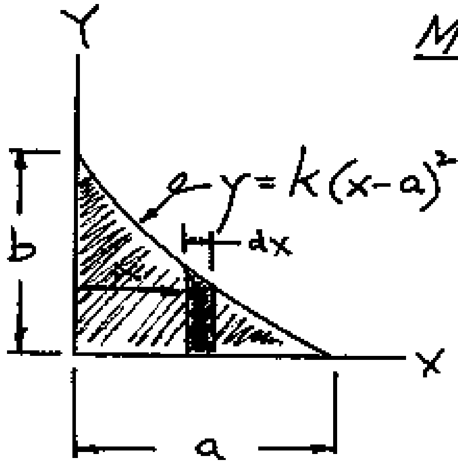
MECH 235

PROB. 9-2 & 9-6 p. 464



MOM. of INERTIA with respect to Y-axis and X-axis

GIVEN: equation of curve $Y = k(x-a)^2$

APPLY B.C.
when $x = 0$, $Y = b$

$\therefore b = k(-a)^2 \quad \therefore k = \frac{b}{a^2}$

$$\boxed{Y = \frac{b}{a^2}(x-a)^2}$$

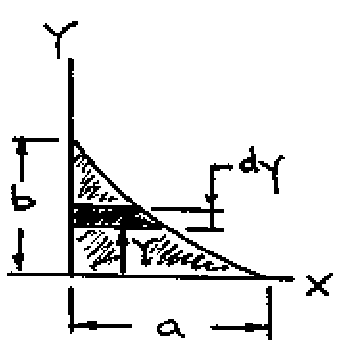
$$dI_Y = x^2 dA = x^2(y\,dx) = x^2\left[\frac{b}{a^2}(x-a)^2\right]dx \qquad \text{EXPAND}$$

$$= x^2 \frac{b}{a^2}(x^2 - 2ax + a^2)dx$$

$$= \frac{b}{a^2}x^4 dx - 2\frac{b}{a^2}a\,x^3 dx + \frac{b}{a^2}a^2 x^2 dx \qquad \text{INTEGRATE}$$

$$I_Y = \int_0^a dI_Y = \left[\frac{b}{a^2}\frac{x^5}{5} - \frac{2b}{a}\frac{x^4}{4} + b\frac{x^3}{3}\right]_0^a = \frac{1}{5}\frac{b}{a^2}a^5 - \frac{1}{2}\frac{b}{a}a^4 + \frac{1}{3}ba^3$$

SIMPLIFY WITH L.C.D. $= \dfrac{6ba^3 - 15ba^3 + 10ba^3}{30} = \boxed{\dfrac{1}{30}ba^3 = I_Y}$



With respect to X-axis, $dI_X = Y^2 dA$
where $dA = x\,dY$
NEED X in terms of Y

$$Y = \frac{b}{a^2}(x-a)^2$$

$$\pm\sqrt{\frac{Ya^2}{b}} = x - a \quad \therefore \boxed{X = \pm\frac{Y^{1/2}a}{b^{1/2}} + a}$$

$$dI_X = Y^2\left[+\frac{Y^{1/2}a}{b^{1/2}} + a\right]dY = Y^{5/2}\frac{a}{\sqrt{b}}\,dY + aY^2 dY \qquad \text{INTEGRATE}$$

$$I_X = \int_0^b dI_X = \left[\frac{a}{\sqrt{b}}\frac{Y^{7/2}}{7/2} + \frac{aY^3}{3}\right]_0^b = \frac{2}{7}\frac{a}{\sqrt{b}}b^{7/2} + \frac{b^3}{3}a$$

$$= \frac{2}{7}ab^3 + \frac{1}{3}ab^3 = \frac{(6+7)ab^3}{21} = \boxed{\frac{13}{21}ab^3 = I_X}$$

FOR POSITIVE ROOT.

PROB. 9-6 cont'd.

CONSIDER THE NEGATIVE ROOT...

$$X = -\frac{Y^{1/2} a}{b^{1/2}} + a$$

$$dI_x = Y^2 \left[-\frac{Y^{1/2}}{b^{1/2}} a + a \right] dY = -Y^{5/2} \frac{a}{\sqrt{b}} dY + aY^2 dY$$

$$I_x = \int_0^b dI_x = \left[\frac{a}{\sqrt{b}} \left(-\frac{Y^{7/2}}{7/2} \right) + \frac{aY^3}{3} \right]_0^b = -\frac{2}{7} \frac{a}{b^{1/2}} b^{7/2} + \frac{ab^3}{3}$$

$$= -\frac{2}{7} ab^3 + \frac{1}{3} ab^3 = \frac{(-6+7)}{21} ab^3 = \boxed{\frac{1}{21} ab^3 = I_x}$$

FOR NEG. ROOT.