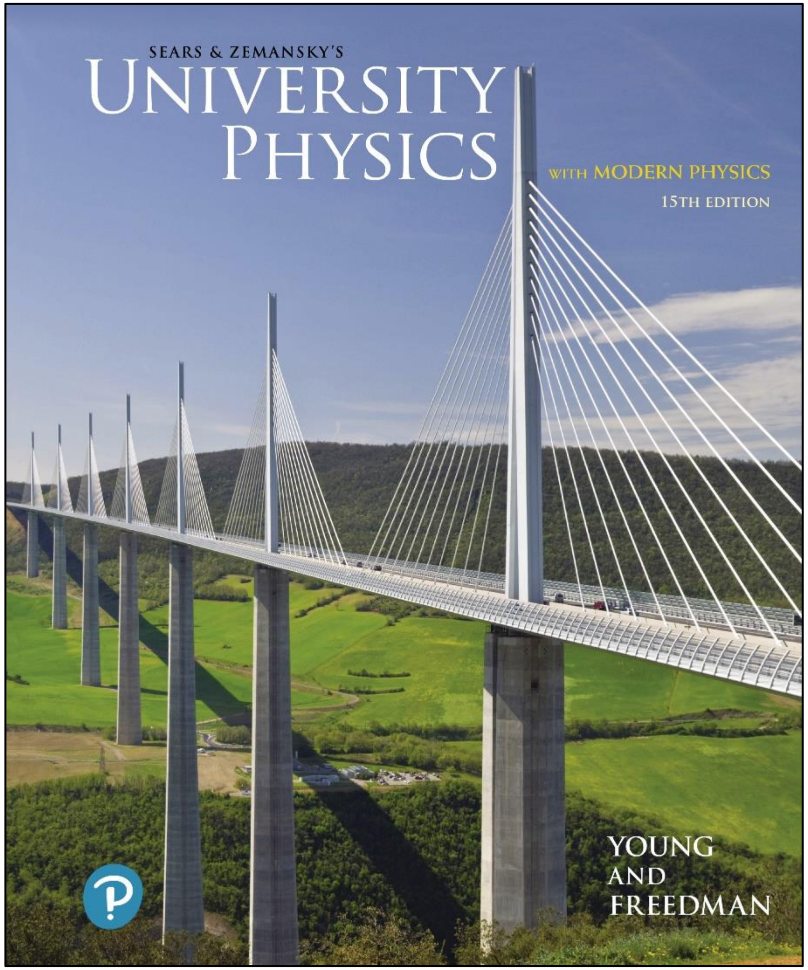
University Physics with Modern Physics

Fifteenth Edition



Chapter 15

Mechanical Waves

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Learning Goals

In this chapter, you'll learn…

- how to use the relationship among speed, frequency, and wavelength for a periodic wave.
- how to calculate the speed of waves on a rope or string.
- what happens when mechanical waves overlap and interfere.
- the properties of standing waves on a string, and how to analyze these waves.
- how stringed instruments produce sounds of specific frequencies.

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Introduction

- Earthquake waves carry enormous power as they travel through the earth.
- Other types of mechanical waves, such as sound waves or the vibration of the strings of a piano, carry far less energy.
- Overlapping waves interfere, which helps us understand musical instruments.

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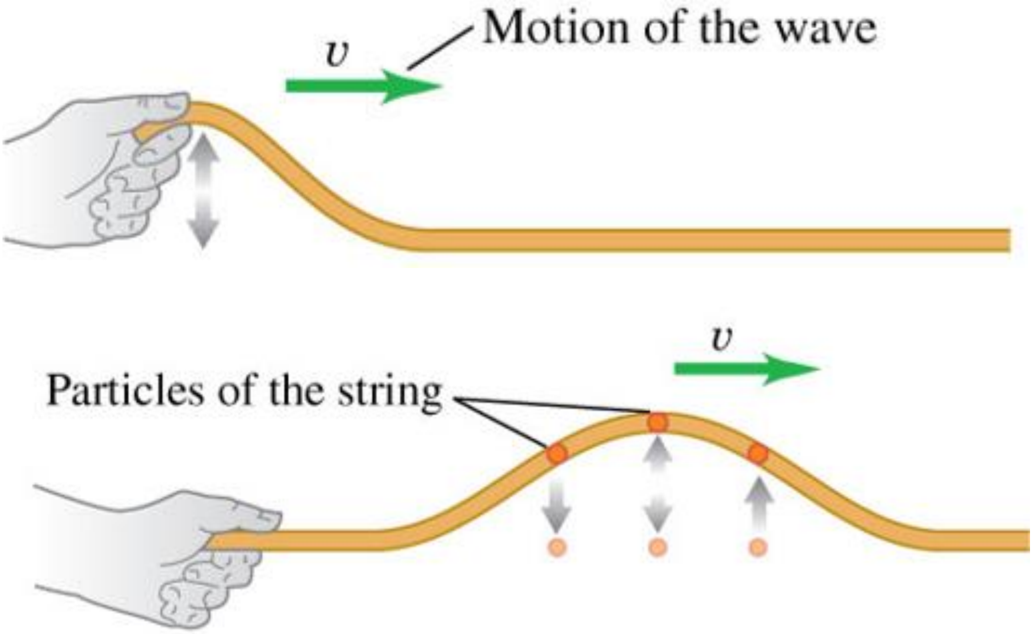
Types of Mechanical Waves (1 of 3)

- A wave on a string is a type of mechanical wave.
- The hand moves the string up and then returns, producing a transverse wave that moves to the right.



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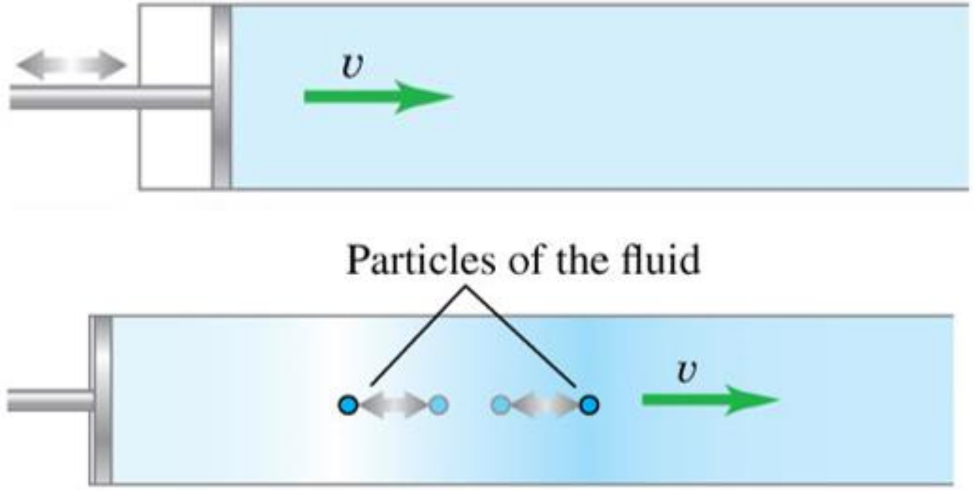
Types of Mechanical Waves (2 of 3)

- A pressure wave in a fluid is a type of mechanical wave.
- The piston moves to the right, compressing the gas or liquid, and then returns, producing a longitudinal wave that moves to the right.



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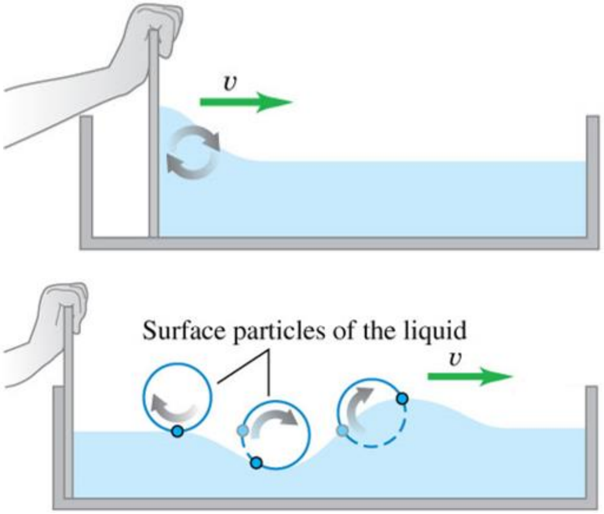
Types of Mechanical Waves (3 of 3)

- A surface wave on a liquid is a type of mechanical wave.
- The board moves to the right and then returns, producing a combination of longitudinal and transverse waves.



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Mechanical Waves

- “Doing the wave” at a sports stadium is an example of a mechanical wave.
- The **disturbance** propagates through the crowd, but there is no transport of matter.
- None of the spectators moves from one seat to another.

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Periodic Waves (1 of 2)

- For a **periodic wave**, each particle of the medium undergoes periodic motion.
- The **wavelength** λ of a periodic wave is the length of one complete wave pattern.
- The **speed** of any periodic wave of frequency f is:

For a **periodic wave:**

$$v = \lambda f$$

- [Video Tutor Solution: Example 15.1]()

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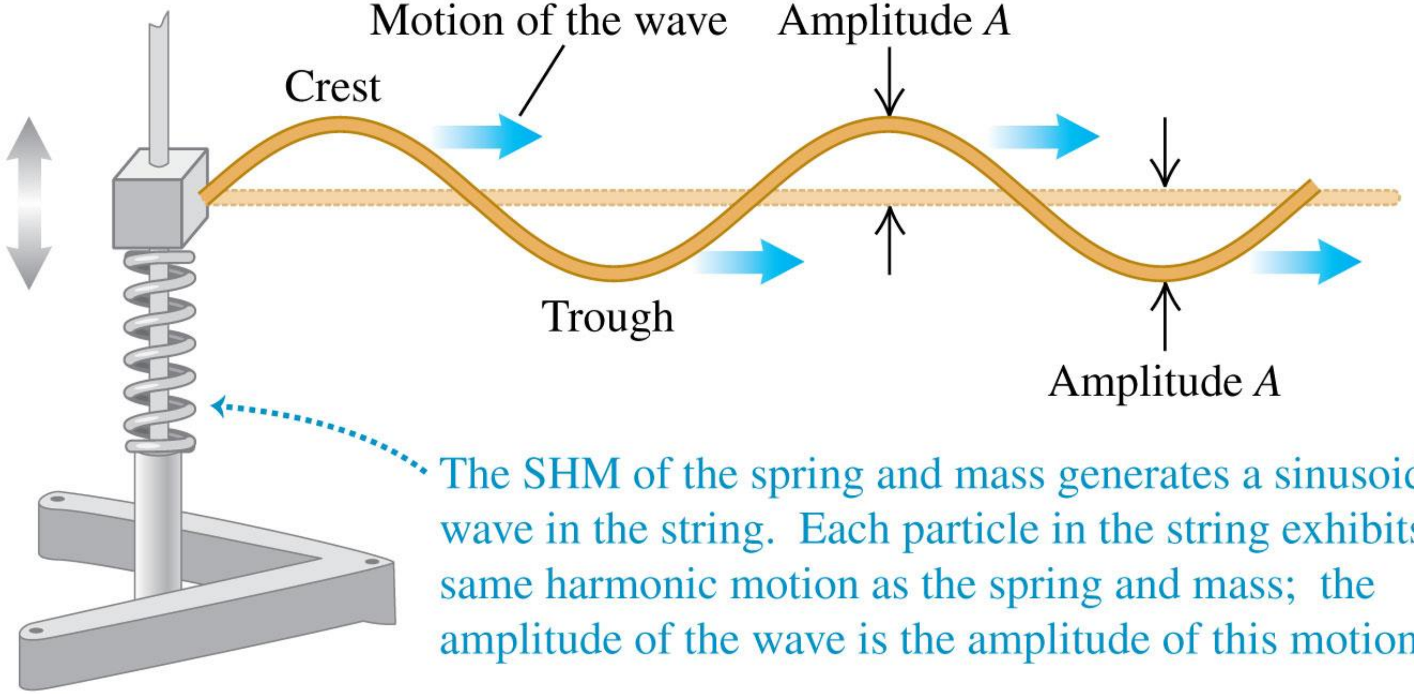
Periodic Transverse Waves

- A mass attached to a spring undergoes simple harmonic motion, producing a sinusoidal wave that travels to the right on the string.



The SHM of the spring and mass generates a sinusoidal wave in the string. Each particle in the string exhibits the same harmonic motion as the spring and mass; the amplitude of the wave is the amplitude of this motion.

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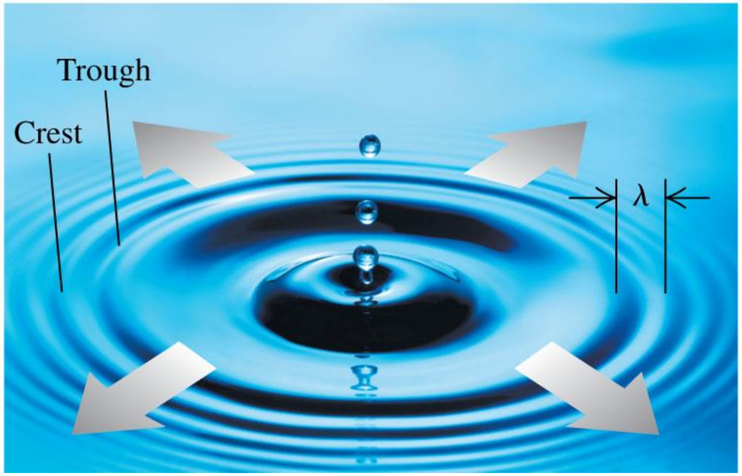
Periodic Waves (2 of 2)

- A series of drops falling into water produces a periodic wave that spreads radially outward.
- The wave crests and troughs are concentric circles.
- The wavelength λ is the radial distance between adjacent crests or adjacent troughs.



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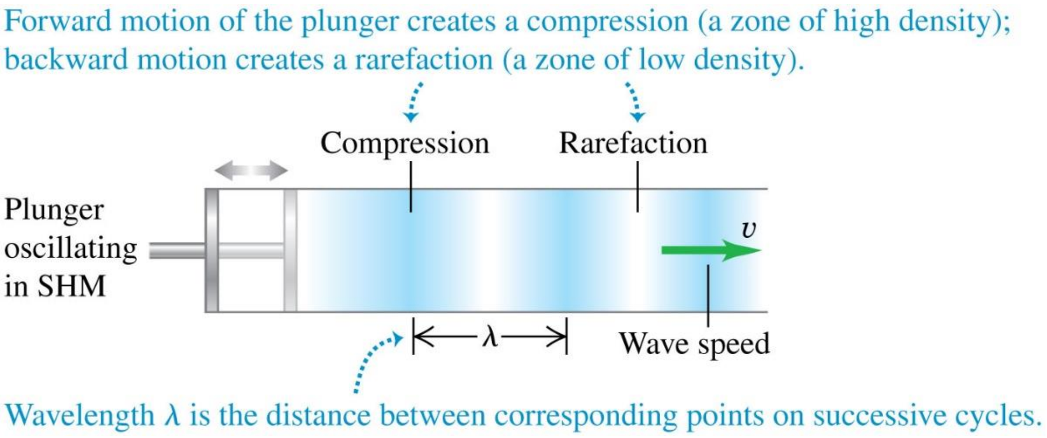
Periodic Longitudinal Waves

- Consider a long tube filled with a fluid, with a piston at the left end.
- If we push the piston in, we compress the fluid near the piston, and this region then pushes against the neighboring region of fluid, and so on, and a wave pulse moves along the tube.



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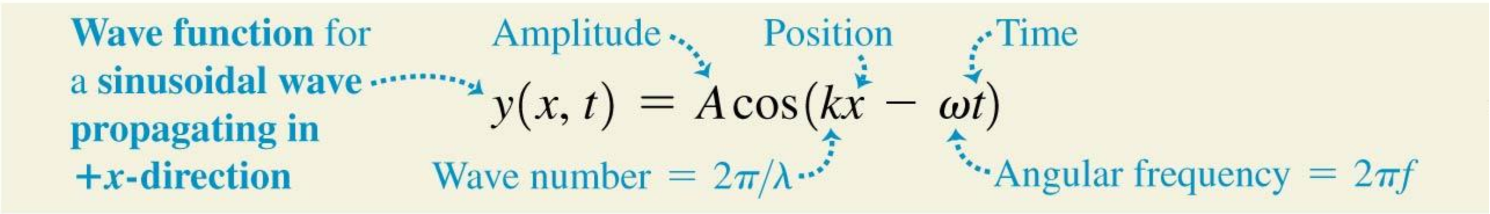
Mathematical Description of a Wave

- The wave function for a sinusoidal wave moving in the +*x*-direction is given by Equation (15.7):



Wave function for **a sinusoidal wave propagating in +*x*-direction**

$$y(x,t) = A\cos(kx - \omega t)$$

Amplitude

Position

Time

Wave number $= 2\pi/\lambda$

Angular frequency $= 2\pi f$

- In this function, *y* is the displacement of a particle at time *t* and position *x*.
- The quantity *A* is the amplitude of the wave.
- The quantity *k* is called the **wave number**, and is defined as $k = \dfrac{2\pi}{\lambda}$.
- The quantity ω is called the **angular frequency**, and is defined as $\omega = 2\pi f = \dfrac{2\pi}{T}$, where *T* is the period.

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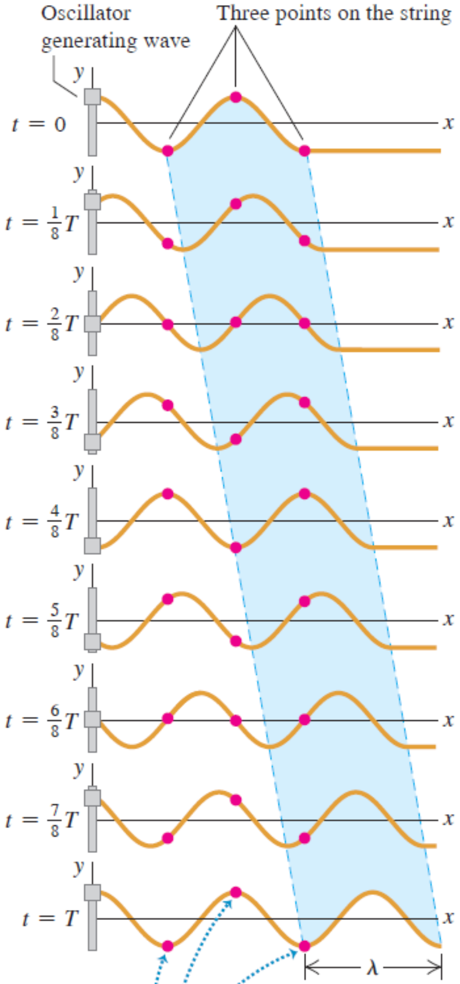
Figure 15.4 A sinusoidal transverse wave traveling to the right along a string. The vertical scale is exaggerated.

The string is shown at time intervals of $\frac{1}{8}$ period for a total of one period T. The highlighting shows the motion of one wavelength of the wave.



The wave advances by one wavelength λ during each period T.

Each point moves up and down in place. Particles one wavelength apart move in phase with each other.

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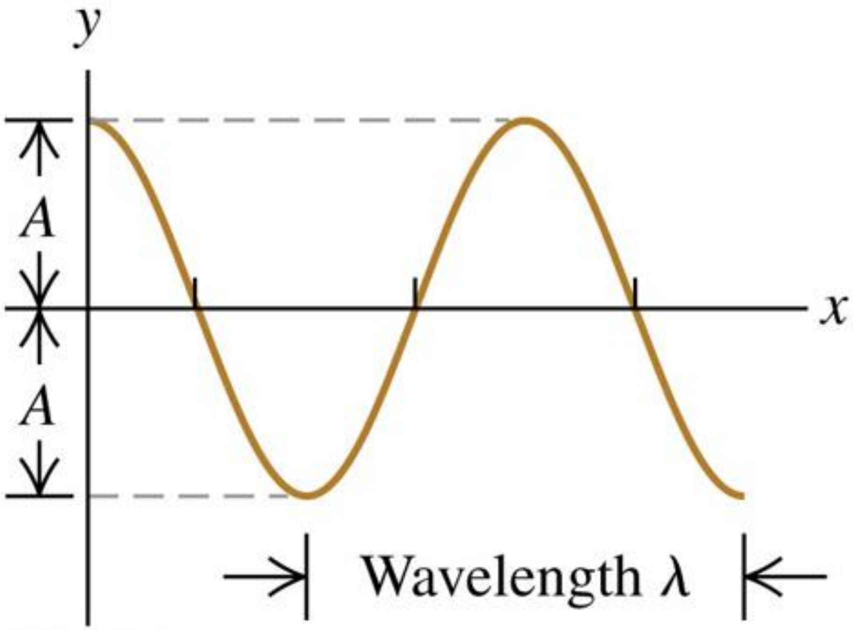
Graphing the Wave Function (1 of 2)

If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.



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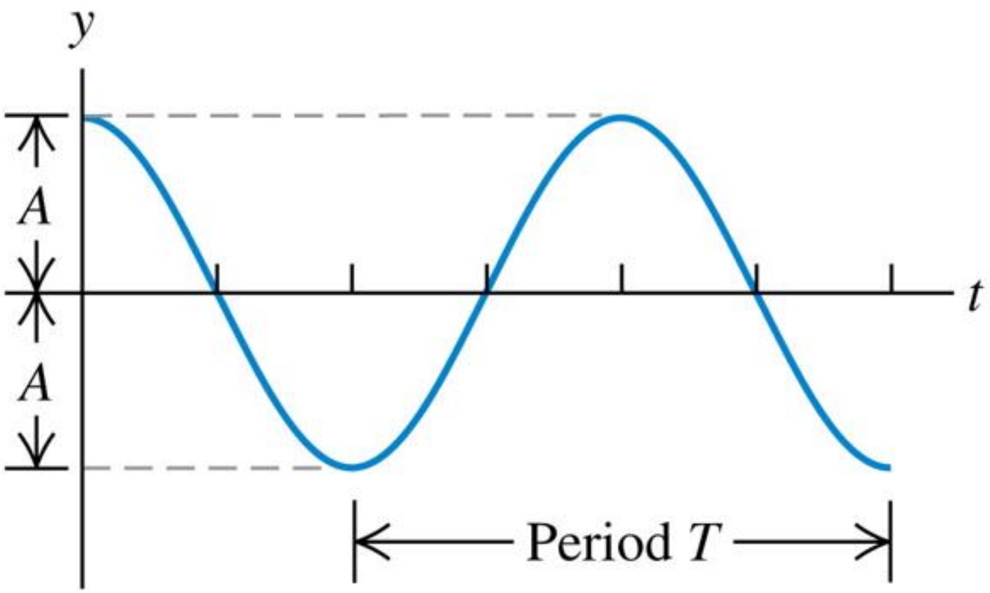
Graphing the Wave Function (2 of 2)

If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.



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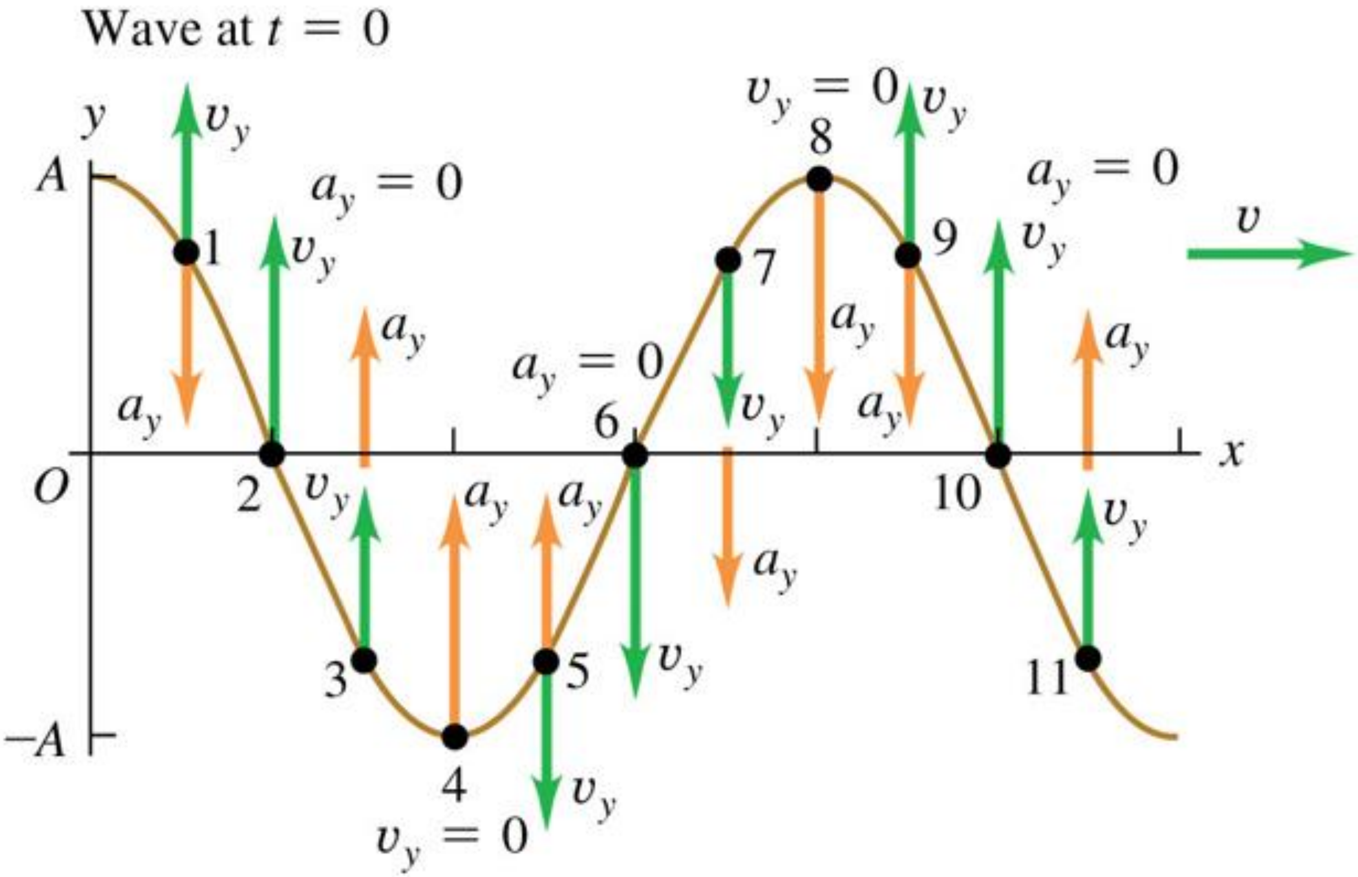
Particle Velocity and Acceleration in a Sinusoidal Wave (1 of 2)



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Particle Velocity and Acceleration in a Sinusoidal Wave (2 of 2)

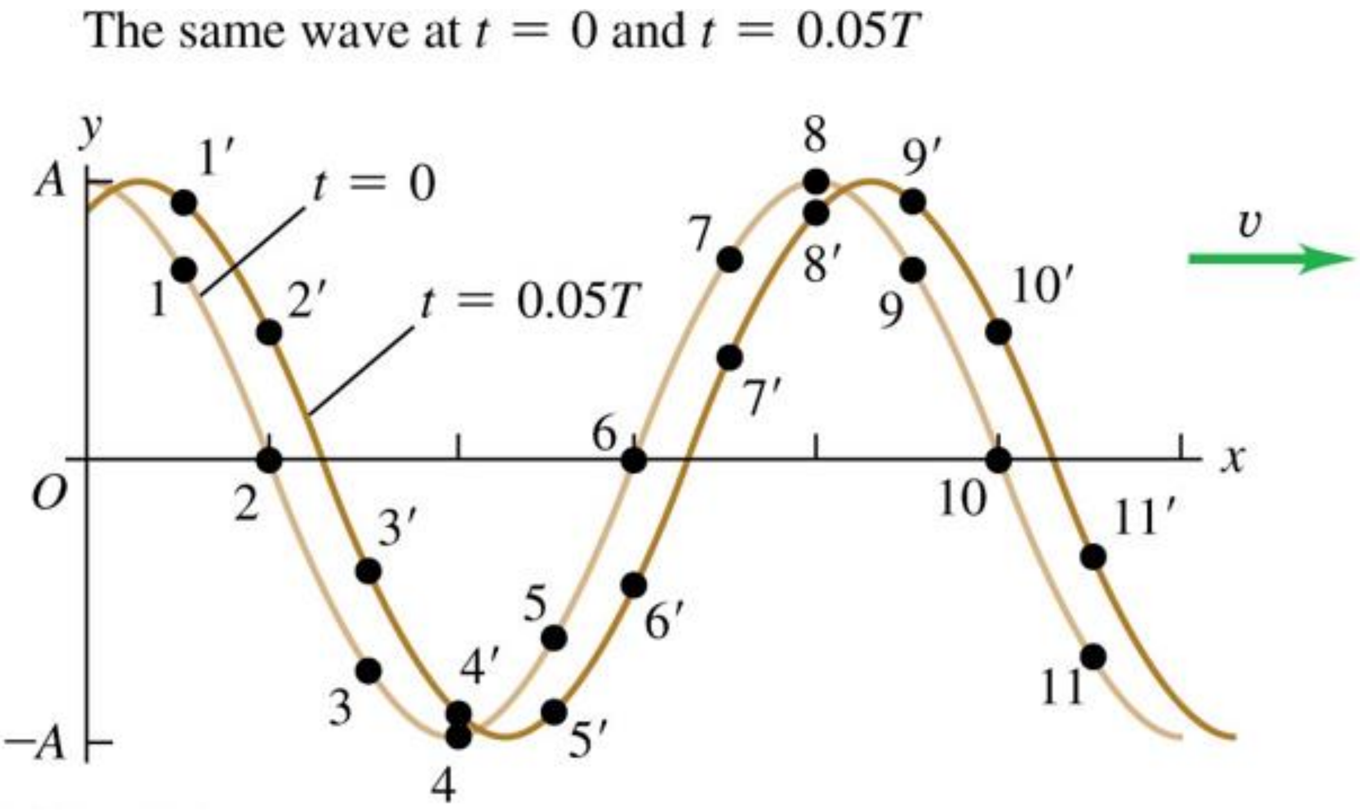

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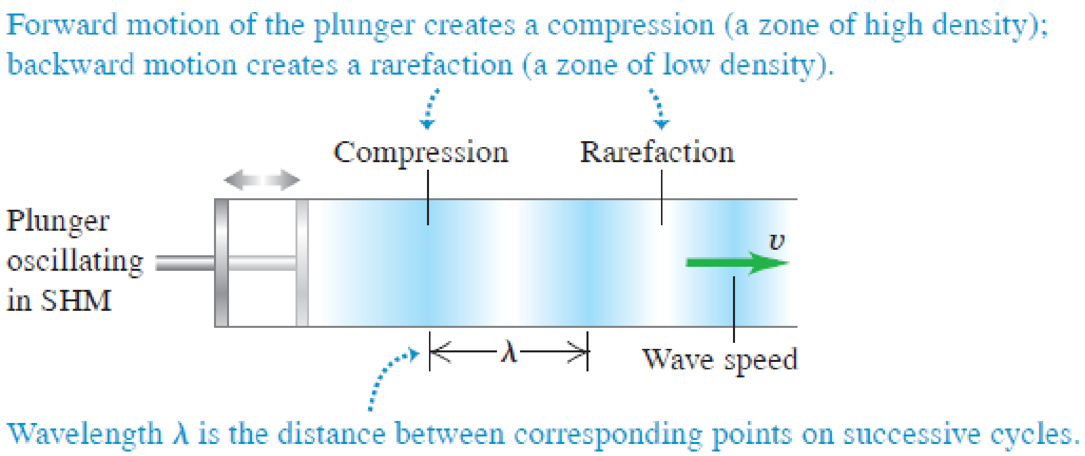
Figure 15.6 Using an oscillating piston to make a sinusoidal longitudinal wave in a fluid.



Forward motion of the plunger creates a compression (a zone of high density); backward motion creates a rarefaction (a zone of low density).

Wavelength λ is the distance between corresponding points on successive cycles.

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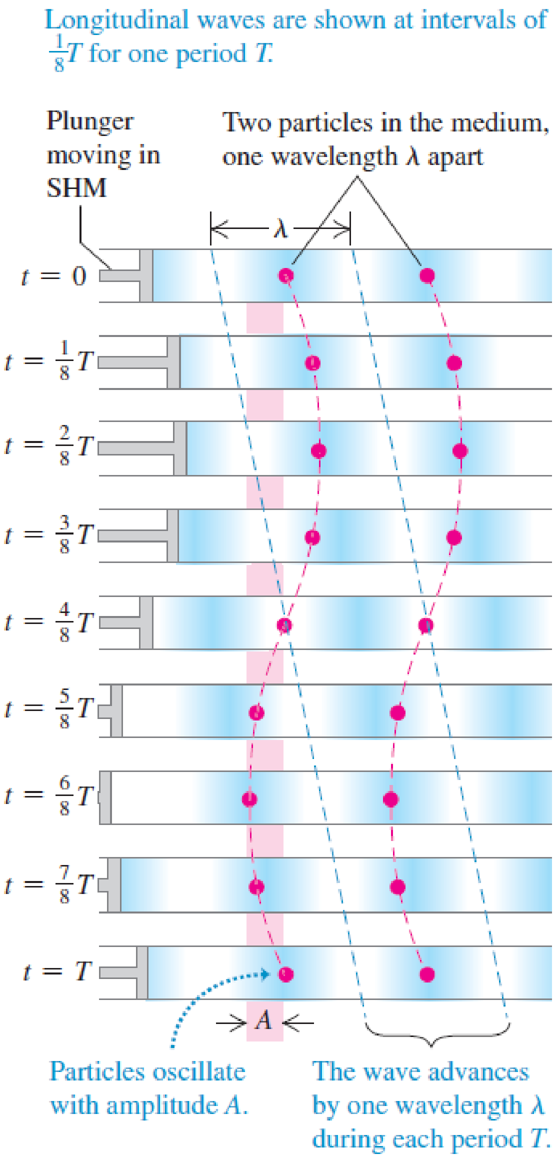
Figure 15.7 A sinusoidal longitudinal wave traveling to the right in a fluid. The wave has the same amplitude A and period T as the oscillation of the piston.



Longitudinal waves are shown at intervals of $\frac{1}{8}T$ for one period T.

Plunger moving in SHM

Two particles in the medium, one wavelength λ apart

Particles oscillate with amplitude A.

The wave advances by one wavelength λ during each period T.

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EXAMPLE 15.1 Wavelength of a musical sound

Sound waves are longitudinal waves in air. The speed of sound depends on temperature; at 20°C it is 344 m/s (1130 ft/s). What is the wavelength of a sound wave in air at 20°C if the frequency is 262 Hz (the approximate frequency of middle C on a piano)?

IDENTIFY and SET UP This problem involves Eq. (15.1), $v = \lambda f$, which relates wave speed v, wavelength λ, and frequency f for a periodic wave. The target variable is the wavelength λ. We are given $v = 344\ \text{m/s}$ and $f = 262\ \text{Hz} = 262\ \text{s}^{-1}$.

EXECUTE We solve Eq. (15.1) for λ:

$$\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{262\ \text{Hz}} = \frac{344\ \text{m/s}}{262\ \text{s}^{-1}} = 1.31\ \text{m}$$

EVALUATE The speed v of sound
quency. Hence $\lambda = v/f$ says tha
proportion to frequency. As an exa
taves above middle C. Each octav
frequency, so the frequency of high
$f = 4(262\ \text{Hz}) = 1048$ Hz. Hence
fourth as large: $\lambda = (1.31\ \text{m})/4 =$

KEYCONCEPT The product of
has the same value no matter what t
the wave speed.

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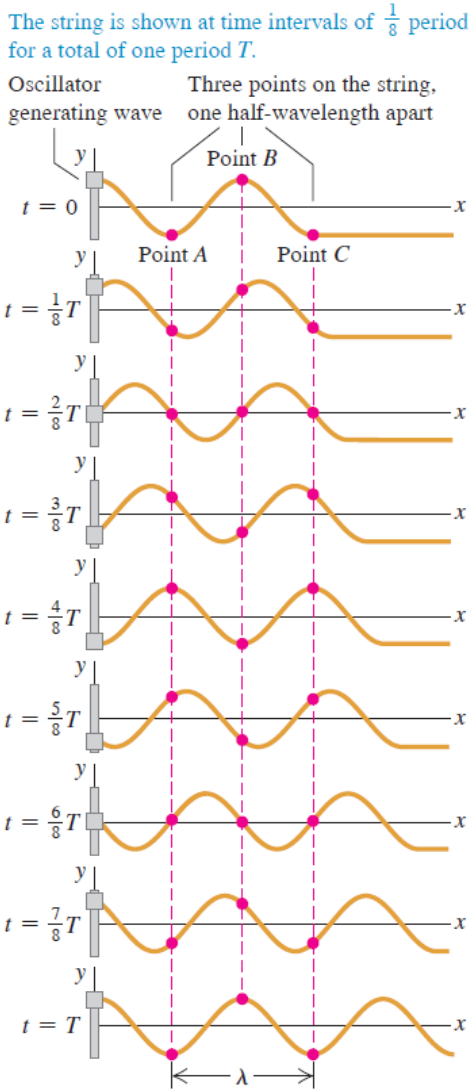
Figure 15.8 Tracking the oscillations of three points on a string as a sinusoidal wave propagates along it.



Suppose that the displacement of a particle at the left end of the string ($x = 0$), where the wave originates, is given by

$$y(x = 0, t) = A \cos \omega t = A \cos 2\pi f t \tag{15.2}$$

That is, the particle oscillates in SHM with amplitude A, frequency f, and angular frequency $\omega = 2\pi f$. The notation $y(x = 0, t)$ reminds us that the motion of this particle is a special case of the wave function $y(x, t)$ that describes the entire wave. At $t = 0$ the particle at $x = 0$ is at its maximum positive displacement ($y = A$) and is instantaneously at rest (because y is a maximum).

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$$y(x,t) = A\cos\left[\omega\left(t - \frac{x}{v}\right)\right]$$

Wave function for a sinusoidal wave propagating in $+x$-direction

$$y(x,t) = \overset{\text{Amplitude}}{A}\cos\left[\underset{\text{Angular frequency} \,=\, 2\pi f}{\omega}\left(\frac{\overset{\text{Position}}{x}}{\underset{\text{Wave speed}}{v}} - \overset{\text{Time}}{t}\right)\right] \tag{15.3}$$

$\lambda = v/f = 2\pi v/\omega$:

Wave function for a sinusoidal wave propagating in $+x$-direction

$$y(x,t) = \overset{\text{Amplitude}}{A}\cos\left[2\pi\left(\frac{\overset{\text{Position}}{x}}{\underset{\text{Wavelength}}{\lambda}} - \frac{\overset{\text{Time}}{t}}{\underset{\text{Period}}{T}}\right)\right] \tag{15.4}$$

It's convenient to define a quantity k, called the **wave number:**

$$k = \frac{2\pi}{\lambda} \qquad (\text{wave number}) \tag{15.5}$$

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$$y(x,t) = A\cos\left[\omega\left(\frac{x}{v} + t\right)\right] = A\cos\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right] = A\cos(kx + \omega t) \quad (15.8)$$

(sinusoidal wave moving in $-x$-direction)

$(kx \pm \omega t)$ is called the **phase**

$$\frac{dx}{dt} = \frac{\omega}{k}$$

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$\begin{aligned} a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} &= -\omega^2 A\cos(kx - \omega t) \\ &= -\omega^2 y(x,t) \end{aligned}$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

Wave equation involves second partial derivatives of wave function:

$$\overbrace{\frac{\partial^2 y(x,t)}{\partial x^2}}^{\text{Second partial derivative with respect to } x} = \frac{1}{\underset{\text{Wave speed}}{v^2}} \underbrace{\frac{\partial^2 y(x,t)}{\partial t^2}}_{\text{Second partial derivative with respect to } t} \quad (15.12)$$

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EXAMPLE 15.2 Wave on a clothesline

Cousin Throckmorton holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12.0\ \text{m/s}$. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (a) Find the wave amplitude A, angular frequency ω, period T, wavelength λ, and wave number k. (b) Write a wave function describing the wave. (c) Write equations for the displacement, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end.

EXECUTE (a) The wave amplitude and frequency are the same as for the oscillations of Throcky's end of the clothesline, $A = 0.075\ \text{m}$ and $f = 2.00\ \text{Hz}$. Hence

$$\omega = 2\pi f = \left(2\pi \frac{\text{rad}}{\text{cycle}}\right)\left(2.00 \frac{\text{cycles}}{\text{s}}\right)$$
$$= 4.00\pi\ \text{rad/s} = 12.6\ \text{rad/s}$$

The period is $T = 1/f = 0.500\ \text{s}$, and from Eq. (15.1),

$$\lambda = \frac{v}{f} = \frac{12.0\ \text{m/s}}{2.00\ \text{s}^{-1}} = 6.00\ \text{m}$$

We find the wave number from Eq. (15.5) or (15.6):

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\ \text{rad}}{6.00\ \text{m}} = 1.05\ \text{rad/m}$$

or

$$k = \frac{\omega}{v} = \frac{4.00\pi\ \text{rad/s}}{12.0\ \text{m/s}} = 1.05\ \text{rad/m}$$

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(b) We write the wave function using Eq. (15.4) and the values of A, T, and λ from part (a):

$$\begin{aligned} y(x,t) &= A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \\ &= (0.075\ \text{m})\cos 2\pi\left(\frac{x}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right) \\ &= (0.075\ \text{m})\cos[(1.05\ \text{rad/m})x - (12.6\ \text{rad/s})t] \end{aligned}$$

We can also get this same expression from Eq. (15.7) by using the values of ω and k from part (a).

(c) We can find the displacement as a function of time at $x = 0$ and $x = +3.00$ m by substituting these values into the wave function from part (b):

$$\begin{aligned} y(x = 0, t) &= (0.075\ \text{m})\cos 2\pi\left(\frac{0}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right) \\ &= (0.075\ \text{m})\cos(12.6\ \text{rad/s})t \\ y(x = +3.00\ \text{m}, t) &= (0.075\ \text{m})\cos 2\pi\left(\frac{3.00\ \text{m}}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right) \\ &= (0.075\ \text{m})\cos[\pi - (12.6\ \text{rad/s})t] \\ &= -(0.075\ \text{m})\cos(12.6\ \text{rad/s})t \end{aligned}$$

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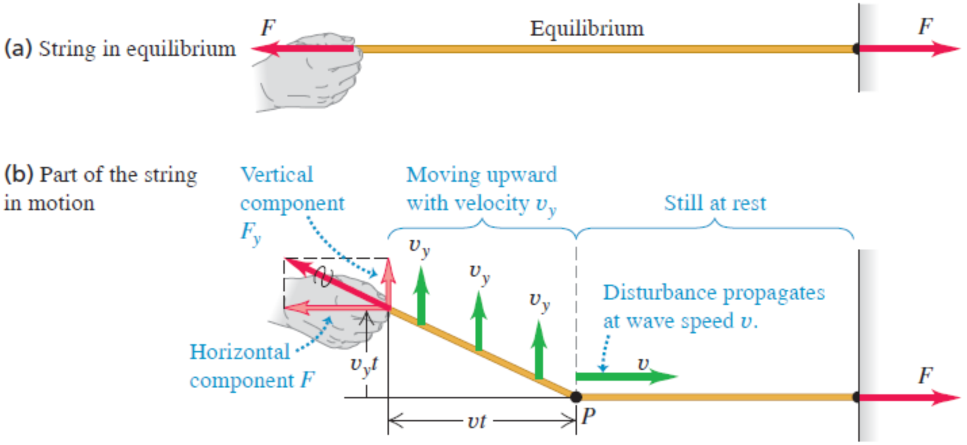

$$\frac{F_y}{F} = \frac{v_y t}{vt} \qquad F_y = F\frac{v_y}{v}$$

$$\text{Transverse impulse} = F_y t = F\frac{v_y}{v}t$$

$$\text{Transverse impulse} = \text{Transverse momentum}$$
$$F_y t = mv_y$$

$$\text{Transverse momentum} = mv_y = (\mu vt)v_y$$

$$F\frac{v_y}{v}t = \mu v t v_y$$

...

Speed of a transverse wave on a string

$$v = \sqrt{\frac{\overset{\text{Tension in string}}{F}}{\underset{\text{Mass per unit length}}{\mu}}} \qquad (15.14)$$

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The Speed of Mechanical Waves

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

Speed of a transverse wave on a string ⋯→

$$v = \sqrt{\frac{F}{\mu}} \qquad (15.14)$$

F ←⋯ Tension in string

μ ←⋯ Mass per unit length

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EXAMPLE 15.3 Calculating wave speed

One end of a 2.00 kg rope is tied to a support at the top of a mine shaft 80.0 m deep (**Fig. 15.14**). The rope is stretched taut by a 20.0 kg box of rocks attached at the bottom. (a) A geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?

EXECUTE (a) The tension in the rope due to the box is

$$F = m_{\text{box}}g = (20.0\ \text{kg})(9.80\ \text{m/s}^2) = 196\ \text{N}$$

and the rope's linear mass density is

$$\mu = \frac{m_{\text{rope}}}{L} = \frac{2.00\ \text{kg}}{80.0\ \text{m}} = 0.0250\ \text{kg/m}$$

Hence, from Eq. (15.14), the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196\ \text{N}}{0.0250\ \text{kg/m}}} = 88.5\ \text{m/s}$$

(b) From Eq. (15.1), the wavelength is

$$\lambda = \frac{v}{f} = \frac{88.5\ \text{m/s}}{2.00\ \text{s}^{-1}} = 44.3\ \text{m}$$

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The Speed of a Wave on a String (1 of 2)

- One of the key properties of any wave is the **wave speed**.
- Consider a string in which the tension is F and the linear mass density (mass per unit length) is μ.
- We expect the speed of transverse waves on the string v should increase when the tension F increases, but it should decrease when the mass per unit length μ increases.
- It is shown in your text that the wave speed is:

Speed of a transverse wave on a string → $v = \sqrt{\dfrac{F}{\mu}}$

F ← Tension in string

μ ← Mass per unit length

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The Speed of a Wave on a String (2 of 2)

- These transmission cables have a relatively large amount of mass per unit length, and a low tension.
- If the cables are disturbed—say, by a bird landing on them—transverse waves will travel along them at a slow speed.

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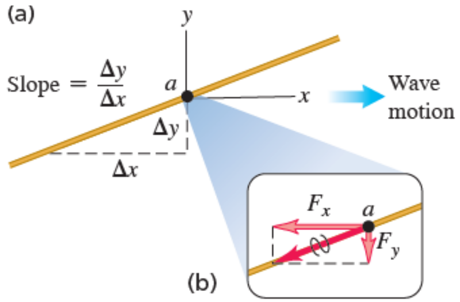

$$F_y(x, t) = -F\frac{\partial y(x, t)}{\partial x} \qquad (15.20)$$

$$y(x, t) = A\cos(kx - \omega t)$$

$$\frac{\partial y(x, t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial y(x, t)}{\partial t} = \omega A\sin(kx - \omega t)$$

$$P(x, t) = Fk\omega A^2\sin^2(kx - \omega t)$$

$$P(x, t) = \sqrt{\mu F}\,\omega^2A^2\sin^2(kx - \omega t) \qquad (15.23)$$

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Wave angular frequency

Average power, sinusoidal wave on a string

$$P_{av} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 \qquad (15.25)$$

Wave amplitude

Mass per unit length

Tension in string

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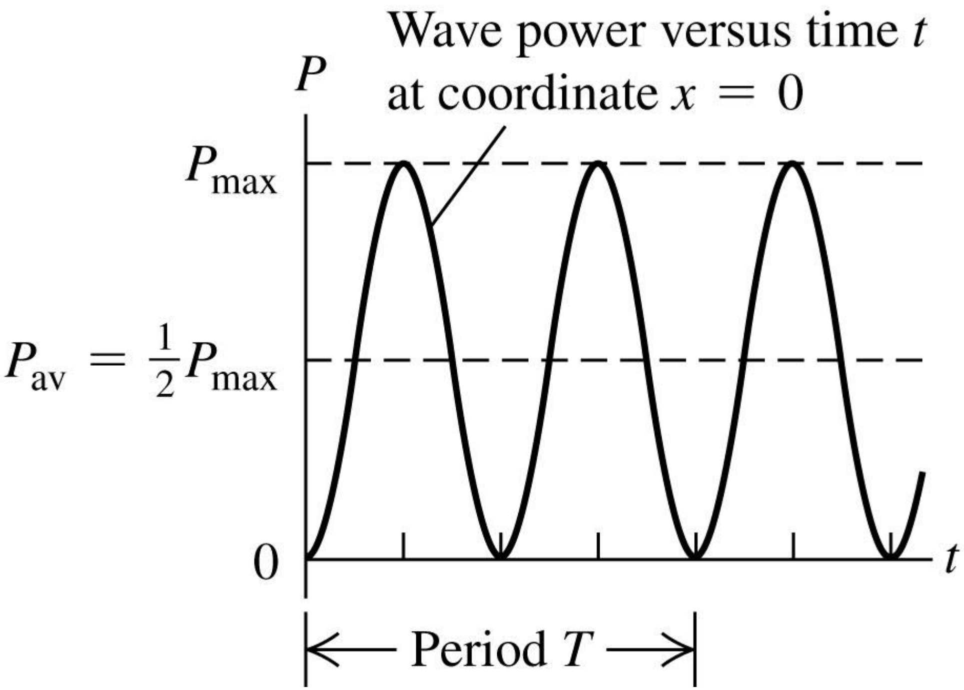
Power in a Wave (1 of 2)

- Shown is the instantaneous power in a sinusoidal wave.
- The power is never negative, which means that energy never flows opposite to the direction of wave propagation.



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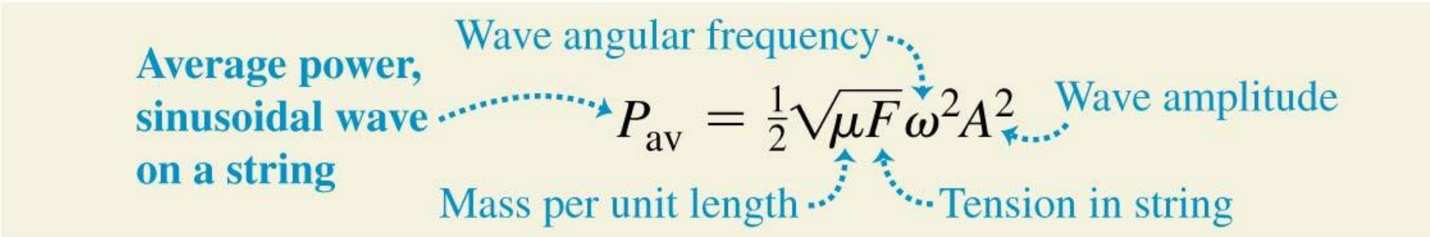
Power in a Wave (2 of 2)

- A wave transfers power along a string because it transfers energy.
- The average power is proportional to the **square** of the amplitude and to the **square** of the frequency.
- This result is true for all waves.
- For a transverse wave on a string, the average power is:



$$P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

Average power, sinusoidal wave on a string

Wave angular frequency

Wave amplitude

Mass per unit length

Tension in string

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EXAMPLE 15.4 Power in a wave

WITH VARIATION PROBLEMS

(a) In Example 15.2 (Section 15.3), at what maximum rate does Throcky put energy into the clothesline? That is, what is his maximum instantaneous power? The linear mass density of the clothesline is $\mu = 0.250\ \text{kg/m}$, and Throcky applies tension $F = 36.0\ \text{N}$. (b) What is his average power? (c) As Throcky tires, the amplitude decreases. What is the average power when the amplitude is 7.50 mm?

IDENTIFY and SET UP In part (a) our target variable is the *maximum instantaneous* power P_{max}, while in parts (b) and (c) it is the *average* power. For part (a) we'll use Eq. (15.24), and for parts (b) and (c) we'll use Eq. (15.25); Example 15.2 gives us all the needed quantities.

EXECUTE (a) From Eq. (15.24),

$$\begin{aligned} P_{\text{max}} &= \sqrt{\mu F}\omega^2 A^2 \\ &= \sqrt{(0.250\ \text{kg/m})(36.0\ \text{N})}(4.00\pi\ \text{rad/s})^2(0.075\ \text{m})^2 \\ &= 2.66\ \text{W} \end{aligned}$$

(b) From Eqs. (15.24) and (15.25), the average power is one-half of the maximum instantaneous power, so

$$P_{\text{av}} = \tfrac{1}{2}P_{\text{max}} = \tfrac{1}{2}(2.66\ \text{W}) = 1.33\ \text{W}$$

Continued

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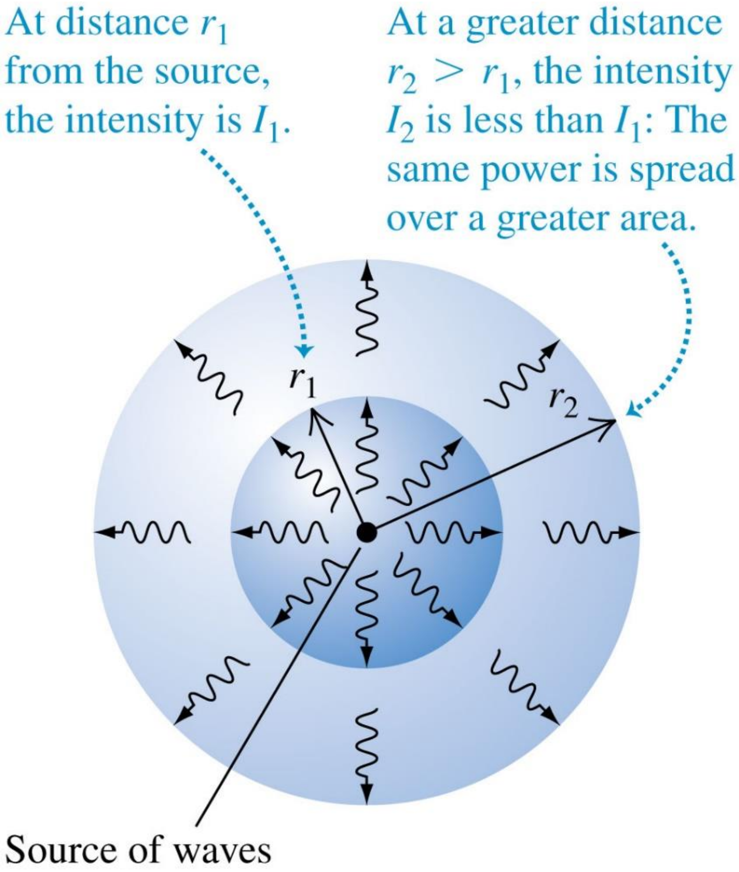
Wave Intensity

- The **intensity** of a wave is the average power it carries per unit area.
- If the waves spread out uniformly in all directions and no energy is absorbed, the intensity I at any distance r from a wave source is inversely proportional to r^2.
- Video Tutor Solution: Example 15.5



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Wave Intensity

Waves on a string carry energy in one dimension (along the direction of the string). But other types of waves, including sound waves in air and seismic waves within the earth, carry energy across all three dimensions of space. For waves of this kind, we define the **intensity** (denoted by I) to be *the time average rate at which energy is transported by the wave, per unit area,* across a surface perpendicular to the direction of propagation. Intensity I is average power per unit area and is usually measured in watts per square meter (W/m^2).

$$I_1 = \frac{P}{4\pi r_1^2}$$

Inverse-square law for intensity:
Intensity is inversely proportional to the square of the distance from source.

Intensity at point 1
Intensity at point 2

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \tag{15.26}$$

Distance from source to point 2
Distance from source to point 1

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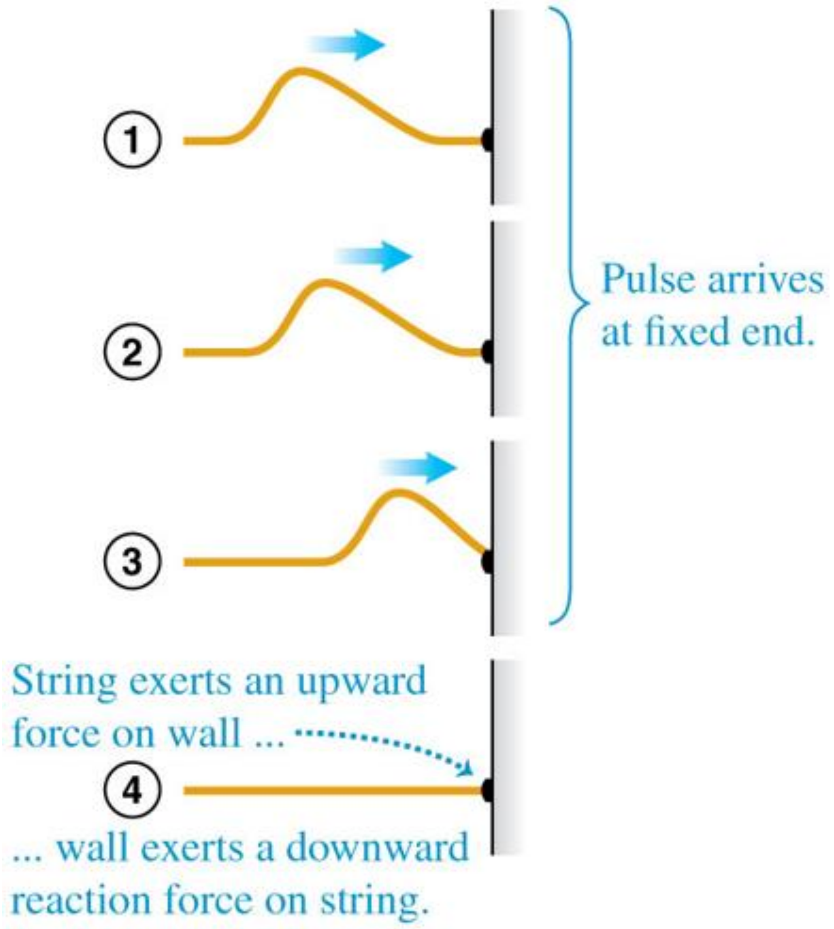
Reflection of a Wave Pulse at a Fixed End of a String (1 of 4)

- What happens when a wave pulse or a sinusoidal wave arrives at the end of the string?
- If the end is fastened to a rigid support, it is a **fixed** end that cannot move.
- The arriving wave exerts a force on the support (drawing 4).



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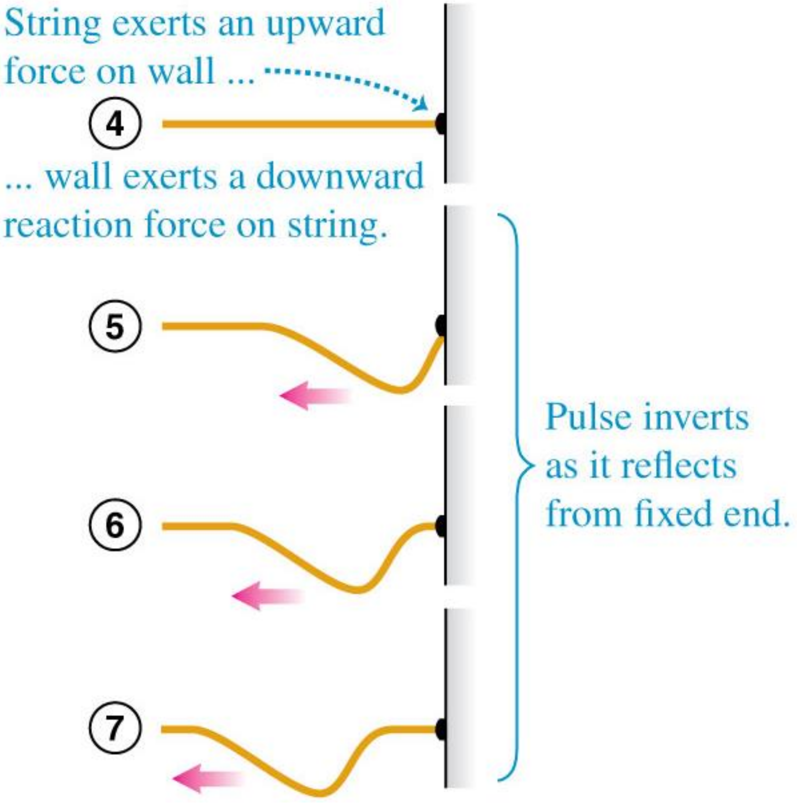
Reflection of a Wave Pulse at a Fixed End of a String (2 of 4)

- The reaction to the force of drawing 4, exerted by the support on the string, “kicks back” on the string and sets up a reflected pulse or wave traveling in the reverse direction.



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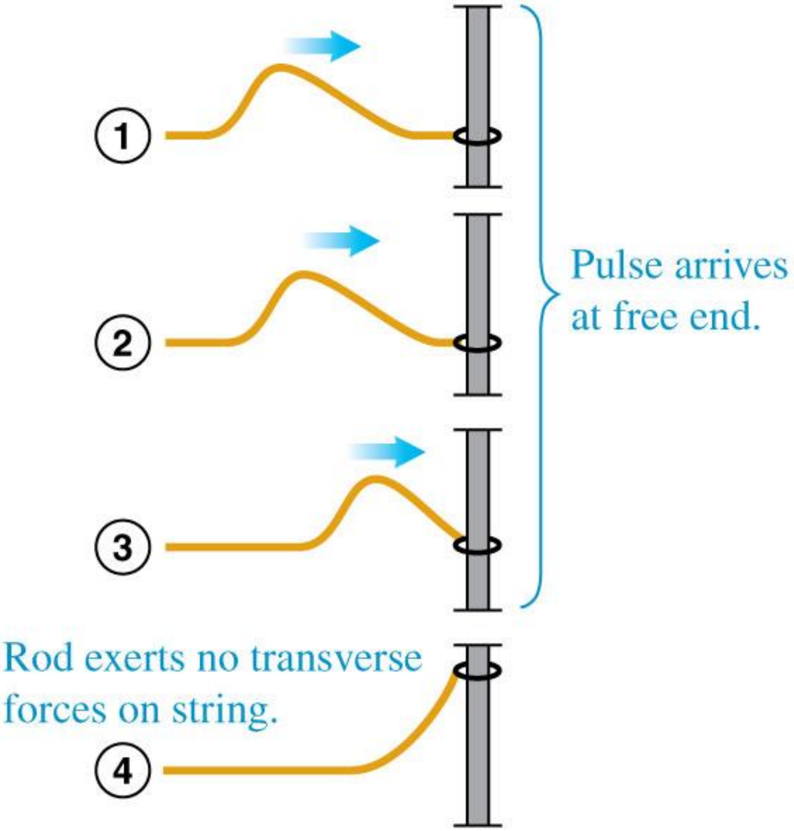
Reflection of a Wave Pulse at a Fixed End of a String (3 of 4)

- A **free end** is one that is perfectly free to move in the direction perpendicular to the length of the string.
- When a wave arrives at this free end, the ring slides along the rod, reaching a maximum displacement, coming momentarily to rest (drawing 4).



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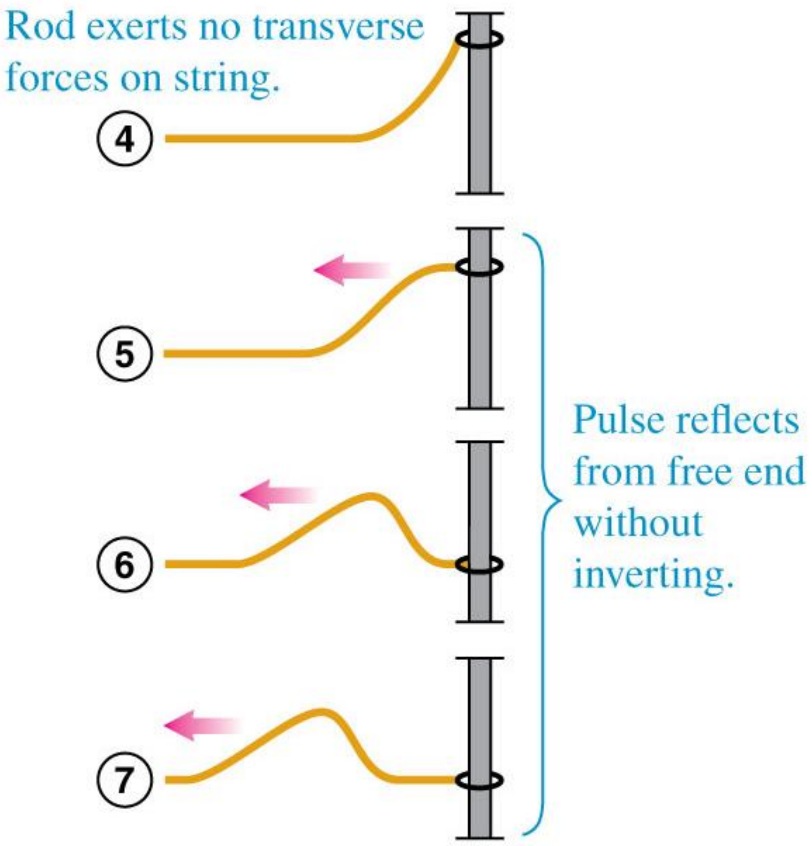
Reflection of a Wave Pulse at a Fixed End of a String (4 of 4)

- In drawing 4, the string is now stretched, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced.



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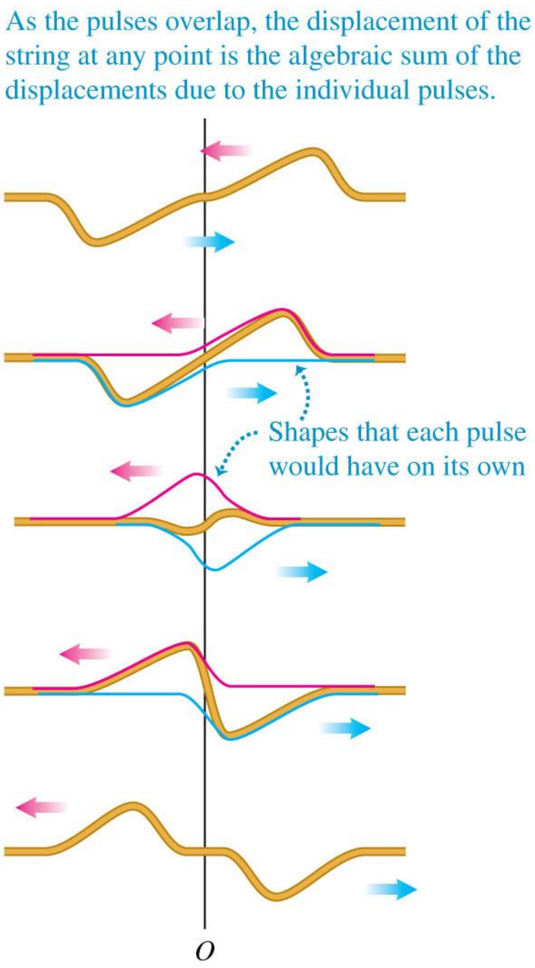
Superposition (1 of 2)

- **Interference** is the result of overlapping waves.
- **Principle of superposition**: When two or more waves overlap, the total displacement is the sum of the displacements of the individual waves.
- Shown is the overlap of two wave pulses—one right side up, one inverted—traveling in opposite directions.
- Time increases from top to bottom.



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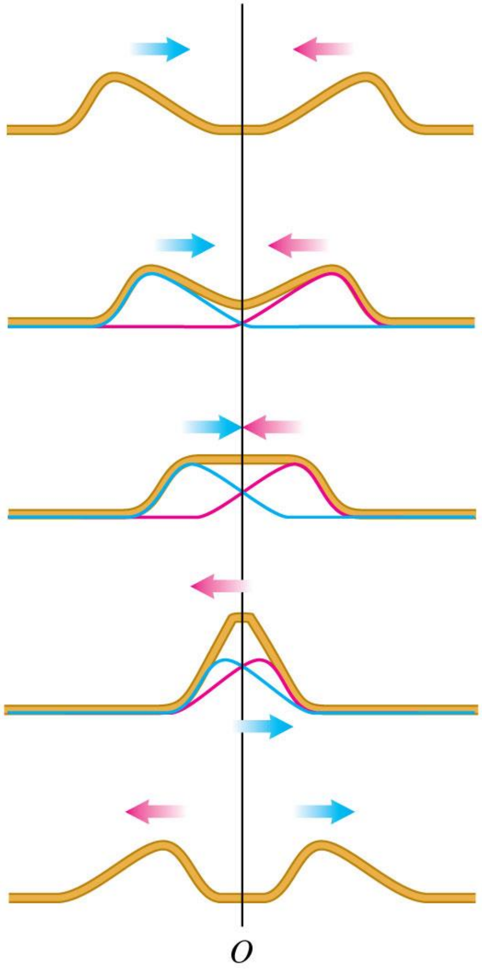
Superposition (2 of 2)

- Overlap of two wave pulses—both right side up—traveling in opposite directions.
- Time increases from top to bottom.



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Standing Waves on a String (1 of 3)

- Waves traveling in opposite directions on a taut string interfere with each other.
- The result is a **standing wave** pattern that does not move on the string.
- **Destructive interference** occurs where the wave displacements cancel, and **constructive interference** occurs where the displacements add.
- At the **nodes** no motion occurs, and at the **antinodes** the amplitude of the motion is greatest.
- Video Tutor Demonstration: Out-of-Phase Speakers

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The Principle of Superposition

Combining the displacements of the separate pulses at each point to obtain the actual displacement is an example of the **principle of superposition:** When two waves overlap, the actual displacement of any point on the string at any time is obtained by adding the displacement the point would have if only the first wave were present and the displacement it would have if only the second wave were present. In other words, the wave function $y(x, t)$ for the resulting motion is obtained by *adding* the two wave functions for the two separate waves:

Wave functions of two overlapping waves

Principle of superposition:

$$y(x, t) = y_1(x, t) + y_2(x, t) \qquad (15.27)$$

Wave function of combined wave = sum of individual wave functions

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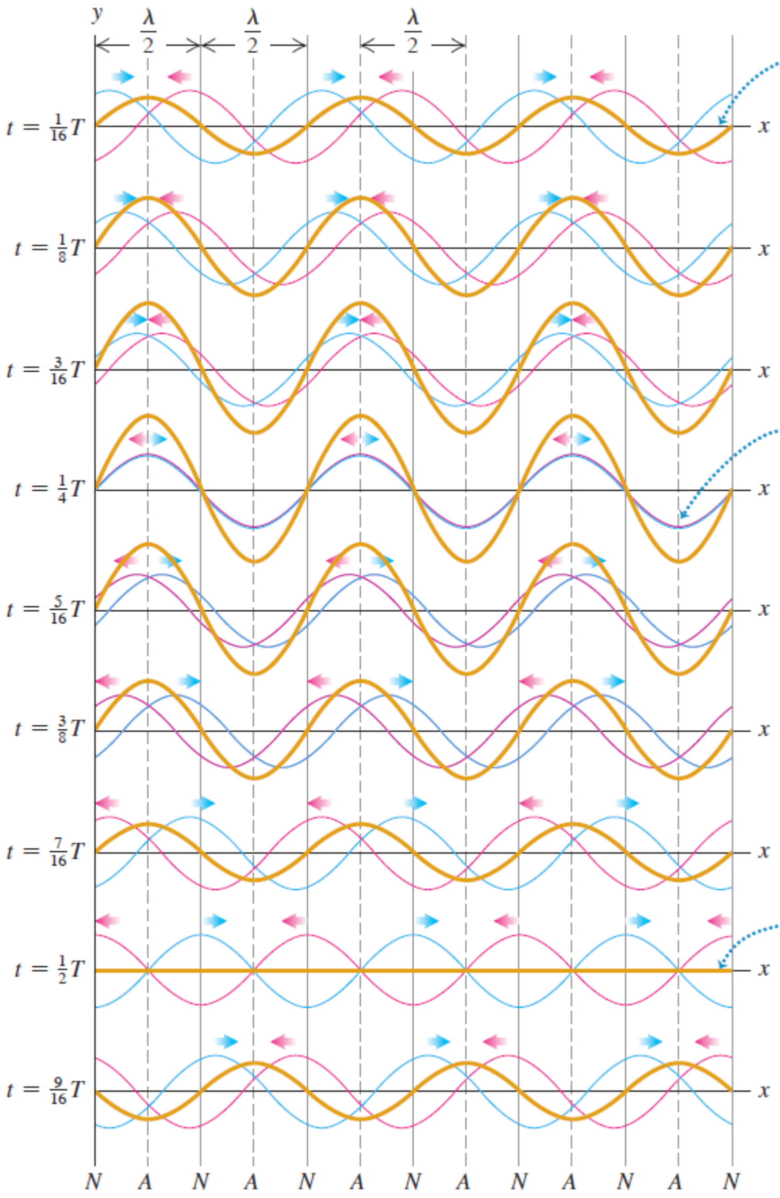

Equilibrium position of string is along the horizontal x-axis.

At this instant the waves coincide, so they add to give maximum string displacement.

At this instant the waves exactly cancel, so the string displacement is zero.

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$$y(x,t) = y_1(x,t) + y_2(x,t) = (2A\sin kx)\sin\omega t \qquad \text{or}$$

Standing wave on a string, fixed end at $x = 0$:

$$\overset{\text{Wave function}}{y(x,t)} = (\overset{\text{Standing-wave amplitude}}{A_{SW}}\sin \underset{\text{Wave number}}{k}\underset{\text{Position}}{x})\sin \underset{\text{Angular frequency}}{\omega} t \leftarrow \text{Time} \qquad (15.28)$$

The standing-wave amplitude A_{SW} is twice the amplitude A of either of the original traveling waves: $A_{SW} = 2A$.

We can use Eq. (15.28) to find the positions of the nodes; these are the points for which $\sin kx = 0$, so the displacement is *always* zero. This occurs when $kx = 0, \pi, 2\pi, 3\pi, \ldots,$ or, using $k = 2\pi/\lambda$,

$$\begin{aligned} x &= 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots \\ &= 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots \end{aligned} \qquad \text{(nodes of a standing wave on a string, fixed end at } x = 0) \qquad (15.29)$$

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A guitar string lies along the x-axis when in equilibrium. The end of the string at $x = 0$ (the bridge of the guitar) is fixed. A sinusoidal wave with amplitude $A = 0.750 \text{ mm} = 7.50 \times 10^{-4}$ m and frequency $f = 440$ Hz, corresponding to the red curves in Fig. 15.24, travels along the string in the $-x$-direction at 143 m/s. It is reflected from the fixed end, and the superposition of the incident and reflected waves forms a standing wave. (a) Find the equation giving the displacement of a point on the string as a function of position and time. (b) Locate the nodes. (c) Find the amplitude of the standing wave and the maximum transverse velocity and acceleration.

EXECUTE (a) The standing-wave amplitude is $A_{SW} = 2A = 1.50 \times 10^{-3}$ m (twice the amplitude of either the incident or reflected wave). The angular frequency and wave number are

$$\omega = 2\pi f = (2\pi \text{ rad})(440 \text{ s}^{-1}) = 2760 \text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{2760 \text{ rad/s}}{143 \text{ m/s}} = 19.3 \text{ rad/m}$$

Equation (15.28) then gives

$$\begin{aligned} y(x, t) &= (A_{SW} \sin kx) \sin \omega t \\ &= [(1.50 \times 10^{-3} \text{ m}) \sin(19.3 \text{ rad/m})x] \sin(2760 \text{ rad/s})t \end{aligned}$$

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(b) From Eq. (15.29), the positions of the nodes are $x = 0$, $\lambda/2$, λ, $3\lambda/2, \ldots.$ The wavelength is $\lambda = v/f = (143\ \text{m/s})/(440\ \text{Hz}) = 0.325\ \text{m}$, so the nodes are at $x = 0, 0.163\ \text{m}, 0.325\ \text{m}, 0.488\ \text{m}, \ldots.$

(c) From the expression for $y(x, t)$ in part (a), the maximum displacement from equilibrium is $A_{SW} = 1.50 \times 10^{-3}\ \text{m} = 1.50\ \text{mm}$.

(c) From the expression for $y(x, t)$ in part (a), the maximum displacement from equilibrium is $A_{SW} = 1.50 \times 10^{-3}\ \text{m} = 1.50\ \text{mm}$.

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This occurs at the *antinodes,* which are midway between adjacent nodes (that is, at $x = 0.081$ m, 0.244 m, 0.406 m, . . .).

For a particle on the string at any point x, the transverse (y-) velocity is

$$\begin{aligned} v_y(x,t) &= \frac{\partial y(x,t)}{\partial t} \\ &= [(1.50 \times 10^{-3}\ \text{m})\sin(19.3\ \text{rad/m})x] \\ &\quad \times [(2760\ \text{rad/s})\cos(2760\ \text{rad/s})t] \\ &= [(4.15\ \text{m/s})\sin(19.3\ \text{rad/m})x]\cos(2760\ \text{rad/s})t \end{aligned}$$

At an antinode, $\sin(19.3\ \text{rad/m})x = \pm 1$ and the transverse velocity varies between $+4.15$ m/s and -4.15 m/s. As is always the case in SHM, the maximum velocity occurs when the particle is passing through the equilibrium position ($y = 0$).

The transverse acceleration $a_y(x,t)$ is the *second* partial derivative of $y(x,t)$ with respect to time. You can show that

$$\begin{aligned} a_y(x,t) &= \frac{\partial v_y(x,t)}{\partial t} = \frac{\partial^2 y(x,t)}{\partial t^2} \\ &= [(-1.15 \times 10^4\ \text{m/s}^2)\sin(19.3\ \text{rad/m})x]\sin(2760\ \text{rad/s})t \end{aligned}$$

At the antinodes, the transverse acceleration varies between $+1.15 \times 10^4\ \text{m/s}^2$ and $-1.15 \times 10^4\ \text{m/s}^2$.

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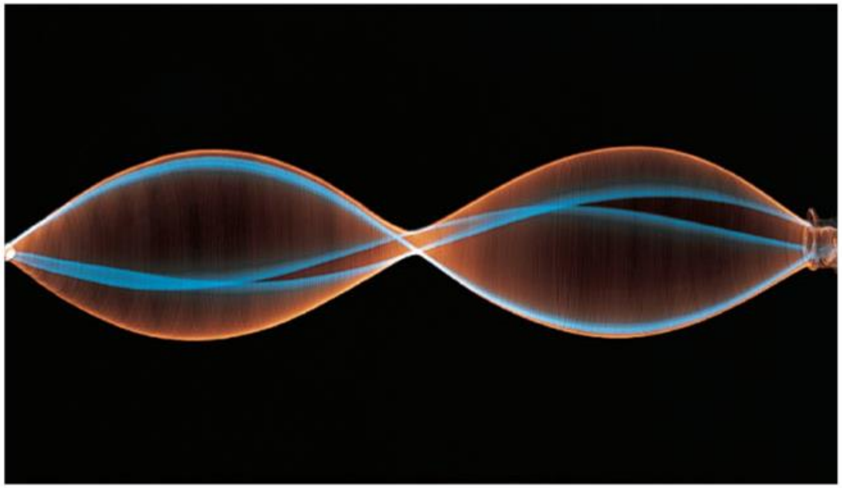
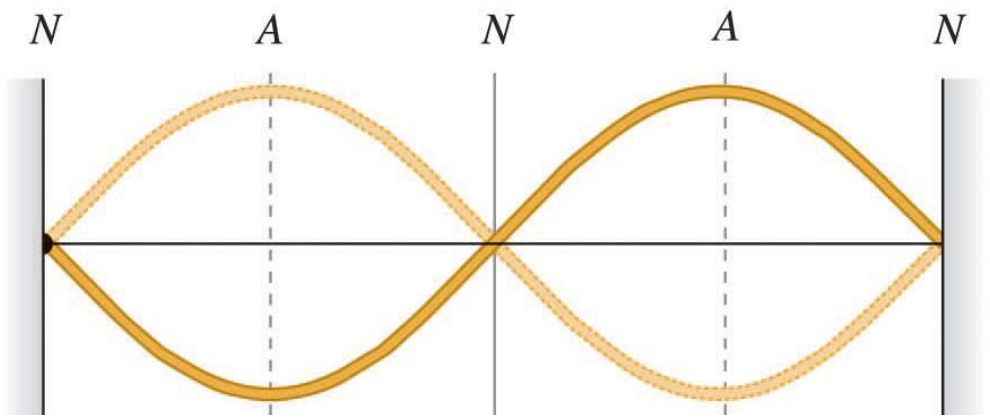
Standing Waves on a String (2 of 3)

- This is a time exposure of a standing wave on a string.
- This pattern is called the **second harmonic**.





N = **nodes:** points at which the string never moves

A = **antinodes:** points at which the amplitude of string motion is greatest

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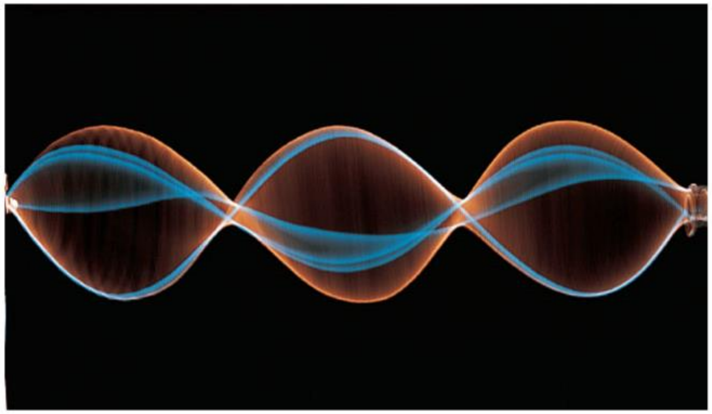
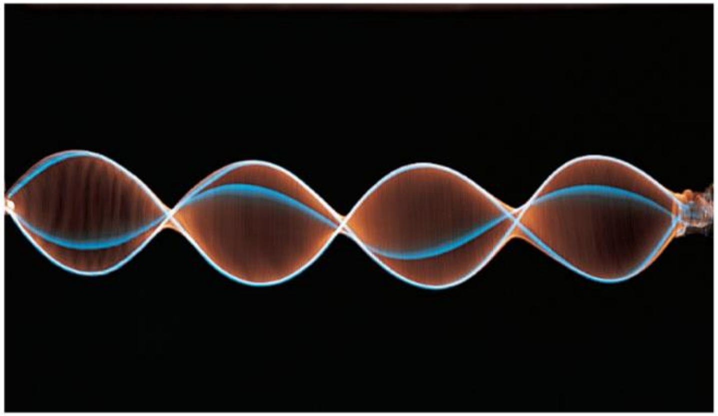
Standing Waves on a String (3 of 3)

- As the frequency of the oscillation of the right-hand end increases, the pattern of the standing wave changes.
- More nodes and antinodes are present in a higher frequency standing wave.





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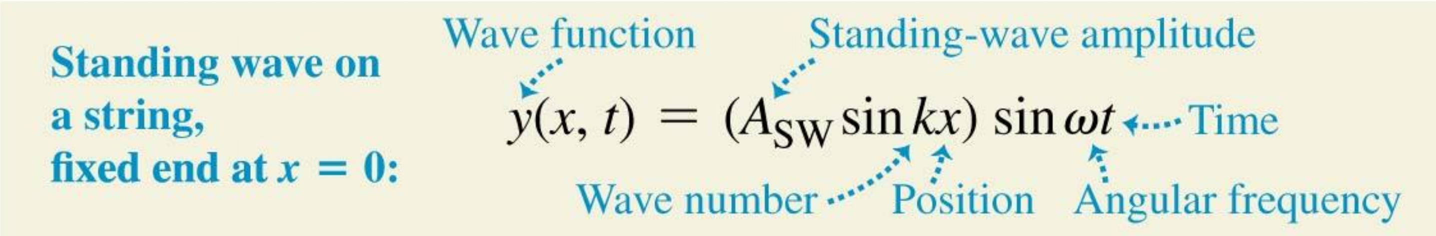
The Mathematics of Standing Waves

- We can derive a wave function for the standing wave by adding the wave functions for two waves with equal amplitude, period, and wavelength traveling in opposite directions.

- The wave function for a standing wave on a string in which $x = 0$ is a fixed end is:



Standing wave on a string, fixed end at $x = 0$:

$$y(x, t) = (A_{SW} \sin kx) \sin \omega t$$

- The standing-wave amplitude A_{SW} is twice the amplitude A of either of the original traveling waves: $A_{SW} = 2A$.

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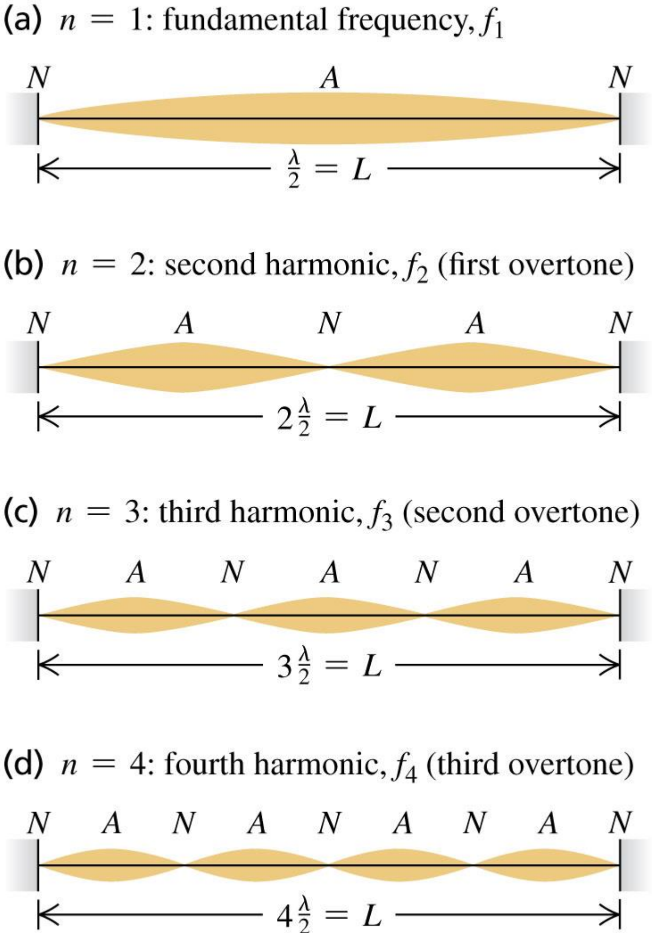
Normal Modes

- For a taut string fixed at both ends, the possible wavelengths are $\lambda_n = \frac{2L}{n}$ and the possible frequencies are $f_n = n\frac{v}{2L} = nf_1$, where n = 1, 2, 3, …
- f_1 is the **fundamental frequency**, f_2 is the second harmonic (first overtone), f_3 is the third harmonic (second overtone), etc.
- The figure illustrates the first four harmonics.



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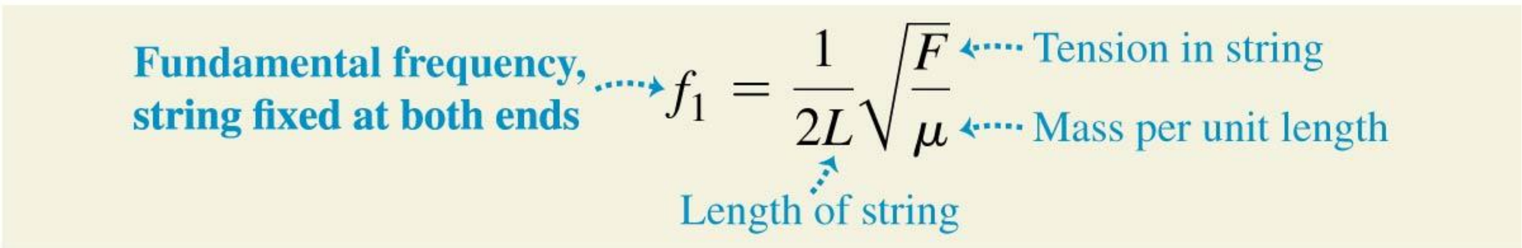
Standing Waves and String Instruments

- When a string on a musical instrument is plucked, bowed or struck, a standing wave with the fundamental frequency is produced:



Fundamental frequency, string fixed at both ends

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Tension in string

Mass per unit length

Length of string

- This is also the frequency of the sound wave created in the surrounding air by the vibrating string.
- Increasing the tension *F* increases the frequency (and the pitch).
- Video Tutor Solution: Example 15.7

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