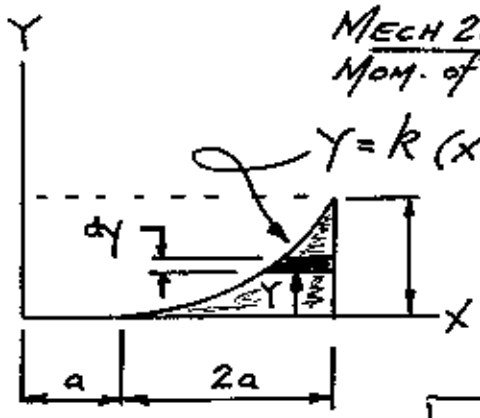
PROB. 9-10 + 9-13 p.464



$Y = k(x-a)^3$

B.C.'s
when $x = a$, $y = 0$
when $x = 3a$, $y = b$

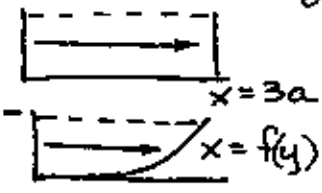
$\therefore b = k(3a-a)^3 = k(2a)^3$

$\therefore k = \frac{b}{(2a)^3} = \frac{b}{8a^3}$

Eq. for curve: $\boxed{y = \frac{b}{8a^3}(x-a)^3}$

With respect to X-axis
$dI_x = y^2 dA = y^2 (x\,dy)$

BUT, Subtract regions



Need $x = f(y)$

$\frac{y 8a^3}{b} = (x-a)^3$

$\left(y\frac{8a^3}{b}\right)^{1/3} = x - a$

substitute

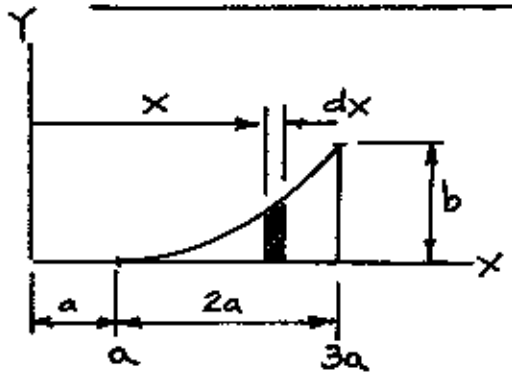
$\boxed{\frac{2a}{b^{1/3}} y^{1/3} + a = x}$

$dI_x = y^2(3a - x')dy = y^2\left[3a - \left(\frac{2a}{b^{1/3}} y^{1/3} + a\right)\right]dy$ EXPAND

$= y^2 2a\,dy - y^2 \frac{2a}{b^{1/3}} y^{1/3} dy = 2ay^2 dy - \frac{2a}{b^{1/3}} y^{7/3} dy$ INTEGRATE

$I_x = \int_0^b dI_y = \left[\frac{2a y^3}{3} - \frac{2a}{b^{1/3}} \frac{y^{10/3}}{10/3}\right]_0^b = \frac{2}{3}ab^3 - \frac{6}{10}\frac{a}{b^{1/3}} b^{10/3}$

$= \frac{2}{3}ab^3 - \frac{6}{10}ab^3 = \frac{20ab^3 - 18ab^3}{30} = \frac{2}{30}ab^3 = \boxed{\frac{1}{15}ab^3 = I_x}$



With respect to Y-axis
$dI_y = x^2 dA = x^2 (y\,dx)$
Y extends from 0 to the curve
X extends from a to 3a

$dI_y = x^2\left(\frac{b}{8a^3}\right)(x-a)^3 dx$ EXPAND

$dI_y = x^2 \frac{b}{8a^3}(x^2 - 2ax + a^2)(x-a)\,dx$

$= \frac{b}{8a^3} x^2 (x^3 - 2ax^2 + a^2x - ax^2 + 2a^2x - a^3)\,dx$

continue →

MOM. of INERTIA WITH RESPECT TO Y-axis

PROB. 9-13 cont'd.

$$dI_Y = \frac{b}{8a^3} x^2 (x^3 - 3ax^2 + 3a^2x - a^3)\,dx$$

$$= \frac{b}{8a^3} \left[x^5 - 3ax^4 + 3a^2x^3 - a^3x^2\right] dx \quad \text{INTEGRATE.}$$

$$I_Y = \int_a^{3a} dI_Y = \frac{b}{8a^3}\left[\frac{x^6}{6} - \frac{3ax^5}{5} + \frac{3a^2x^4}{4} - \frac{a^3x^3}{3}\right]_a^{3a}$$

$$= \frac{b}{8a^3}\left[\frac{3^6a^6}{6} - \frac{3a\,3^5a^5}{5} + \frac{3a^2\,3^4a^4}{4} - \frac{a^3\,3^3a^3}{3}\right] -$$

$$\frac{b}{8a^3}\left[\frac{a^6}{6} - \frac{3a^6}{5} + \frac{3a^6}{4} - \frac{a^6}{3}\right]$$

$$= \frac{1}{8} ba^3 \left[\frac{(3^6-1)}{6} - \frac{3}{5}(3^5-1) + \frac{3}{4}(3^4-1) - \frac{1}{3}(3^3-1)\right]$$

$$= \frac{1}{8} ba^3 \left[121.33 - 145.2 + 60 - 8.667\right]$$

$$\boxed{I_Y = 3.433\, ba^3} \quad \text{and} \quad \boxed{I_X = \frac{1}{15} ab^3}$$