MECH 236

IMPULSE and MOMENTUM p.806 - 807

IMPACT = COLLISION OF TWO BODIES.

IN MECHANICS, there is a QUANTITY that defines A MEASURE OF REBOUND or RESILIENCE after a collision:

COEFFICIENT of RESTITUTION $= e$

= RATIO OF 2 IMPULSES

or NEG. RATIO OF $\dfrac{\text{Relative Velocity of 2 Bodies AFTER IMPACT}}{\text{Relative Velocity of 2 Bodies BEFORE IMPACT}}$ and

$$e = -\frac{V_{BA_f}}{V_{BA_i}} = \frac{-(V_{B_f} - V_{A_f})}{(V_{B_i} - V_{A_i})} = \frac{V_{B_f} - V_{A_f}}{V_{A_i} - V_{B_i}} \quad \frac{\text{OUTPUT}}{\text{INPUT}}$$

$$\boxed{0 \le e \le 1}$$ LIMITING CASES

$e = 0$ PERFECTLY PLASTIC IMPACT

After impact: 2 BODIES MOVE TOGETHER AT SAME SPEED

$\therefore V_{BA_f} = 0$ since $V_{B_f} = V_{A_f}$

$e = 1$ PERFECTLY ELASTIC IMPACT

MECHANICAL ENERGY IS CONSERVED

$$V_{B_f} - V_{A_f} = V_{A_i} - V_{B_i}$$ INITIAL = FINAL

Rearrange $$V_{B_f} + V_{B_i} = V_{A_i} + V_{A_f} \qquad (1.)$$

CONSERVATION OF MOMENTUM:

$$m_A V_{A_i} + m_B V_{B_i} = m_A V_{A_f} + m_B V_{B_f}$$

Rearrange $$m_A (V_{A_i} - V_{A_f}) = m_B (V_{B_f} - V_{B_i}) \qquad (2.)$$

MULT. (1) * (2) $$m_A (V_{A_i}^2 - V_{A_f}^2) = m_B (V_{B_f}^2 - V_{B_i}^2)$$

GROUP LIKE TERMS

INITIAL = FINAL

$$m_A V_{A_i}^2 + m_B V_{B_i}^2 = m_A V_{A_f}^2 + m_B V_{B_f}^2$$

If you divide thru by 2, you'll see K.E.

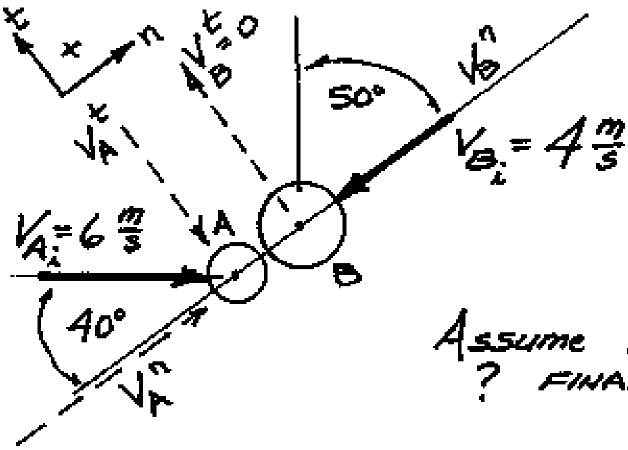
$$\left(\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2\right)_{initial} = \left(\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2\right)_{final}$$

K.E. OF THE PAIR OF "PARTICLES" IS CONSERVED

$$KE_i = KE_f \quad \therefore \quad \Delta KE = 0$$

when $e = 1$

WHEN 2 PARTICLES COLLIDE OBLIQUELY, COEF. of RESTITUTION, e, RELATES ONLY THE COMPONENTS OF MOTION along the line of IMPACT!



PROB. 13.165
p. 809

$m_A = 600\,g = 0.6\ kg$

$m_B = 1\ kg$

$e = 0.8$

Assume No FRICTION $\therefore \mu = 0$

? FINAL VELOCITIES V_{A_f}, V_{B_f}

- Select Normal (//) and tangential (⊥) DIRECTIONS

- FOLLOW SIGN CONVENTION

TOTAL MOMENTUM CONSERVED:

t-dir. $\quad m_A V_{A_i}^t + m_B V_{B_i}^t = m_A V_{A_f}^t + m_B V_{B_f}^t$

$$(.6\,kg)(-6 \sin 40°) + (1\,kg)(0) = .6\, V_{A_f}^t + 1\, V_{B_f}^t \qquad (1)$$

$$-2.314 = .6\, V_{A_f}^t + V_{B_f}^t$$

TOTAL MOMENTUM CONSERVED:

n-dir. $\quad m_A V_{A_i}^n + m_B V_{B_i}^n = m_A V_{A_f}^n + m_B V_{B_f}^n$

$$(.6\,kg)(6\cos 40^\circ) + (1\,kg)(-4\tfrac{m}{s}) = .6\,V_{A_f}^n + 1\,V_{B_f}^n \qquad (2)$$

RELATIVE VELOCITIES & RESTITUTION along line of impact

$$0.8 = -\frac{V_{BA_f}}{V_{BA_i}} = \frac{V_{B_f}^n - V_{A_f}^n}{-[(-4\frac{m}{s}) - (+6\frac{m}{s}\cos 40^\circ)]}$$

$$0.8[4 + 4.596] = 6.877 = V_{B_f}^n - V_{A_f}^n \qquad (3)$$

CAN SOLVE (2) & (3) SIMULTANEOUSLY

$$(2) \quad 2.75776 - 4 =$$

$$-1.242 = .6\,V_{A_f}^n + \cancel{V_{B_f}^n}$$

$$(3) \quad 6.877 = -V_{A_f}^n + \cancel{V_{B_f}^n} \qquad \text{SUBTRACT}$$

$$-8.119 = 1.6\,V_{A_f}^n$$

$$\therefore \boxed{V_{A_f}^n = -5.075\tfrac{m}{s}}$$

BACK SUB. (3) WATCH your signs

$$6.877 = V_{B_f}^n - (-5.075\tfrac{m}{s})$$

$$\therefore V_{B_f}^n = 6.877 - 5.075 \Rightarrow \boxed{V_{B_f}^n = 1.802\tfrac{m}{s} \text{ POSIT. n-dir.}}$$

BALL A, MOMENTUM CONSERVED, t dir.

$$\cancel{m_A} V_{A_i}^t = \cancel{m_A} V_{A_f}^t \qquad \therefore \boxed{V_{A_f}^t = -6\tfrac{m}{s}\sin 40^\circ = -3.857\tfrac{m}{s}}$$

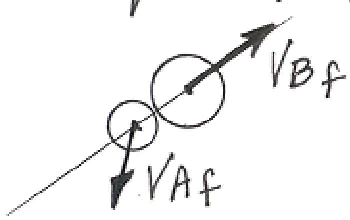
BACK SUB. (1) $\; -2.314 = .6(-3.857\tfrac{m}{s}) + V_{B_f}^t$

$$\therefore V_{B_f}^t = 2\times 10^{-4} \approx 0 \qquad \boxed{V_{B_f}^t = 0}$$

But you already knew this.

$$\therefore V_A = \sqrt{(V_A^n)^2 + (V_A^t)^2}$$

$$= \sqrt{(5.075)^2 + (3.857)^2} = 6.374\tfrac{m}{s} \angle \tan^{-1}\frac{t}{n} = 37.23^\circ + 180$$



$$V_{A_f} = 6.374\tfrac{m}{s} \angle 217.2^\circ$$

$$V_{B_f} = 1.802\tfrac{m}{s} \quad \text{POSIT. n-dir.}$$