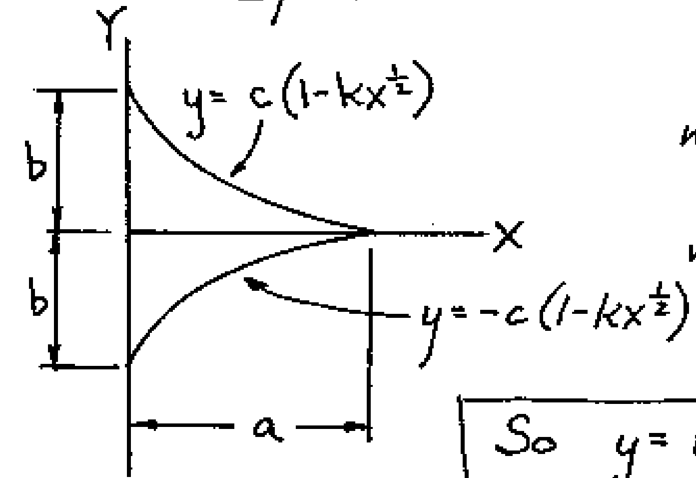
MECH 234 + 235

PROB 9.9
p. 464

MOMENTS of INERTIA or
2nd. Moment of Area

where $dI_x = y^2 dA = y(y\,dA) = y^2(x\,dy)$ where $x = f(y)$

$dI_y = x^2 dA = x(x\,dA) = x^2(y\,dx)$ where $y = f(x)$



APPLY LIMITS

when $x = 0$, $y = b$

$\therefore b = c(1-0) \quad \therefore \boxed{c = b}$

when $y = 0$, $x = a$

$\therefore 0 = c(1 - ka^{1/2})$

$-1 = -ka^{1/2} \quad \therefore \boxed{k = \frac{1}{a^{1/2}}}$

$$\boxed{\text{So } y = b\left(1 - \frac{1}{\sqrt{a}}\sqrt{x}\right)}$$

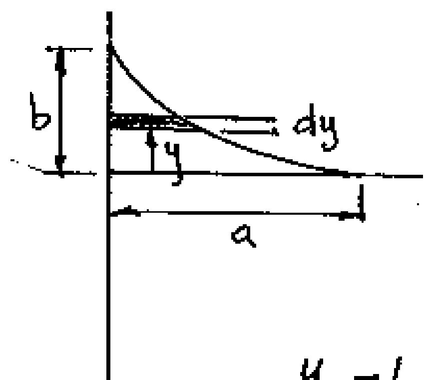
Lower Curve: when $x = 0$, $y = -b$

$\therefore -b = -c(1-0) \quad \therefore \boxed{c = b}$

when $y = 0$, $x = a$

$\therefore 0 = -c(1 - ka^{1/2}) \quad \therefore \boxed{k = \frac{1}{a^{1/2}}}$

SAME RESULTS, SOLVE for I_x for one area above X-axis and double.



$dI_x = y^2 dA = y^2[x\,dy]$

NEED $x = f(y)$

IF $y = b\left(1 - \frac{\sqrt{x}}{\sqrt{a}}\right)$

$\frac{y}{b} = 1 - \frac{\sqrt{x}}{\sqrt{a}}$

$\frac{y}{b} - 1 = \frac{y-b}{b} = -\frac{\sqrt{x}}{\sqrt{a}}$

$\left(\frac{b-y}{b}\right)\sqrt{a} = \sqrt{x}$

$\left(\frac{b-y}{b}\right)^2 a = \boxed{x = \frac{a}{b^2}(b-y)^2}$

substitute for x above

$$dI_x = y^2[x\,dy] = y^2\left[\frac{a}{b^2}(b-y)^2\right]dy$$ EXPAND

$$= \frac{a}{b^2}\left[y^2(b^2-2by+y^2)\right]dy = \frac{a}{b^2}\left[b^2y^2-2by^3+y^4\right]dy$$

$$= ay^2dy - 2\frac{a}{b}y^3dy + \frac{a}{b^2}y^4dy$$ INTEGRATE.

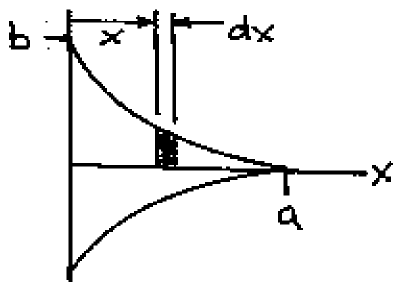
$$I_x = \int_0^b dI_x = \left[\frac{a}{3}y^3 - \frac{2}{4}\frac{a}{b}y^4 + \frac{a}{5b^2}y^5\right]_0^b$$ APPLY LIMITS.

$$= \frac{ab^3}{3} - \frac{1}{2}\frac{a}{b}b^4 + \frac{a}{5b^2}b^5 = \frac{ab^3}{3} - \frac{ab^3}{2} + \frac{ab^3}{5}$$

L.C.D. $$= \frac{10ab^3 - 15ab^3 + 6ab^3}{30} = \frac{1}{30}ab^3$$ ×2 areas

$$\therefore \boxed{I_x = \frac{1}{15}ab^3}$$

REDO FOLLOWING SAMPLE 9.3 and p. 459 USING SAME ELEMENT FOR I_x and I_y



$$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}\left[b\left(1-\frac{\sqrt{x}}{\sqrt{a}}\right)\right]^3dx$$

$$= \frac{1}{3}b^3\left[1-\frac{\sqrt{x}}{\sqrt{a}}\right]^3dx$$ EXPAND.

$$= \frac{1}{3}b^3\left(1-2\frac{\sqrt{x}}{\sqrt{a}}+\frac{x}{a}\right)\left(1-\frac{\sqrt{x}}{\sqrt{a}}\right)dx$$

$$= \frac{1}{3}b^3\left[1-2\frac{\sqrt{x}}{\sqrt{a}}+\frac{x}{a}-\frac{\sqrt{x}}{\sqrt{a}}+2\frac{x}{a}-\left(\frac{x}{a}\right)^{3/2}\right]dx$$ GROUP.

$$= \frac{1}{3}b^3\left[1-3\frac{\sqrt{x}}{\sqrt{a}}+3\frac{x}{a}-\left(\frac{x}{a}\right)^{3/2}\right]dx$$ INTEGRATE.

$$I_x = \int_0^a dI_y = \frac{1}{3}b^3\left[x - \frac{3x^{3/2}}{\frac{3}{2}\sqrt{a}} + \frac{3x^2}{2a} - \frac{x^{5/2}}{\frac{5}{2}a^{3/2}}\right]_0^a$$ × 2 areas

$$= \frac{2}{3}b^3\left[a - 2\frac{\sqrt{a}^3}{\sqrt{a}} + \frac{3a^2}{2a} - \frac{2}{5}\frac{a^{5/2}}{a^{3/2}}\right]$$

$$= \frac{2}{3}b^3\left[a - 2a + \frac{3}{2}a - \frac{2}{5}a\right] = \frac{2}{3}b^3\left[\frac{1}{2}a - \frac{2}{5}a\right]$$

L.C.D. $$= \frac{2}{3}b^3\left[\frac{5a-4a}{10}\right] = \frac{\cancel{2}}{3}\frac{ab^3}{\cancel{10}_{5}} = \boxed{I_x = \frac{1}{15}ab^3}$$

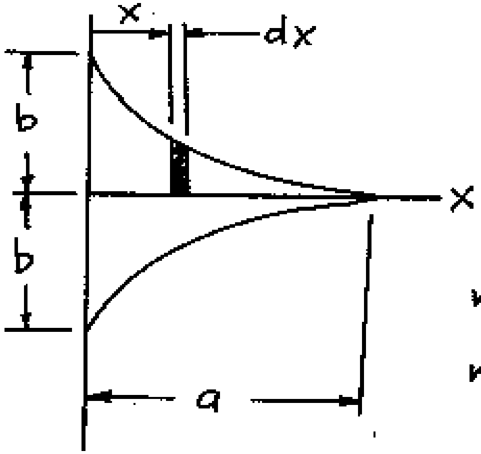
SAME ANSWER.

MECH 234 + 235

PROB. 9-12

Mom. of Inertia = 2^{nd} Mom. of Area

Follows 9.9



Moment of Inertia with respect to Y-axis $= dI_y = x^2\, dA$

$= x^2 [y\, dx]$

where $y = c(1 - kx^{1/2})$

when $x = 0,\ y = b \quad \therefore \quad c = b$

when $y = 0,\ x = a \quad \therefore \quad k = \frac{1}{a^{1/2}}$

$$\therefore \boxed{y = b\left(1 - \frac{\sqrt{x}}{\sqrt{a}}\right)}$$

$$dI_y = x^2\left[b\left(1 - \frac{\sqrt{x}}{\sqrt{a}}\right)\right]dx = b\left[x^2 - \frac{x^{5/2}}{a^{1/2}}\right]dx$$

INTEGRATE.

$$I_y = \int_0^a dI_y = \int_0^a b\left[x^2 dx - \frac{x^{5/2}}{a^{1/2}}\, dx\right] = b\left[\frac{x^3}{3} - \frac{x^{7/2}}{\frac{7}{2} a^{1/2}}\right]_0^a$$

$$= \frac{ba^3}{3} - \frac{2}{7} \frac{ba^{7/2}}{a^{1/2}} = \frac{ba^3}{3} - \frac{2}{7} ba^3$$

SIMPLIFY.

L.C.D. $= \frac{7ba^3 - 6ba^3}{21}$ $\quad \therefore \boxed{I_y = \frac{ba^3}{21}} \times 2$ areas

$$\boxed{I_x = \frac{1}{15} ab^3 , \qquad I_y = \frac{2}{21} ba^3}$$