diameter, coil diameter, number of coils, material of spring

$k = f$[displacement or STRETCH]





slope $= \dfrac{\Delta F}{\Delta X}$ = CONSTANT

$= k$ = PROPORTIONALITY CONSTANT

$\therefore \; F \propto X$

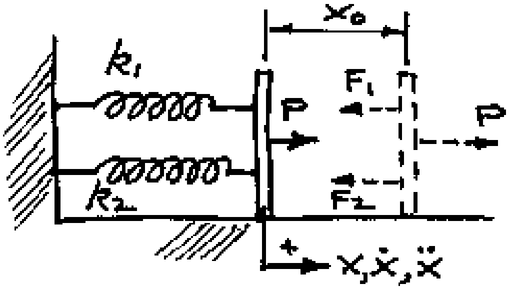
$\therefore \; \boxed{F = kX}$ SPRING FORCE

The amount of FORCE required to STRETCH (or COMPRESS) a spring depends on the geometry and material of the spring, k.

COMPOSITE SPRINGS

SPRINGS IN PARALLEL are physically parallel

NOTE: they SHARE THE DEFLECTION



P = PULLING FORCE TO STRETCH SPRINGS

X_o = DEFLECTION or "STRETCH" OF SPRINGS

$\Sigma \vec{F}_{\substack{on\\mass}} = m\vec{a} = m\ddot{x}$ where $m \approx 0$

$-F_1 - F_2 + P = 0$ define F_1 and F_2

$-(k_1 X_o) - (k_2 X_o) = -P$ both springs are stretched same amount, X_o

$\underbrace{(k_1 + k_2)}\, X_o = P$

$k_{eq.} \, X_o = P$

$\therefore \; \boxed{k_{eq.} = k_1 + k_2 \quad \text{PARALLEL}}$

NOTE: SIMILAR TO ELECT. RESISTORS IN SERIES.

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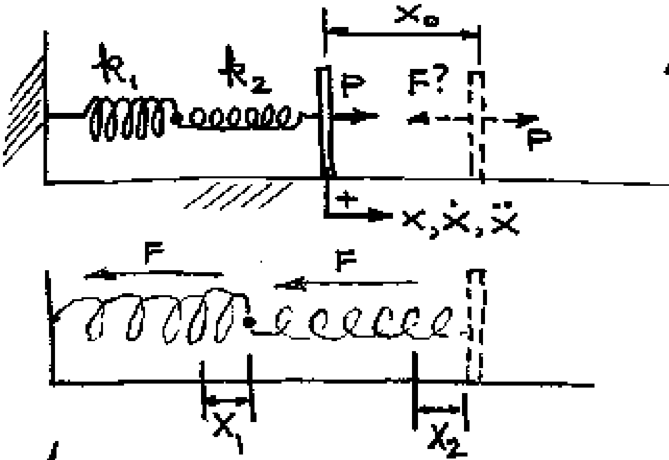
Springs in Series

are physically connected in series



Note: they SHARE THE FORCE

$$\sum \vec{F}_{\text{on mass}} = m\vec{a} = m\ddot{x} \quad \text{where } m \approx 0$$

$$-F + P = 0$$

? only ONE FORCE !

k_1 STRETCHES X_1 $\therefore F_1 = k_1 X_1$

k_2 STRETCHES X_2 $\therefore F_2 = k_2 X_2$

$$X_1 + X_2 = X_{TOT} = X_0$$

IMPORTANT:

P = FORCE NEEDED TO STRETCH k_2

F_2 = RESISTING FORCE in SPRING k_2 $\boxed{F_2 = k_2 x_2}$

when $F_2 = P$ the FORCE is passed on

P = FORCE NEEDED TO STRETCH k_1

F_1 = RESISTING FORCE in SPRING k_1 $\boxed{F_1 = k_1 x_1}$

LIKE CURRENT in a CIRCUIT $F_1 = F_2 = F = P$

$\therefore$ Look at their DEFLECTIONS where $\boxed{F = k X_0 \; ?}$

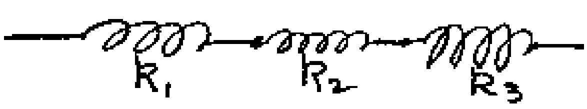
$$X_1 + X_2 = X_0$$

$$\frac{F_1}{k_1} + \frac{F_2}{k_2} = \frac{F}{k_{EQ}} \quad \text{SAME FORCE} \; \div F$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{EQ}}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{k_{eq}} \Rightarrow \boxed{k_{eq} = \frac{k_1 k_2}{k_1 + k_2}}$$

Note: RESISTORS in PARALLEL



$$\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{k_{EQ}} = \frac{k_2 k_3 + k_1 k_3 + k_1 k_2}{k_1 k_2 k_3}$$

$$k_{eq} = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}$$