

Mom. of INERTIA with respect to Y-axis

At max. pt.  $x=a$ ,  $y=b$

$\therefore$  for  $y = kx^3$

$$k = \frac{y}{x^3} = \frac{b}{a^3} \therefore y = \frac{b}{a^3} x^3$$

By definition,  $dI_y = x^2 dA$

where  $dA = (y' - y) dx$   
from straight line to curve.

$$dI_y = x^2 [b - y] dx \quad \text{sub. for the function}$$

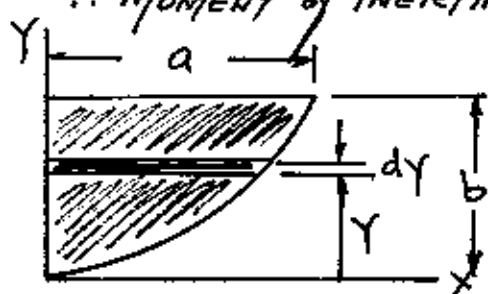
$$= x^2 \left[ b - \frac{b}{a^3} x^3 \right] dx = bx^2 dx - \frac{bx^5}{a^3} dx$$

$$I_y = \int dI_y = \int_0^a bx^2 dx - \int_0^a \frac{bx^5}{a^3} dx$$

$$= \left[ \frac{bx^3}{3} \right]_0^a - \left[ \frac{bx^6}{6a^3} \right]_0^a = \frac{ba^3}{3} - \frac{ba^6}{6a^3} = \frac{ba^3}{6}$$

$\therefore$  MOMENT of INERTIA with respect to Y-axis =

$$I_y = \frac{1}{6} a^3 b$$



Mom. of INERTIA with respect to X-axis

SINCE  $y = \frac{b}{a^3} x^3 \therefore x = \left[ \frac{y a^3}{b} \right]^{1/3}$

By definition,  $dI_x = y^2 dA$

where  $dA = (x dy)$

from straight line to curve.

$$dI_x = y^2 \left[ \frac{a y^{1/3}}{b^{1/3}} \right] dy = \frac{a}{b^{1/3}} y^{(2+1/3)} dy = \frac{a}{b^{1/3}} y^{7/3} dy$$

$$I_x = \int dI_x = \int_0^b \frac{a}{b^{1/3}} y^{7/3} dy = \frac{a}{b^{1/3}} \left[ \frac{y^{10/3}}{10/3} \right]_0^b = \frac{3}{10} \frac{a}{b^{1/3}} b^{10/3}$$

$$= \frac{3}{10} a b^{9/3} = \frac{3}{10} ab^3 = I_x = \text{Mom. of INERTIA with respect to X-axis}$$