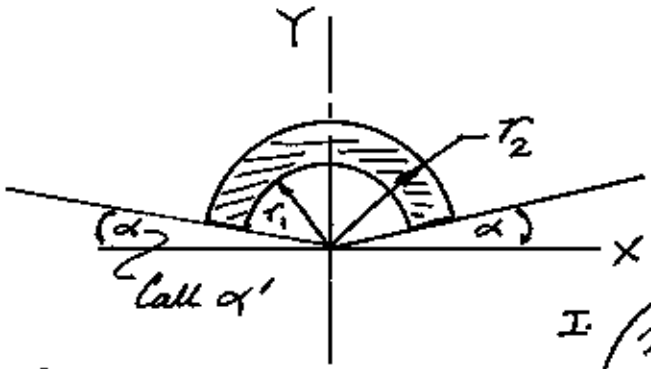
GM

MECH 234 + 235

CENTROIDS

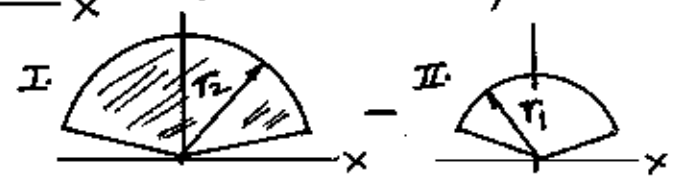
PROB. 5-17

P. 223



Due to symmetry, centroid in X-dir. is zero. $\bar{X} = 0$

Determine $\bar{Y}$



Refer to p. 215

CIRCULAR SECTOR (turned 90°) $\bar{Y} = \dfrac{2r\sin\alpha}{3\alpha}$ } See figure.

$A = \alpha r^2$ }

where this $\alpha = [90° - \alpha'] = \left[\dfrac{\pi}{2} - \alpha'\right]$

$\therefore$

SECT.	AREA	$\bar{Y}$	$A\bar{Y}$
I.	$\left[\frac{\pi}{2} - \alpha'\right] r_2^2$	$\dfrac{2r_2 \sin\left[\frac{\pi}{2} - \alpha'\right]}{3\left[\frac{\pi}{2} - \alpha'\right]}$	$\Rightarrow \dfrac{2}{3} r_2^3 \sin\left[\frac{\pi}{2} - \alpha'\right]$
II.	$\left[\frac{\pi}{2} - \alpha'\right] r_1^2$	$\dfrac{2r_1 \sin\left[\frac{\pi}{2} - \alpha'\right]}{3\left[\frac{\pi}{2} - \alpha'\right]}$	$\Rightarrow \dfrac{2}{3} r_1^3 \sin\left[\frac{\pi}{2} - \alpha'\right]$

I-II. ΣA

$\left(r_2^2 - r_1^2\right)\left[\frac{\pi}{2} - \alpha'\right]$

I-II $\Sigma A\bar{Y}$

$\dfrac{2}{3} \sin\left[\frac{\pi}{2} - \alpha'\right]\left(r_2^3 - r_1^3\right)$

$$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A} = \frac{2}{3} \frac{\sin\left[\frac{\pi}{2} - \alpha'\right]\left(r_2^3 - r_1^3\right)}{\left[\frac{\pi}{2} - \alpha'\right]\left(r_2^2 - r_1^2\right)}$$