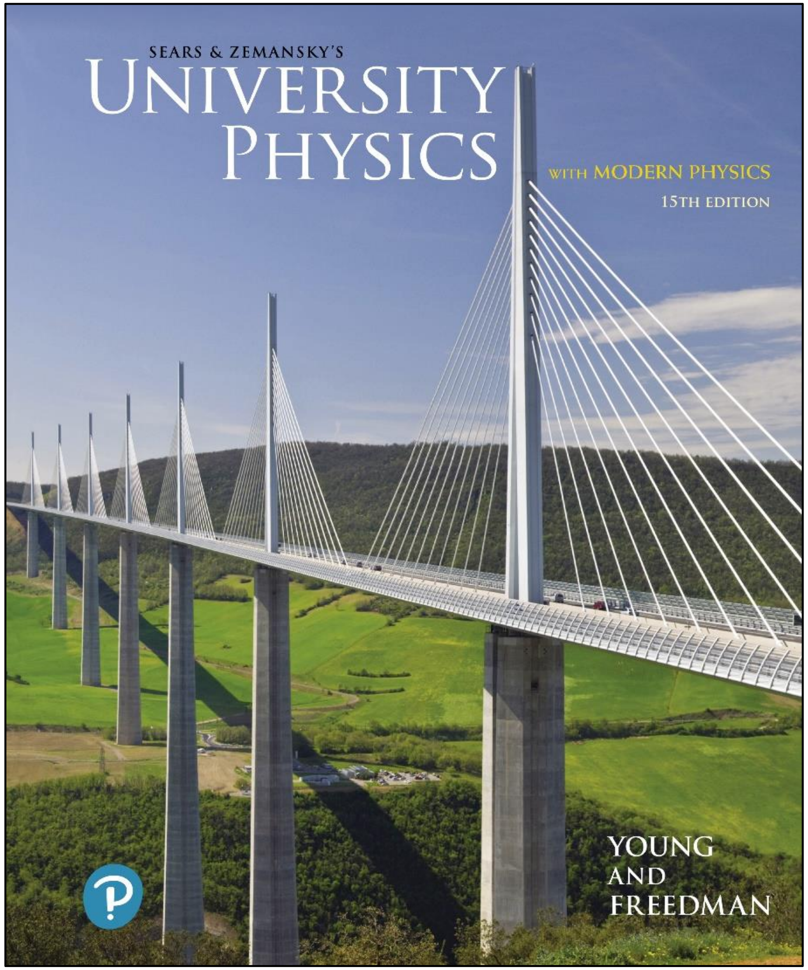
University Physics with Modern Physics

Fifteenth Edition



Chapter 36

Diffraction

Pearson

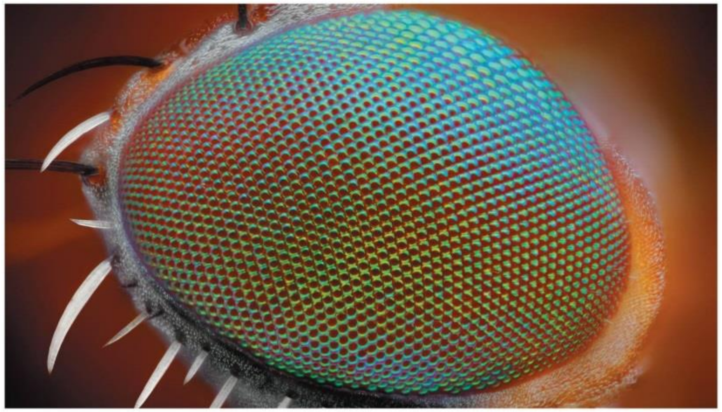
Learning Outcomes

In this chapter, you'll learn…

- how to calculate the intensity at various points in a single-slit diffraction pattern.
- what happens when coherent light shines on an array of narrow, closely spaced slits.
- how x-ray diffraction reveals the arrangement of atoms in a crystal.
- how diffraction sets limits on the smallest details that can be seen with an optical system.
- how holograms work.

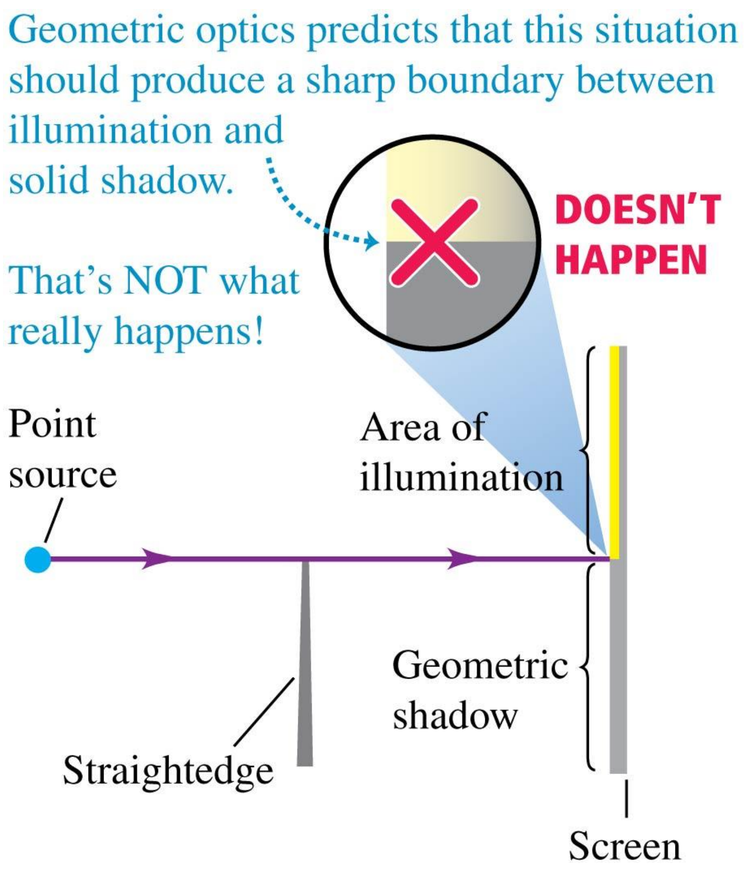
Introduction

- Flies have compound eyes with thousands of miniature lenses, each only about 20 μm in diameter.
- Due to the wave-nature of light, the ability of a lens to resolve fine details improves as the lens diameter *D* increases.
- Each miniature lens in a fly's eye has very poor resolution, compared to those produced by a human eye, because the lens is so small.
- We'll continue our exploration of the wave nature of light with **diffraction**.



Diffraction

- According to geometric optics, when an opaque object is placed between a point light source and a screen, the shadow of the object forms a perfectly sharp line.
- However, the wave nature of light causes interference patterns, which blur the edge of the shadow.
- This is one effect of **diffraction**.



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In Fig. 36.1, both the point source and the screen are relatively close to the obstacle forming the diffraction pattern. This situation is described as *near-field diffraction* or **Fresnel diffraction,** pronounced “Freh-nell” (after the French scientist Augustin Jean Fresnel, 1788–1827). By contrast, we use the term **Fraunhofer diffraction** (after the German physicist Joseph von Fraunhofer, 1787–1826) for situations in which the source, obstacle, and screen are far enough apart that we can consider all lines from the source to the obstacle to be parallel, and can likewise consider all lines from the obstacle to a given point on the screen to be parallel. We’ll restrict the following discussion to Fraunhofer diffraction, which is usually simpler to analyze in detail than Fresnel diffraction.

Diffraction and Huygen's Principle

- This photograph was made by placing a razor blade halfway between a pinhole, illuminated by monochromatic light, and a photographic film.
- The film recorded the shadow cast by the blade.
- Note the fringe pattern around the blade outline, which is caused by diffraction.

(a)

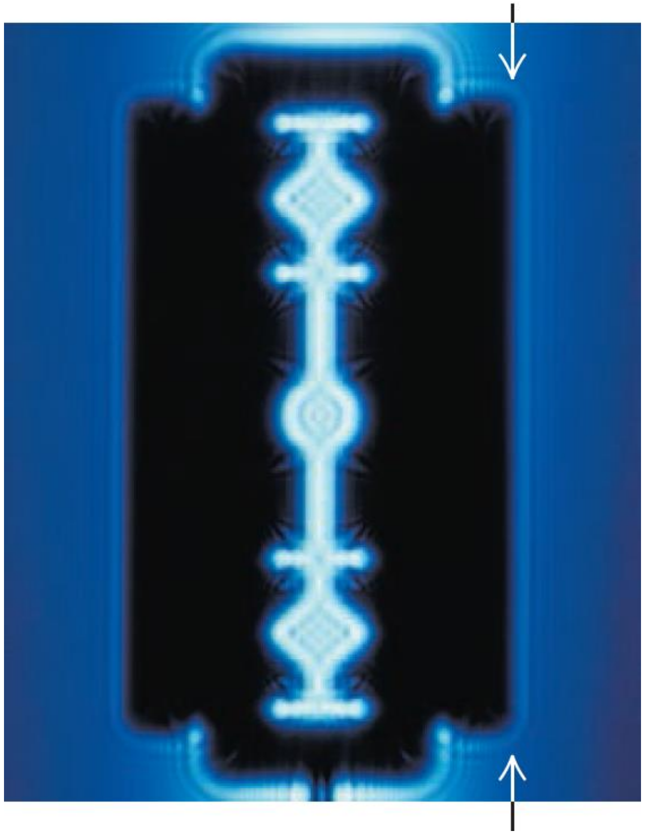
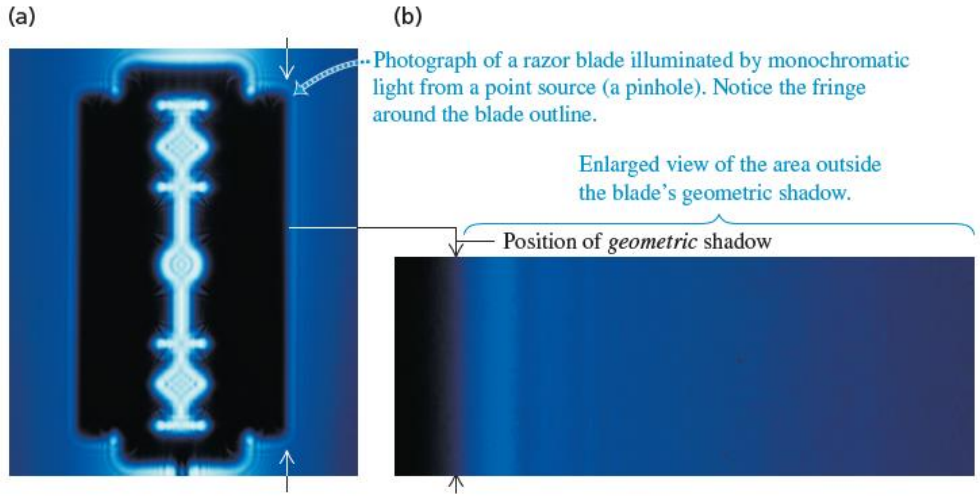


Figure 36.2 An example of diffraction.



Diffraction from a Single Slit (1 of 2)

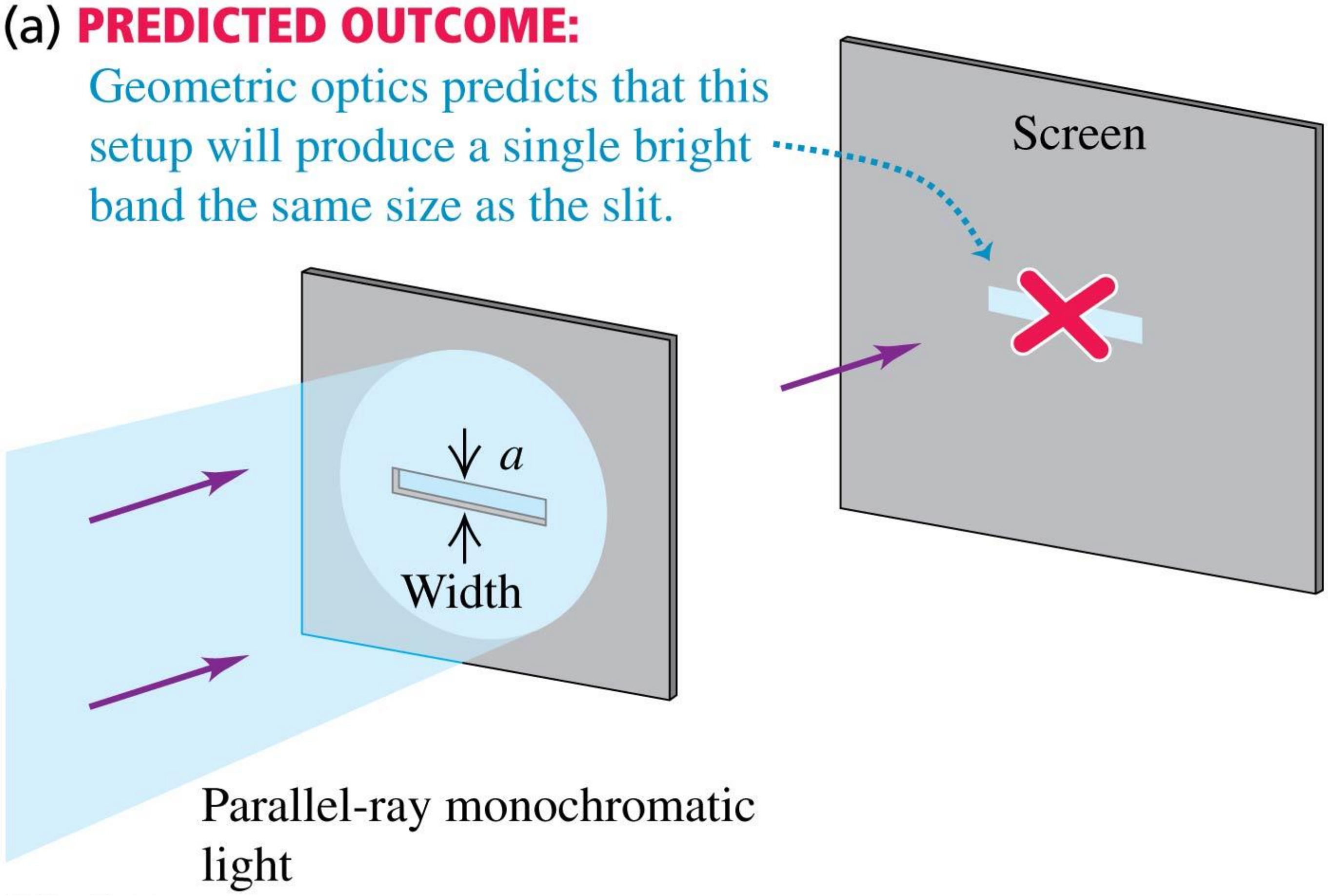
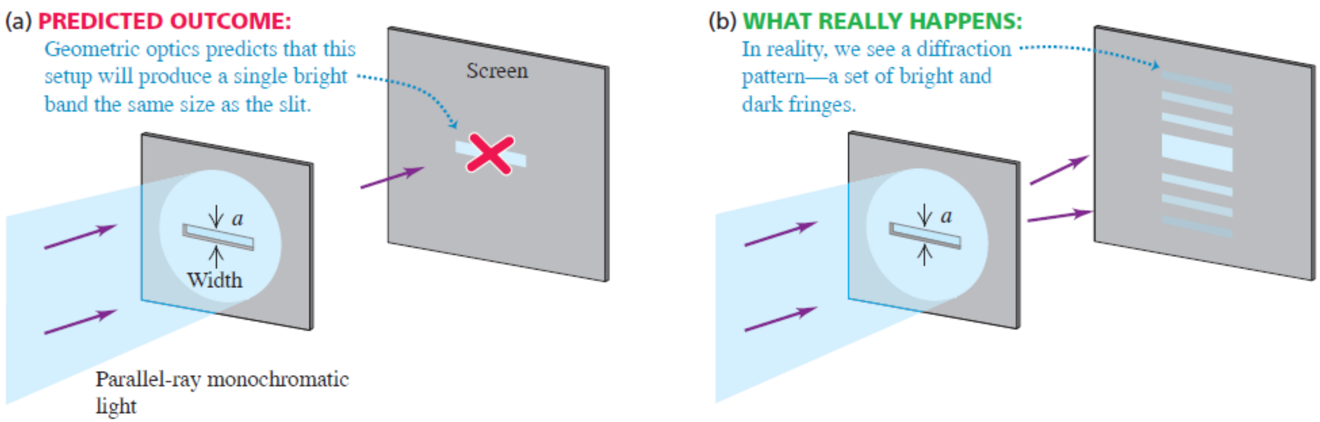
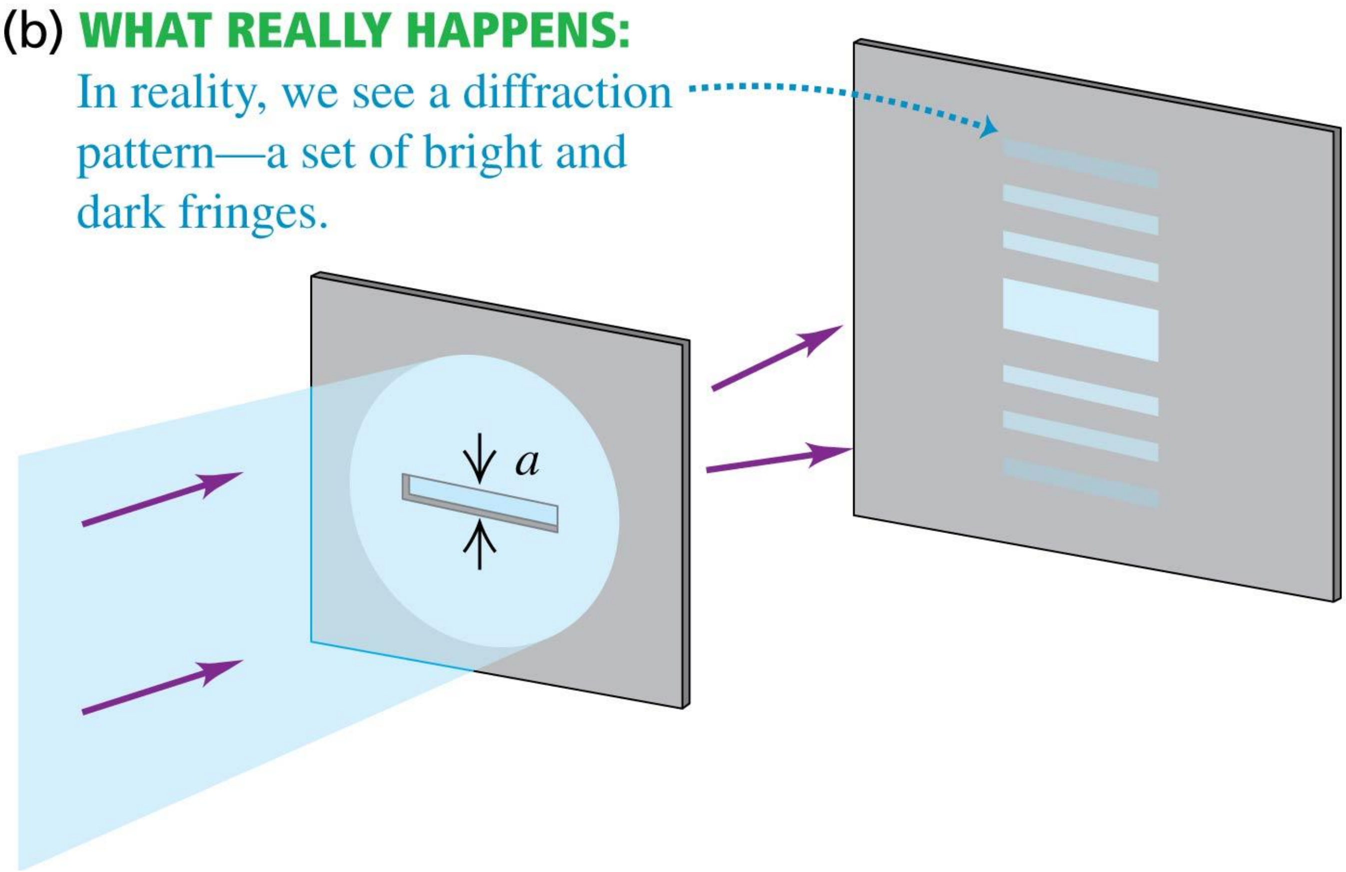


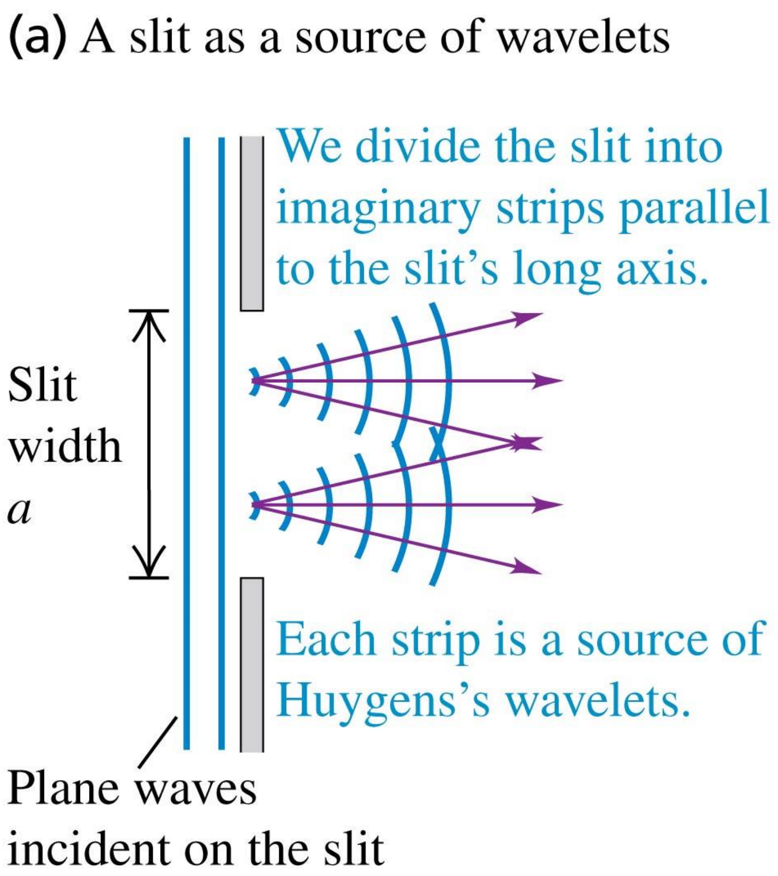
Figure 36.3 (a) The "shadow" of a horizontal slit as incorrectly predicted by geometric optics.
(b) A horizontal slit actually produces a diffraction pattern. The slit width has been greatly exaggerated.



Diffraction from a Single Slit (2 of 2)



Fresnel Diffraction by a Single Slit



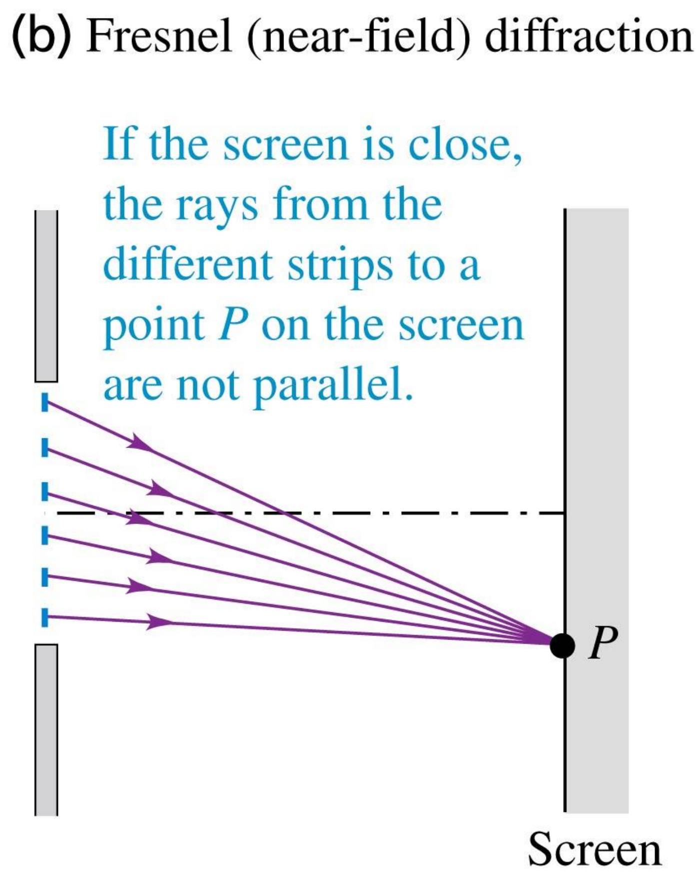
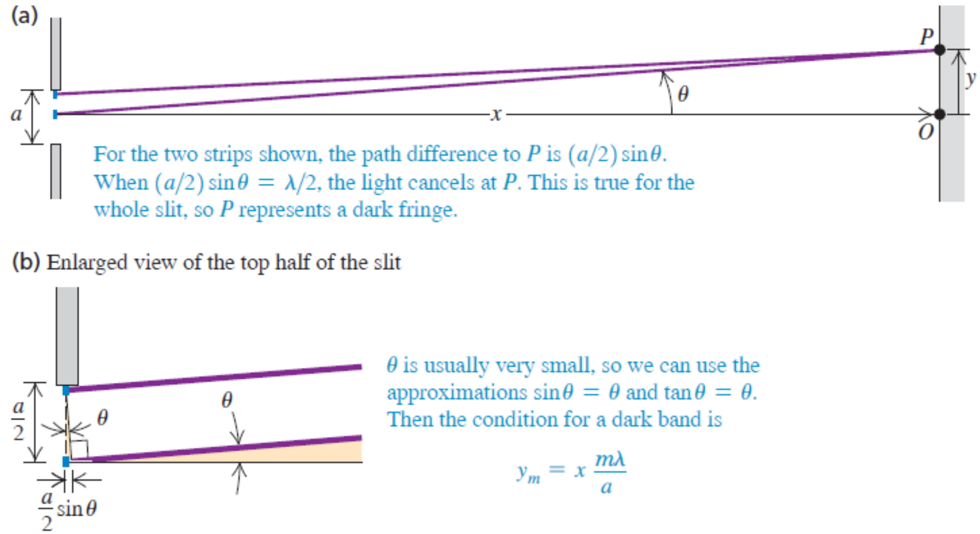


Figure 36.5 Side view of a horizontal slit. When the distance x to the screen is much greater than the slit width a, the rays from a distance $a/2$ apart may be considered parallel.



Dark fringes, single-slit diffraction:

Angle of line from center of slit to mth dark fringe on screen

$$\sin\theta = \frac{m\lambda}{a} \quad (m = \pm1, \pm2, \pm3, \dots) \tag{36.2}$$

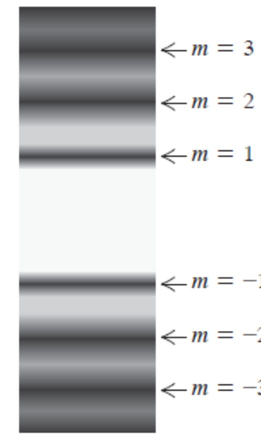
Slit width

Wavelength

With light, the wavelength λ is of the order of 500 nm $= 5 \times 10^{-7}$ m. This is often much smaller than the slit width a; a typical slit width is 10^{-2} cm $= 10^{-4}$ m. Therefore the values of θ in Eq. (36.2) are often so small that the approximation $\sin\theta \approx \theta$ (where θ is in radians) is a very good one. In that case we can rewrite this equation as

$$\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \ldots) \qquad \text{(for small angles } \theta \text{ in radians)}$$

Figure 36.6 Photograph of the Fraunhofer diffraction pattern of a single horizontal slit.



Also, if the distance from slit to screen is x, as in Fig. 36.5a, and the vertical distance of the mth dark band from the center of the pattern is y_m, then $\tan\theta = y_m/x$. For small θ we may also approximate $\tan\theta$ by θ (in radians). We then find

$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x) \qquad (36.3)$$

EXAMPLE 36.1 Single-slit diffraction

You pass 633 nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (**Fig. 36.7**, next page). How wide is the slit?

IDENTIFY and SET UP This problem involves the relationship between the positions of dark fringes in a single-slit diffraction pattern and the slit width a (our target variable). The distances between fringes on the screen are much smaller than the slit-to-screen distance, so the angle θ shown in Fig. 36.5a is very small and we can use Eq. (36.3) to solve for a.

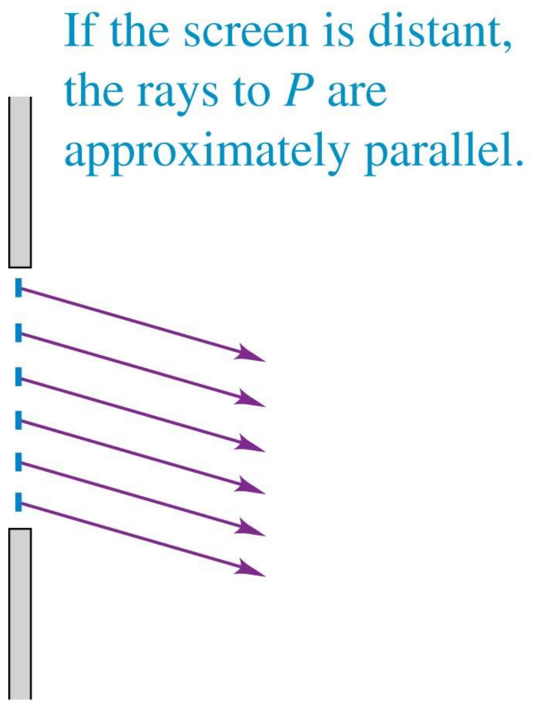
EXECUTE The first minimum corresponds to $m = 1$ in Eq. (36.3). The distance y_1 from the central maximum to the first minimum on either side is half the distance between the two first minima, so $y_1 = (32\ \text{mm})/2 = 16\ \text{mm}$. Solving Eq. (36.3) for a, we find

$$a = \frac{x\lambda}{y_1} = \frac{(6.0\ \text{m})(633 \times 10^{-9}\ \text{m})}{16 \times 10^{-3}\ \text{m}} = 2.4 \times 10^{-4}\ \text{m} = 0.24\ \text{mm}$$

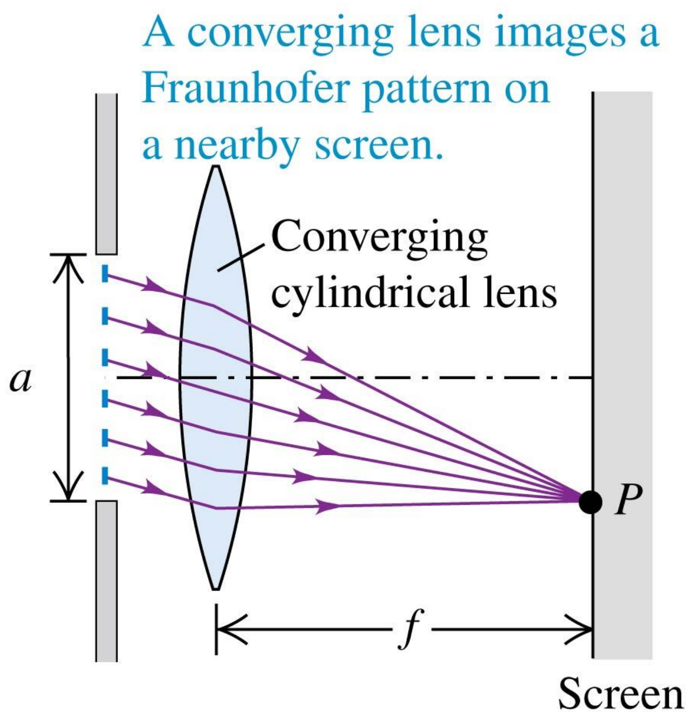
EVALUATE The angle θ is small only if the wavelength is small compared to the slit width. Since $\lambda = 633\ \text{nm} = 6.33 \times 10^{-7}\ \text{m}$ and we have found $a = 0.24\ \text{mm} = 2.4 \times 10^{-4}\ \text{m}$, our result is consistent with this: The wavelength is $(6.33 \times 10^{-7}\ \text{m})/(2.4 \times 10^{-4}\ \text{m}) = 0.0026$ as large as the slit width. Can you show that the distance between the *second* minima on either side is $2(32\ \text{mm}) = 64\ \text{mm}$, and so on?

Fraunhofer Diffraction by a Single Slit

(c) Fraunhofer (far-field) diffraction



(d) Imaging Fraunhofer diffraction



Locating the Dark Fringes

- Shown is the Fraunhofer diffraction pattern from a single horizontal slit.
- It is characterized by a central bright fringe centered at $\theta = 0$, surrounded by a series of dark fringes.
- The central bright fringe is twice as wide as the other bright fringes.

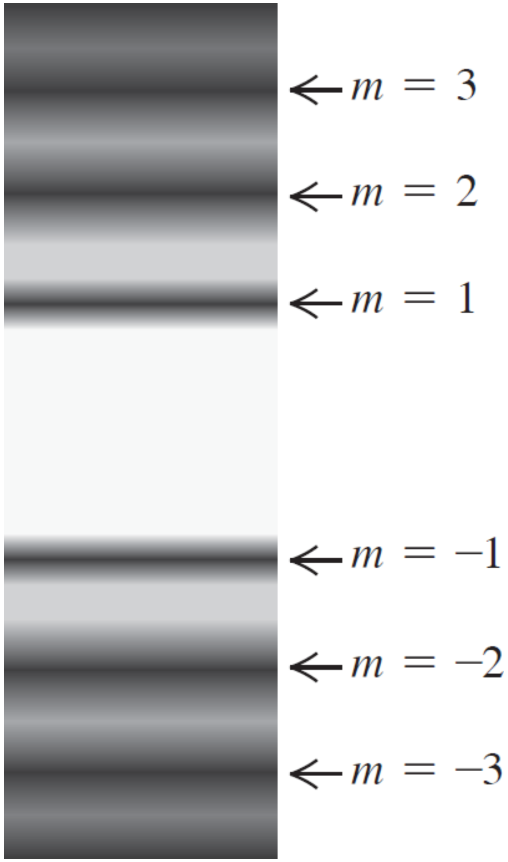
Dark fringes, single-slit diffraction:

Angle of line from center of slit to mth dark fringe on screen

$$\sin\theta = \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \ldots)$$

Slit width

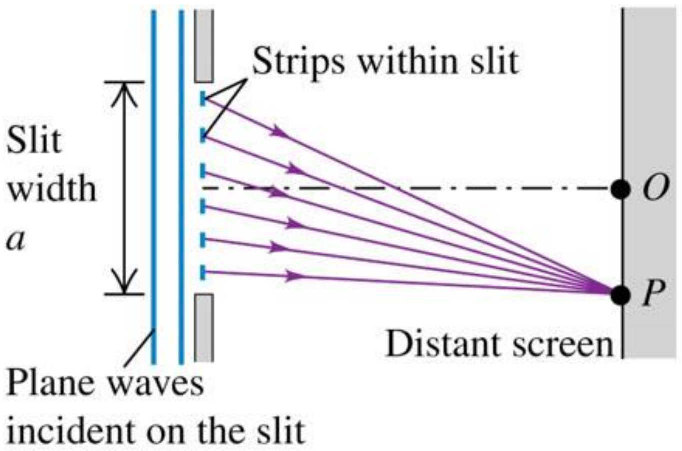
Wavelength



- Video Tutor Solution: Example 36.1

Intensity in the Single-Slit Pattern (1 of 2)

- We can derive an expression for the intensity distribution for the single-slit diffraction pattern by using phasor-addition.
- We imagine a plane wave front at the slit subdivided into a large number of strips.
- At the point *O*, the phasors are all **in phase**.



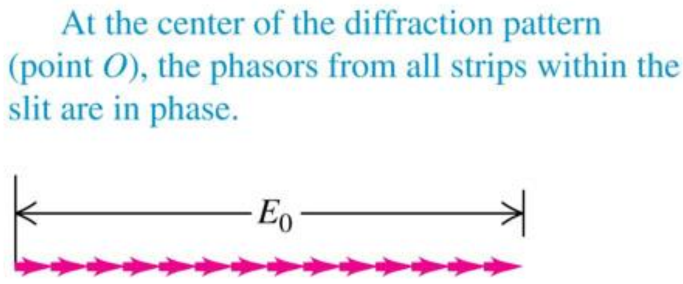
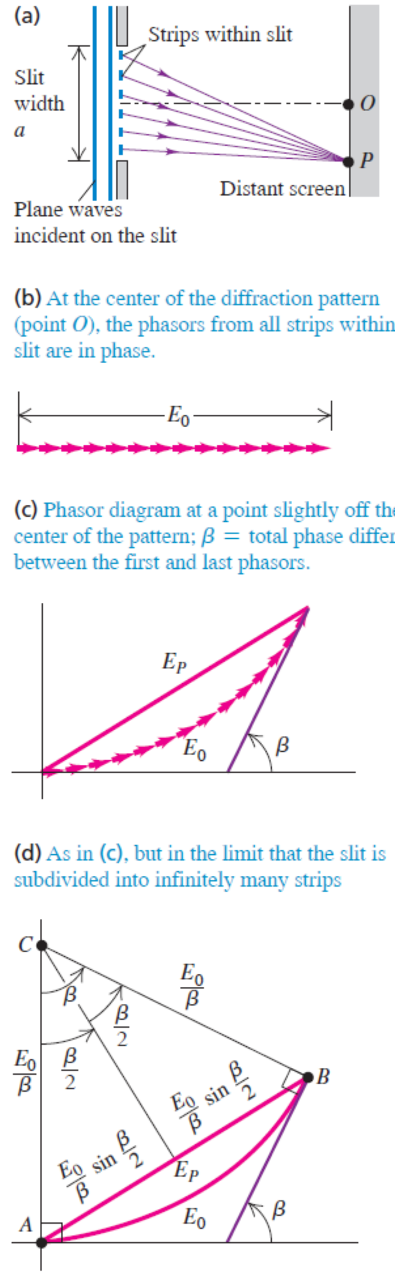


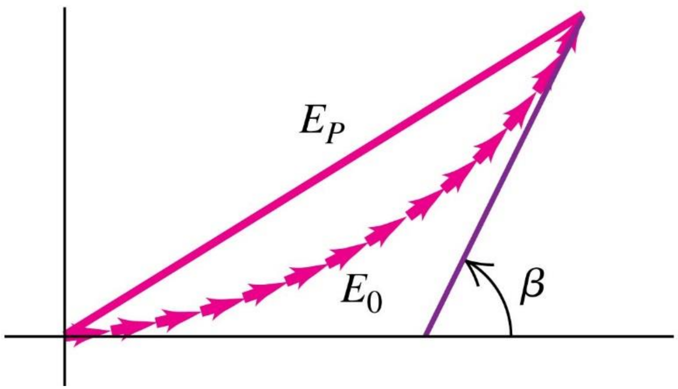
Figure 36.8 Using phasor diagrams tc find the amplitude of the $\vec{E}$ field in sin slit diffraction. Each phasor represents $\vec{E}$ field from a single strip within the sl



Intensity in the Single-Slit Pattern (2 of 2)

- Now consider wavelets arriving from different strips at point *P*.
- Because of the differences in path length, there are now phase differences between wavelets coming from adjacent strips.
- The vector sum of the phasors is now part of the perimeter of a many-sided polygon.

(c) Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.



E_P of the resultant electric field at P is equal to the chord AB, which is $2(E_0/\beta)\sin(\beta/2)$. (Note that β *must* be in radians!) We then have

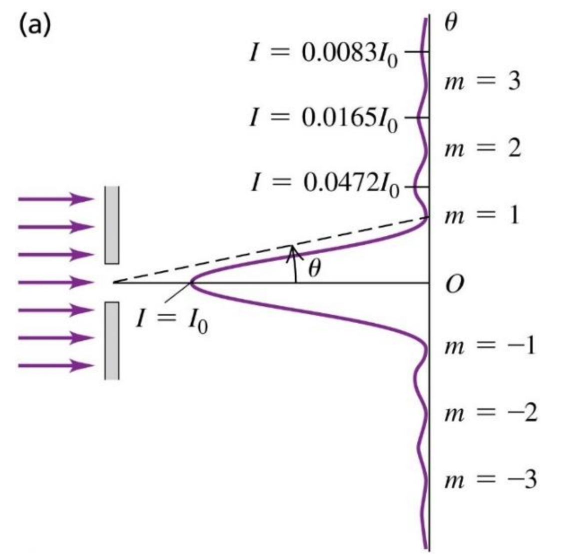
$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2} \qquad \text{(amplitude in single-slit diffraction)} \tag{36.4}$$

The intensity at each point on the screen is proportional to the square of the amplitude given by Eq. (36.4). If I_0 is the intensity in the straight-ahead direction where $\theta = 0$ and $\beta = 0$, then the intensity I at any point is

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \qquad \text{(intensity in single-slit diffraction)} \tag{36.5}$$

Intensity Maxima in a Single-Slit Pattern

- Shown is the intensity versus angle in a single-slit diffraction pattern.
- Most of the wave power goes into the central intensity peak (between the *m* = 1 and *m* = −1 intensity minima).



Angle of line from center of slit to position on screen

Intensity in single-slit diffraction

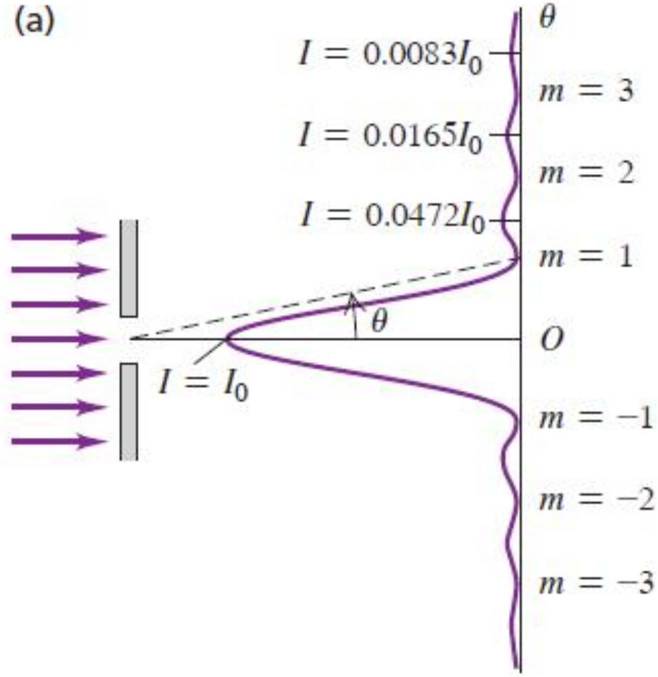
$$I = I_0\left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2$$

Intensity at $\theta = 0$ Slit width Wavelength

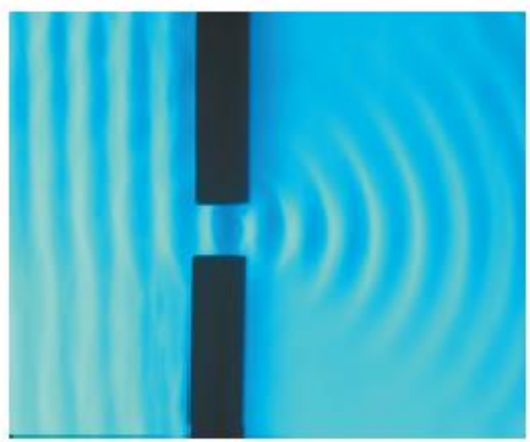
- [Video Tutor Solution: Example 36.2]()

Pearson

(a)



(b)



The dark fringes in the pattern are the places where $I = 0$. These occur at points for which the numerator of Eq. (36.5) is zero so that β is a multiple of 2π. From Eq. (36.6) this corresponds to

$$\frac{a \sin\theta}{\lambda} = m \qquad (m = \pm 1, \pm 2, \ldots)$$

$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \ldots) \tag{36.8}$$

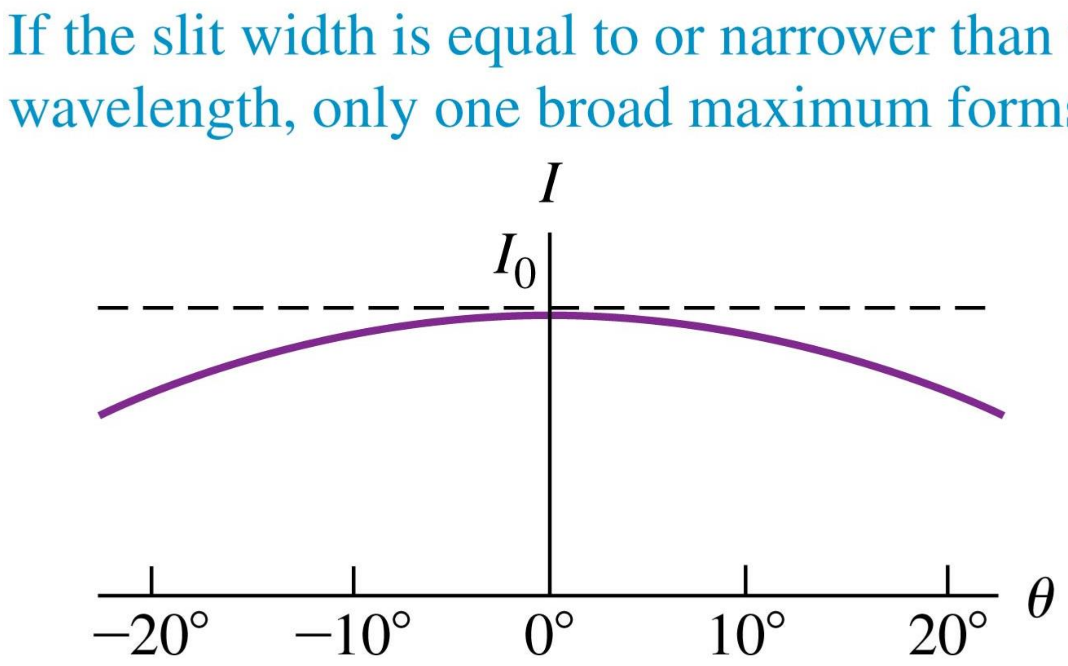
Intensity Maxima in the Single-Slit Pattern

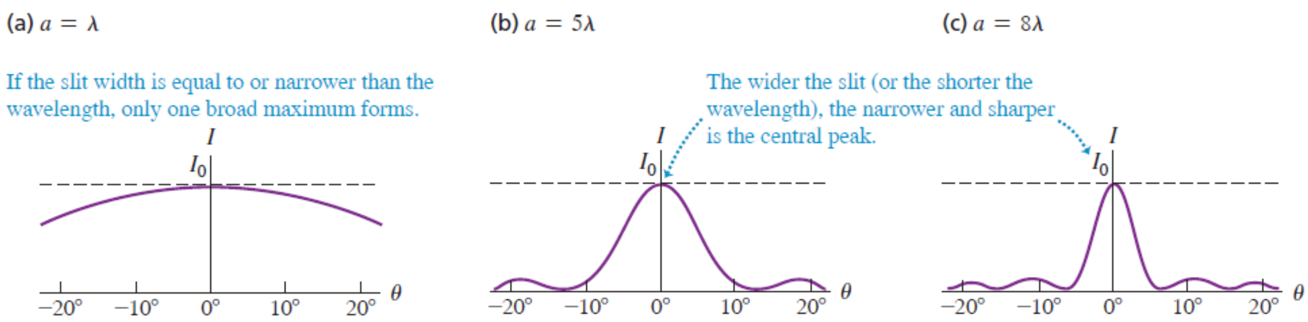
We can also use Eq. (36.5) to calculate the positions of the peaks, or *intensity maxima,* and the intensities at these peaks. This is not quite as simple as it may appear. We might expect the peaks to occur where the sine function reaches the value ± 1—namely, where $\beta = \pm\pi, \pm 3\pi, \pm 5\pi$, or in general,

$$\beta \approx \pm(2m + 1)\pi \qquad (m = 0, 1, 2, \ldots) \tag{36.9}$$

Width of the Single-Slit Pattern (1 of 2)

- The single-slit diffraction pattern depends on the ratio of the slit width *a* to the wavelength λ.
- Below is the pattern when $a = \lambda$.

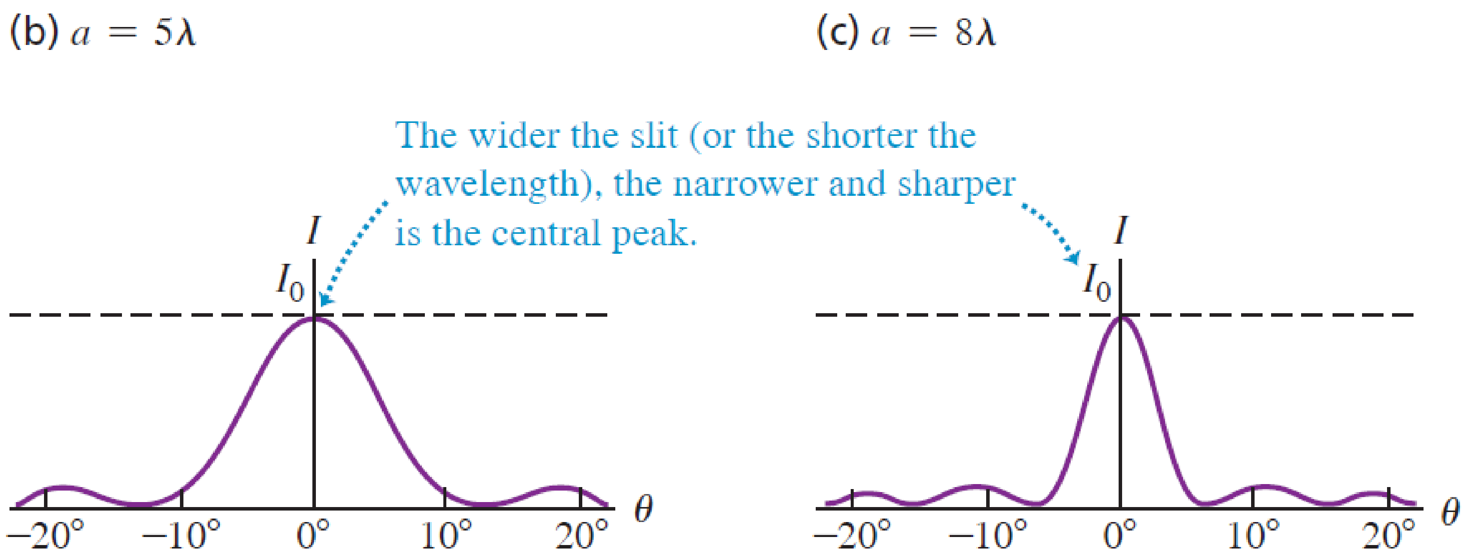




(a) $a = \lambda$
(b) $a = 5\lambda$
(c) $a = 8\lambda$
If the slit width is equal to or narrower than the wavelength, only one broad maximum forms.
The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.
I
I_0
θ
−20°
−10°
0°
10°
20°

Width of the Single-Slit Pattern (2 of 2)

- The single-slit diffraction pattern depends on the ratio of the slit width *a* to the wavelength λ.
- Below are the patterns when $a = 5\lambda$ (left) and $a = 8\lambda$ (right).



EXAMPLE 36.2 Single-slit diffraction: Intensity I

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66 radian phase difference between wavelets from the two edges of the slit? (b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

EXAMPLE 36.2 Single-slit diffraction: Intensity I

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66 radian phase difference between wavelets from the two edges of the slit? (b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

EXECUTE (a) We have $\beta/2 = 33$ rad, so from Eq. (36.5),

$$I = I_0\left[\frac{\sin(33\text{ rad})}{33\text{ rad}}\right]^2 = (9.2 \times 10^{-4})I_0$$

(b) From Eq. (36.6),

$$\frac{a}{\lambda} = \frac{\beta}{2\pi\sin\theta} = \frac{66\text{ rad}}{(2\pi\text{ rad})\sin 7.0^\circ} = 86$$

For example, for 550 nm light the slit width is $a = (86)(550\text{ nm}) = 4.7 \times 10^{-5}\text{ m} = 0.047\text{ mm}$, or roughly $\frac{1}{20}$ mm.

EXAMPLE 36.3 Single-slit diffraction: Intensity II

In the experiment described in Example 36.1 (Section 36.2), the intensity at the center of the pattern is I_0. What is the intensity at a point on the screen 3.0 mm from the center of the pattern?

EXAMPLE 36.3 Single-slit diffraction: Intensity II

In the experiment described in Example 36.1 (Section 36.2), the intensity at the center of the pattern is I_0. What is the intensity at a point on the screen 3.0 mm from the center of the pattern?

EXECUTE Referring to Fig. 36.5a, we have $y = 3.0\ \text{mm}$ and $x = 6.0\ \text{m}$, so $\tan\theta = y/x = (3.0 \times 10^{-3}\ \text{m})/(6.0\ \text{m}) = 5.0 \times 10^{-4}$. This is so small that the values of $\tan\theta$, $\sin\theta$, and θ (in radians) are all nearly the same. Then, from Eq. (36.7),

$$\frac{\pi a \sin\theta}{\lambda} = \frac{\pi(2.4 \times 10^{-4}\ \text{m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7}\ \text{m}} = 0.60$$

$$I = I_0\left(\frac{\sin 0.60}{0.60}\right)^2 = 0.89 I_0$$

Two Slits of Finite Width (1 of 2)

- Figure (a) shows the intensity in a single-slit diffraction pattern with slit width *a*.
- The diffraction minima are labeled by the integer $m_d = \pm 1, \pm 2, \ldots$ ("d" for "diffraction").
- Figure (b) shows the pattern formed by two very narrow slits with distance *d* between slits, where *d* is four times as great as the single-slit width *a*.
- "i" is for "interference."

(a) Single-slit diffraction pattern for a slit width *a*

(b) Two-slit interference pattern for narrow slits whose separation *d* is four times the width of the slit in **(a)**

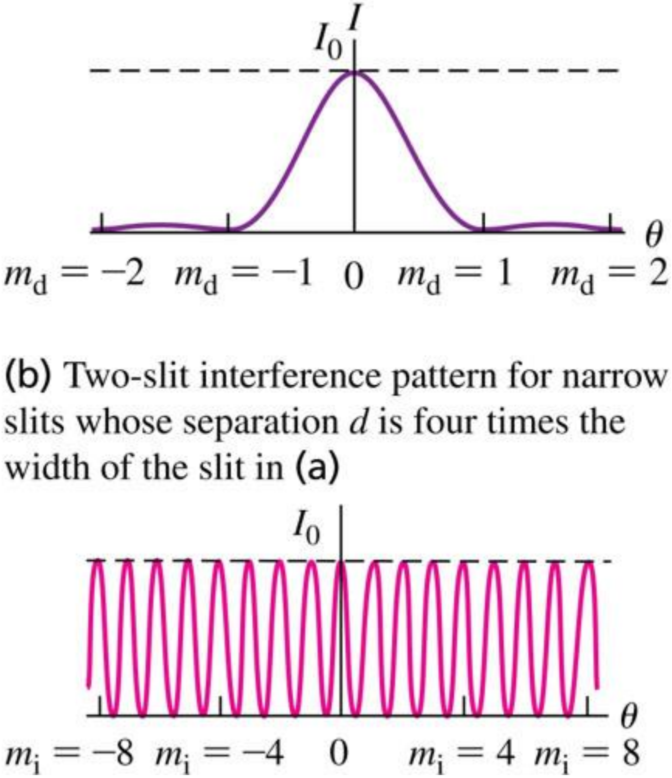
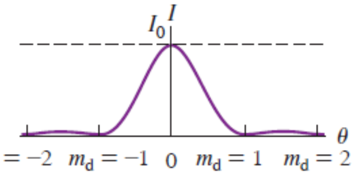
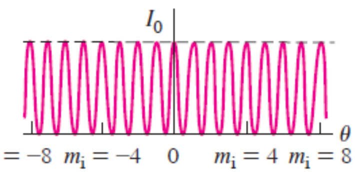


Figure 36.12 Finding the intensity pattern for two slits of finite width.

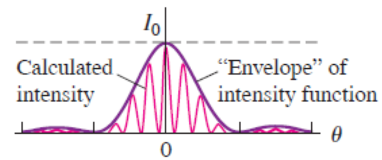
(a) Single-slit diffraction pattern for a slit width a



(b) Two-slit interference pattern for narrow slits whose separation d is four times the width of the slit in (a)



(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



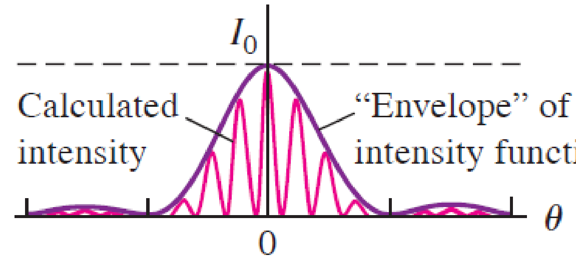
(d) Photograph of the pattern calculated in (c)

For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing.

Two Slits of Finite Width (2 of 2)

- Figure (c) shows the pattern from two slits with width *a*, separated by a distance (between centers) $d = 4a$.
- The two-slit peaks are in the same positions as before, but their intensities are modulated by the single-slit pattern, which acts as an "envelope" for the intensity function.
- Figure (d) shows the pattern, which is both from diffraction *and* interference.

(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects

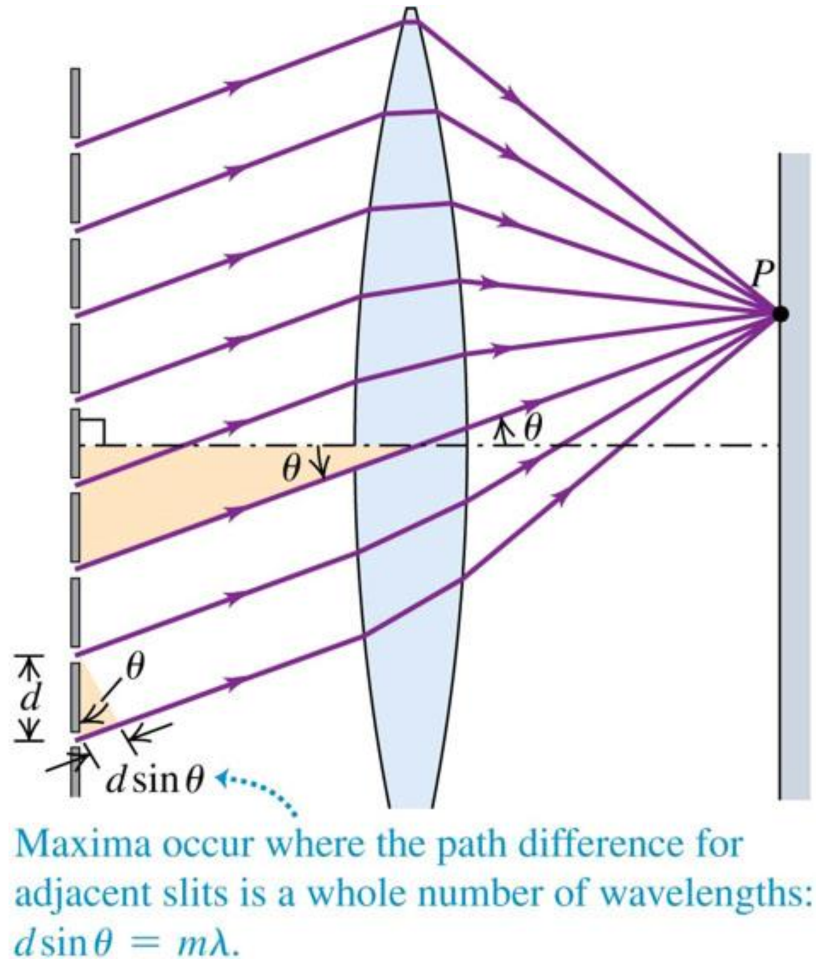


(d) Photograph of the pattern calculated in (c)

For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing.

Several Slits

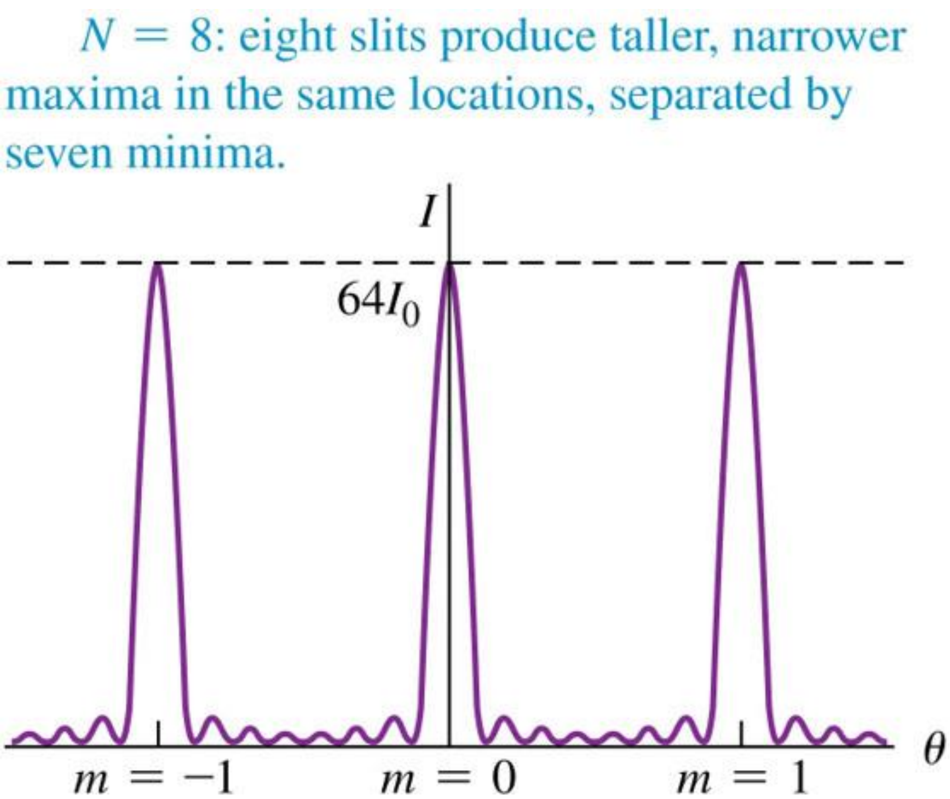
- Shown is an array of eight narrow slits, with distance *d* between adjacent slits.
- Constructive interference occurs for rays at angle θ to the normal that arrive at point *P* with a path difference between adjacent slits equal to an integer number of wavelengths.



$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Interference Pattern of Several Slits (1 of 2)

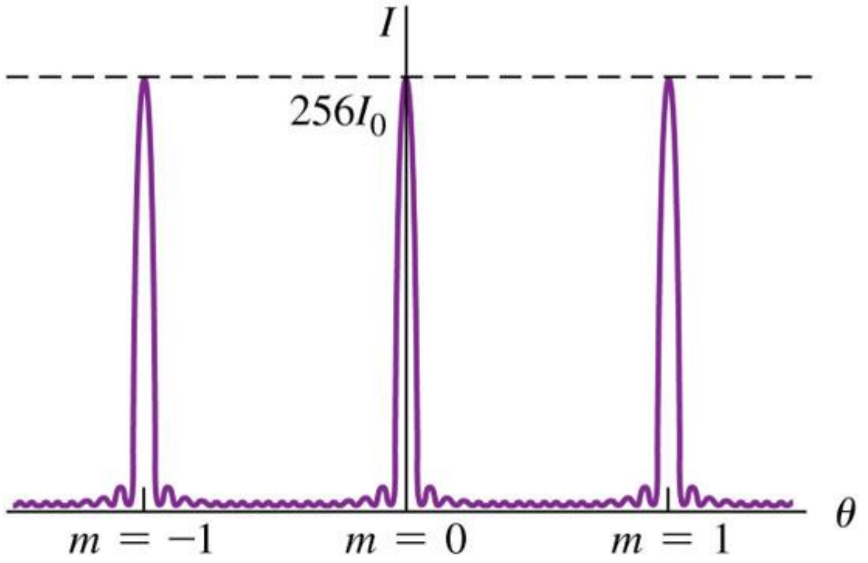
- Shown is the result of a detailed calculation of the eight-slit pattern.
- The large maxima, called principal maxima, are in the same positions as for a two-slit pattern, but are much narrower.



Interference Pattern of Several Slits (2 of 2)

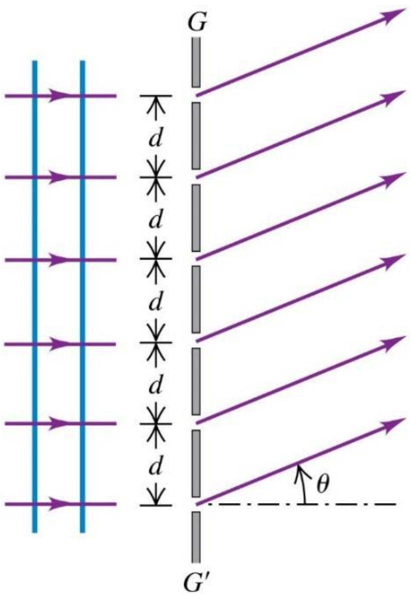
- Shown is the result for 16 slits.
- The height of each principal maximum is proportional to N^2, so from energy conservation, the width of each principal maximum must be proportional to $\frac{1}{N}$.

$N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



The Diffraction Grating

- An array of a large number of parallel slits is called a **diffraction grating**.
- In the figure, GG' is a cross section of a transmission grating.
- The slits are perpendicular to the plane of the page.
- The diagram shows only six slits; an actual grating may contain several thousand.



Distance between slits   Wavelength

Intensity maxima, multiple slits: $d\sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$

Angle of line from center of slit array to mth bright region on screen

- Video Tutor Solution: Example 36.4

EXAMPLE 36.4 Width of a grating spectrum

The wavelengths of the visible spectrum are approximately 380 nm (violet) to 750 nm (red). (a) Find the angular limits of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating. (b) Do the first-order and second-order spectra overlap? What about the second-order and third-order spectra? Do your answers depend on the grating spacing?

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EXECUTE (a) The grating spacing is

$$d = \frac{1}{600 \text{ slits/mm}} = 1.67 \times 10^{-6} \text{ m}$$

We solve Eq. (36.13) for θ:

$$\theta = \arcsin \frac{m\lambda}{d}$$

Then for $m = 1$, the angular deviations θ_{v1} and θ_{r1} for violet and red light, respectively, are

$$\theta_{v1} = \arcsin\left(\frac{380 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = 13.2°$$

$$\theta_{r1} = \arcsin\left(\frac{750 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = 26.7°$$

(b) With $m = 2$ and $m = 3$, our equation $\theta = \arcsin(m\lambda/d)$ for 380 mm violet light yields

$$\theta_{v2} = \arcsin\left(\frac{2(380 \times 10^{-9}\ \text{m})}{1.67 \times 10^{-6}\ \text{m}}\right) = 27.1°$$

$$\theta_{v3} = \arcsin\left(\frac{3(380 \times 10^{-9}\ \text{m})}{1.67 \times 10^{-6}\ \text{m}}\right) = 43.0°$$

For 750 nm red light, this same equation gives

$$\theta_{r2} = \arcsin\left(\frac{2(750 \times 10^{-9}\ \text{m})}{1.67 \times 10^{-6}\ \text{m}}\right) = 63.9°$$

$$\theta_{r3} = \arcsin\left(\frac{3(750 \times 10^{-9}\ \text{m})}{1.67 \times 10^{-6}\ \text{m}}\right) = \arcsin(1.35) = \text{undefined}$$

The Reflection Grating

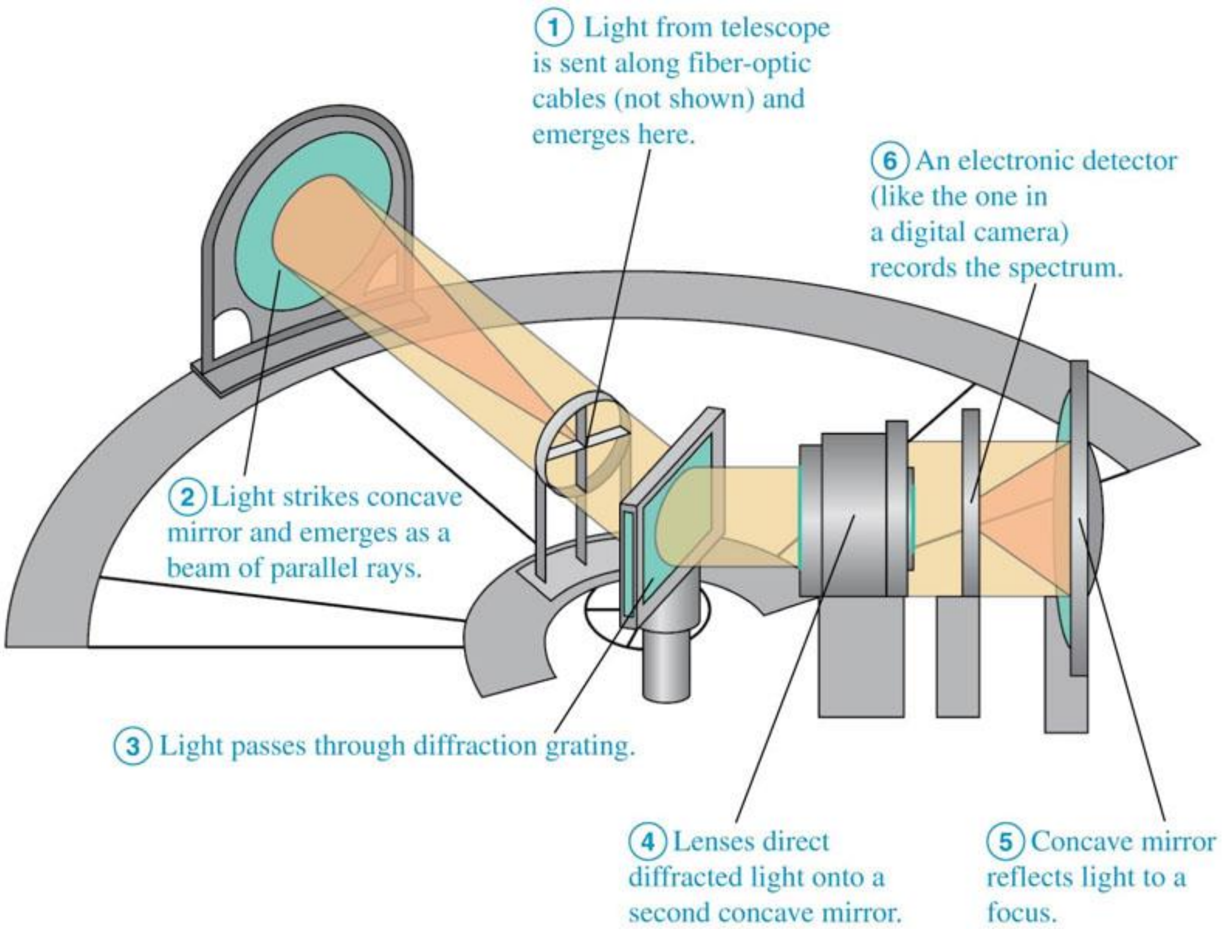
- The rainbow-colored reflections from the surface of a DVD are a reflection-grating effect.
- The “grooves” are tiny pits 0.12 mm deep in the surface of the disc, with a uniform radial spacing of 0.74 mm = 740 nm.
- Information is coded on the DVD by varying the length of the pits.
- The reflection-grating aspect of the disc is merely an aesthetic side benefit.

Resolution of a Grating Spectrograph

In spectroscopy it is often important to distinguish slightly differing wavelengths. The minimum wavelength difference $\Delta\lambda$ that can be distinguished by a spectrograph is described by the **chromatic resolving power** R, defined as

$$R = \frac{\lambda}{\Delta\lambda} \qquad \text{(chromatic resolving power)} \qquad (36.14)$$

Diagram of a Grating Spectrograph



Resolution of a Grating Spectrograph

- In spectroscopy it is often important to distinguish slightly differing wavelengths.
- The minimum wavelength difference $\Delta\lambda$ that can be distinguished by a spectrograph is described by the **chromatic resolving power** *R*.
- For a grating spectrograph with a total of *N* slits, used in the m^{th} order, the chromatic resolving power is:

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

From our discussion in Section 36.4 the mth-order maximum occurs when the phase difference ϕ for adjacent slits is $\phi = 2\pi m$. The first minimum beside that maximum occurs when $\phi = 2\pi m + 2\pi/N$, where N is the number of slits. The phase difference is also given by $\phi = (2\pi d \sin\theta)/\lambda$, so the angular interval $d\theta$ corresponding to a small increment $d\phi$ in the phase shift can be obtained from the differential of this equation:

$$d\phi = \frac{2\pi d \cos\theta \, d\theta}{\lambda}$$

When $d\phi = 2\pi/N$, this corresponds to the angular interval $d\theta$ between a maximum and the first adjacent minimum. Thus $d\theta$ is given by

$$\frac{2\pi}{N} = \frac{2\pi d \cos\theta \, d\theta}{\lambda} \qquad \text{or} \qquad d \cos\theta \, d\theta = \frac{\lambda}{N}$$

Now we need to find the angular spacing $d\theta$ between maxima for two slightly different wavelengths. The positions of these maxima are given by $d \sin\theta = m\lambda$, and the differential of this equation gives

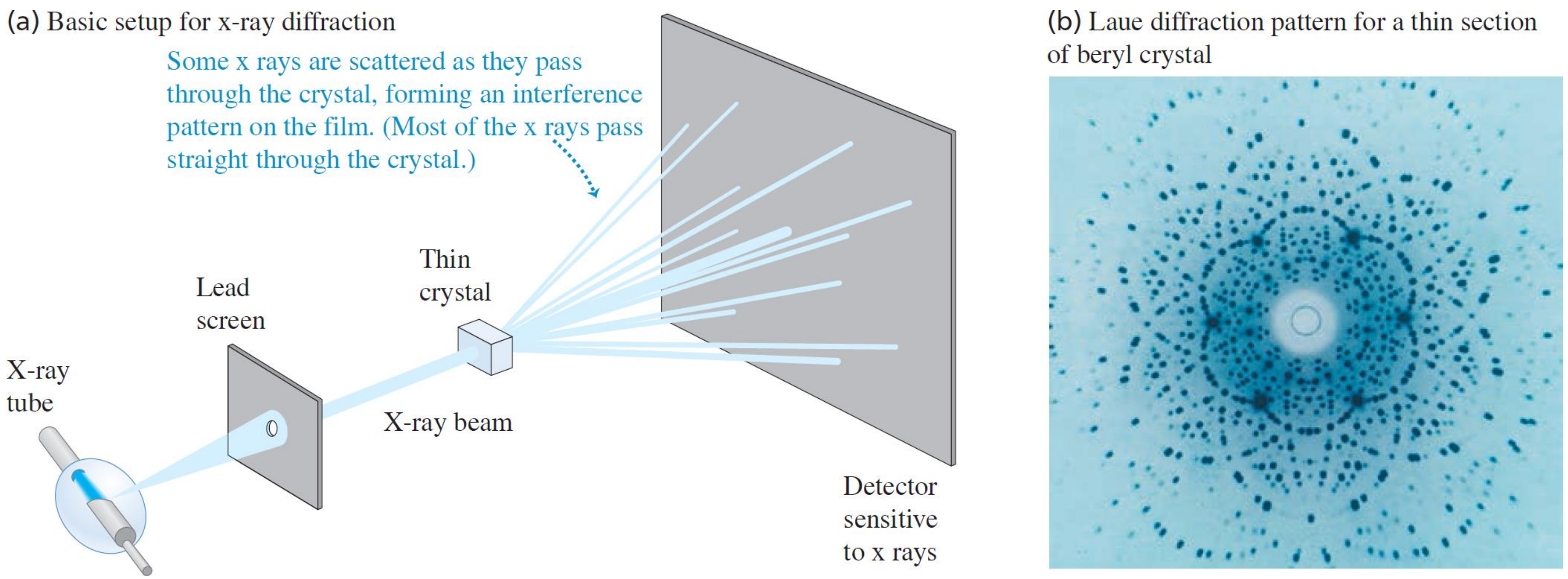
$$d \cos\theta \, d\theta = m \, d\lambda$$

According to our criterion, the limit or resolution is reached when these two angular spacings are equal. Equating the two expressions for the quantity $(d \cos\theta \, d\theta)$, we find

$$\frac{\lambda}{N} = m \, d\lambda \qquad \text{and} \qquad \frac{\lambda}{d\lambda} = Nm$$

X-Ray Diffraction

- When x rays pass through a crystal, the crystal behaves like a diffraction grating, causing **x-ray diffraction**.



- Video Tutor Solution: Example 36.5

Pearson

Figure 36.21 Model of the arrangement of ions in a crystal of NaCl (table salt). The spacing of adjacent atoms is 0.282 nm. (The electron clouds of the atoms actually overlap slightly.)

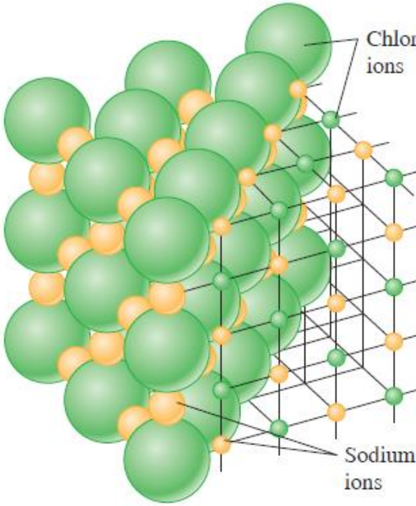
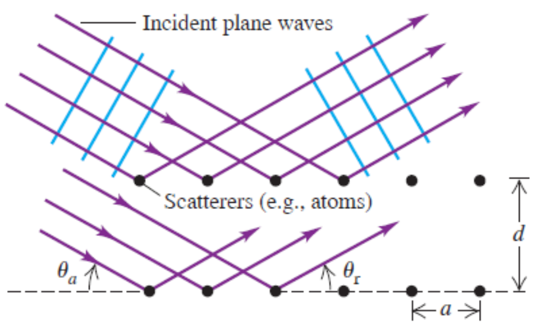


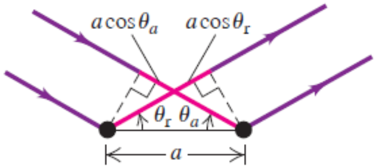
Figure 36.22 A two-dimensional model of scattering from a rectangular array. The distance between adjacent atoms in a horizontal row is a; the distance between adjacent rows is d. The angles in (b) are measured from the *surface* of the array, not from its normal.

(a) Scattering of waves from a rectangular array



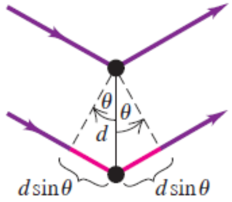
(b) Scattering from adjacent atoms in a row

Interference from adjacent atoms in a row is constructive when the path lengths $a\cos\theta_a$ and $a\cos\theta_r$ are equal, so that the angle of incidence θ_a equals the angle of reflection (scattering) θ_r.



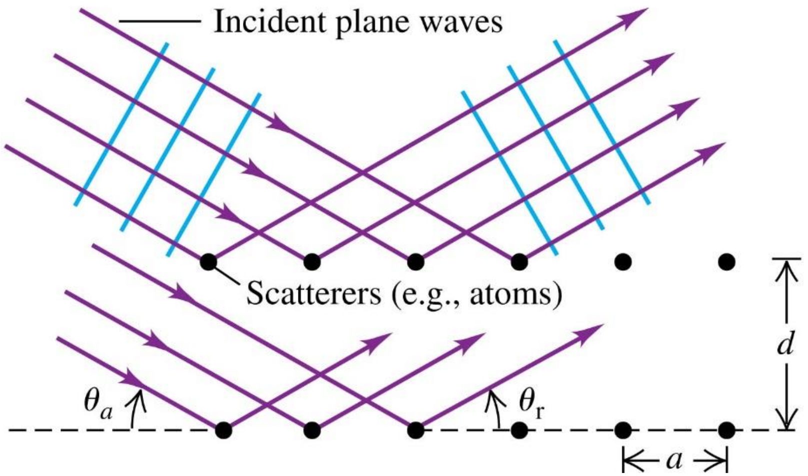
(c) Scattering from atoms in adjacent rows

Interference from atoms in adjacent rows is constructive when the path difference $2d\sin\theta$ is an integer number of wavelengths, as in Eq. (36.16).



A Simple Model of X-Ray Diffraction

- To better understand x-ray diffraction, we consider a two-dimensional scattering situation.
- The path length from source to observer is the same for all the scatterers in a single row if $\theta_a = \theta_r = \theta$.



Bragg condition for constructive interference from an array:

Distance between adjacent rows in array

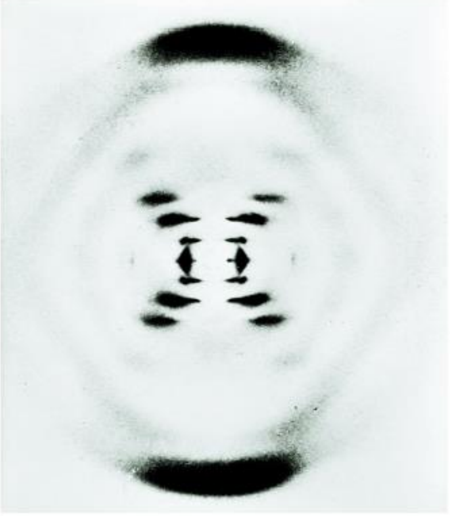
Wavelength

$$2d\sin\theta = m\lambda \quad (m = 1, 2, 3, \ldots)$$

Angle of line from surface of array to *m*th bright region on screen

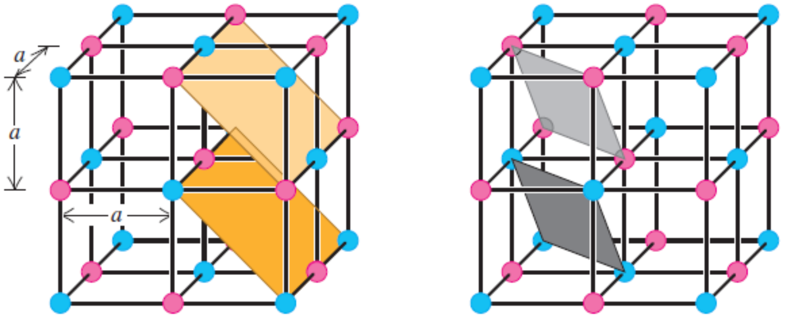
CAUTION **Bragg *reflection* is really Bragg *interference*** While we are using the term *reflection*, remember that we are dealing with an *interference* effect. The reflections from various planes are closely analogous to interference effects in thin films (see Section 35.4).

Figure 36.24 The British scientist Rosalind Franklin made this groundbreaking x-ray diffraction image of DNA in 1953. The dark bands arranged in a cross provided the first evidence of the helical structure of the DNA molecule.



(a) Spacing of planes is $d = a/\sqrt{2}$.

(b) Spacing of planes is $d = a/\sqrt{3}$.



EXAMPLE 36.5 X-ray diffraction

You direct a beam of 0.154 nm x rays at certain planes of a silicon crystal. As you increase the angle of incidence of the beam from zero, the first strong interference maximum occurs when the beam makes an angle of 34.5° with the planes. (a) How far apart are the planes? (b) Will you find other interference maxima from these planes at larger angles of incidence?

EXECUTE (a) We solve Eq. (36.16) for d and set $m = 1$:

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.154\text{ nm})}{2\sin 34.5^\circ} = 0.136\text{ nm}$$

This is the distance between adjacent planes.

(b) To calculate other angles, we solve Eq. (36.16) for $\sin\theta$:

$$\sin\theta = \frac{m\lambda}{2d} = m\frac{0.154\text{ nm}}{2(0.136\text{ nm})} = m(0.566)$$

Values of m of 2 or greater give values of $\sin\theta$ greater than unity, which is impossible. Hence there are *no* other angles for interference maxima for this particular set of crystal planes.

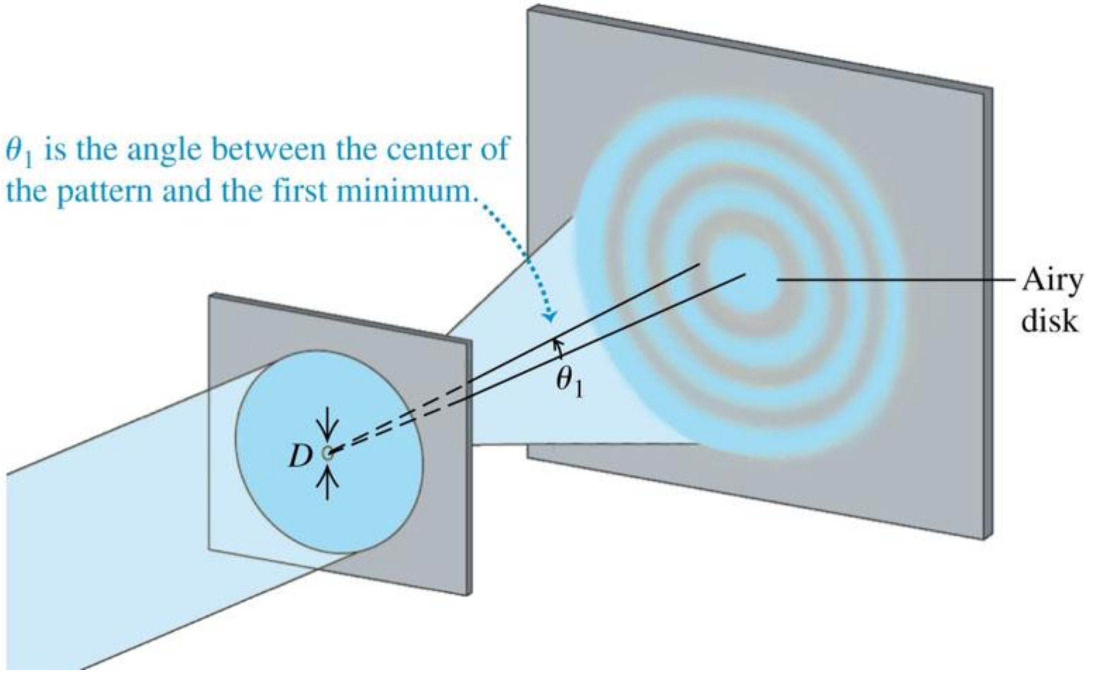
TEST YOUR UNDERSTANDING OF SECTION 36.6 You are doing an x-ray diffraction experiment with a crystal in which the atomic planes are 0.200 nm apart. You are using x rays of wavelength 0.0900 nm. What is the highest-order maximum present in the diffraction pattern? (i) Third; (ii) fourth; (iii) fifth; (iv) sixth; (v) seventh.

(ii) The angular position of the mth maximum is given by Eq. (36.16), $2d\sin\theta = m\lambda$. This gives $m = (2d\sin\theta)/\lambda$. The sine function can never be greater than 1, so the largest value of m in the pattern can be no greater than $2d/\lambda = 2(0.200\text{ nm})/(0.0900\text{ nm}) = 4.44$. Since m must be an integer, the highest-order maximum in the pattern is $m = 4$ (fourth order). The $m = 5, 6, 7, \ldots$ maxima do not appear.

ANSWER

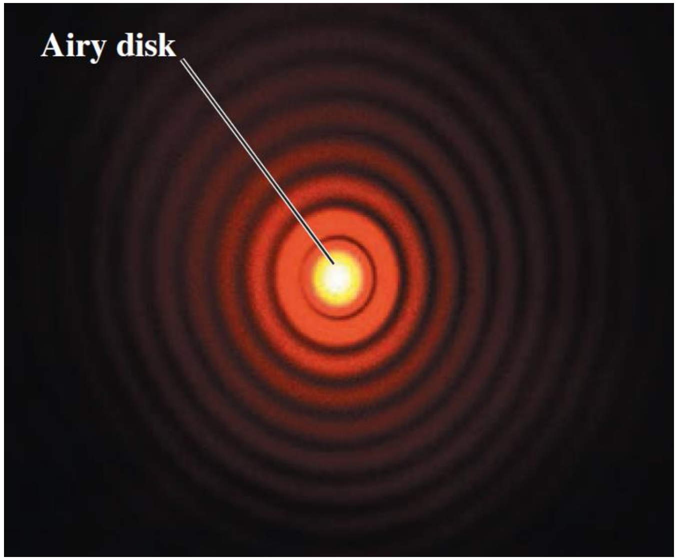
Circular Apertures

- The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings.



Diffraction by a Circular Aperture

- The central bright spot in the diffraction pattern of a circular aperture is called the Airy disk.
- We can describe the radius of the Airy disk by the angular radius θ_1 of the first dark ring:



Diffraction by a circular aperture:

Angular radius of first dark ring = angular radius of Airy disk

$$\sin\theta_1 = 1.22\frac{\lambda}{D}$$

λ: Wavelength

D: Aperture diameter

The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings, as **Fig. 36.25** shows. We can describe the pattern in terms of the angle θ, representing the angular radius of each ring. The angular radius θ_1 of the first *dark* ring is given by

Diffraction by a circular aperture:

$$\underbrace{\sin\theta_1}_{\text{Angular radius of first dark ring = angular radius of Airy disk}} = 1.22\frac{\overbrace{\lambda}^{\text{Wavelength}}}{\underbrace{D}_{\text{Aperture diameter}}} \qquad (36.17)$$

The angular radii of the next two dark rings are given by

$$\sin\theta_2 = 2.23\frac{\lambda}{D} \qquad \sin\theta_3 = 3.24\frac{\lambda}{D} \qquad (36.18)$$

The central bright spot is called the **Airy disk,** in honor of Sir George Airy (1801–1892), who first derived the expression for the intensity in the pattern. The angular radius of the Airy disk is that of the first dark ring, given by Eq. (36.17). The angular radii of the first three *bright* rings outside the Airy disk are

$$\sin\theta = 1.63\frac{\lambda}{D}, \qquad 2.68\frac{\lambda}{D}, \qquad 3.70\frac{\lambda}{D} \qquad (36.19)$$

vA camera lens with focal length $f = 50$ mm and maximum aperture $f/2$ forms an image of an object 9.0 m away. (a) If the resolution is limited by diffraction, what is the minimum distance between two points on the object that are barely resolved? What is the corresponding distance between image points? (b) How does the situation change if the lens is "stopped down" to $f/16$? Use $\lambda = 500$ nm in both cases.

EXECUTE (a) The aperture diameter is $D = f/(f\text{-number}) = (50\text{ mm})/2 = 25\text{ mm} = 25 \times 10^{-3}$ m. From Eq. (36.17) the angular separation θ of two object points that are barely resolved is

$$\theta \approx \sin\theta = 1.22\frac{\lambda}{D} = 1.22\frac{500 \times 10^{-9}\text{ m}}{25 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-5}\text{ rad}$$

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$$\theta \approx \sin\theta = 1.22\frac{\lambda}{D} = 1.22\frac{500 \times 10^{-9}\text{ m}}{25 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-5}\text{ rad}$$

We know from our thin-lens analysis in Section 34.4 that, apart from sign, $y/s = y'/s'$ [see Eq. (34.14)]. Thus the angular separations of the object points and the corresponding image points are both equal to θ. Because the object distance s is much greater than the focal length $f = 50$ mm, the image distance s' is approximately equal to f. Thus

We know from our thin-lens analysis in Section 34.4 that, apart from sign, $y/s = y'/s'$ [see Eq. (34.14)]. Thus the angular separations of the object points and the corresponding image points are both equal to θ. Because the object distance s is much greater than the focal length $f = 50$ mm, the image distance s' is approximately equal to f. Thus

$$\frac{y}{9.0\text{ m}} = 2.4 \times 10^{-5} \qquad y = 2.2 \times 10^{-4}\text{ m} = 0.22\text{ mm}$$

$$\frac{y'}{50\text{ mm}} = 2.4 \times 10^{-5} \qquad y' = 1.2 \times 10^{-3}\text{ mm}$$

$$= 0.0012\text{ mm} \approx \tfrac{1}{800}\text{ mm}$$

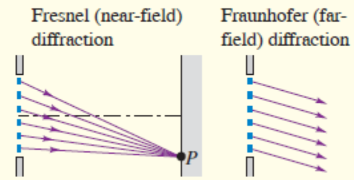
(b) The aperture diameter is now $(50\ \text{mm})/16$, or one-eighth as large as before. The angular separation between barely resolved points is eight times as great, and the values of y and y' are also eight times as great as before:

$$y = 1.8\ \text{mm} \qquad y' = 0.0096\ \text{mm} = \tfrac{1}{100}\ \text{mm}$$

Only the best camera lenses can approach this resolving power.

CHAPTER 36 SUMMARY

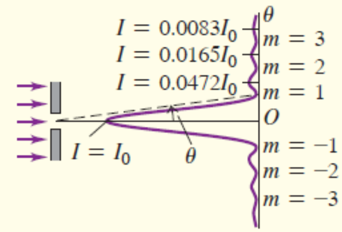
Fresnel and Fraunhofer diffraction: Diffraction occurs when light passes through an aperture or around an edge. When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction. When the source or the observer is relatively close to the obstructing surface, it is Fresnel diffraction.



Single-slit diffraction: Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen. Equation (36.2) gives the condition for destructive interference (a dark fringe) at a point P in the pattern at angle θ. Equation (36.7) gives the intensity in the pattern as a function of θ. (See Examples 36.1–36.3.)

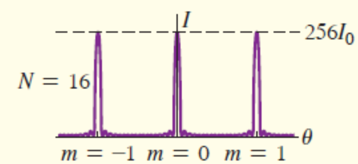
$$\sin\theta = \frac{m\lambda}{a} \qquad (36.2)$$
$$(m = \pm 1, \pm 2, \pm 3, \ldots)$$

$$I = I_0 \left\{ \frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda} \right\}^2 \qquad (36.7)$$



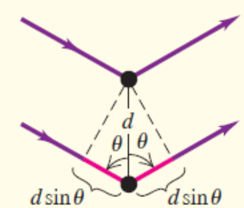
Diffraction gratings: A diffraction grating consists of a large number of thin parallel slits, spaced a distance d apart. The condition for maximum intensity in the interference pattern is the same as for the two-source pattern, but the maxima for the grating are very sharp and narrow. (See Example 36.4.)

$$d\sin\theta = m\lambda \qquad (36.13)$$
$$(m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$



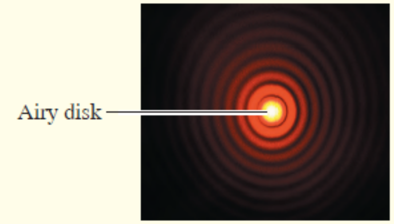
X-ray diffraction: A crystal serves as a three-dimensional diffraction grating for x rays with wavelengths of the same order of magnitude as the spacing between atoms in the crystal. For a set of crystal planes spaced a distance d apart, constructive interference occurs when the angles of incidence and scattering (measured from the crystal planes) are equal and when the Bragg condition [Eq. (36.16)] is satisfied. (See Example 36.5.)

$$2d\sin\theta = m\lambda \qquad (36.16)$$
$$(m = 1, 2, 3, \ldots)$$



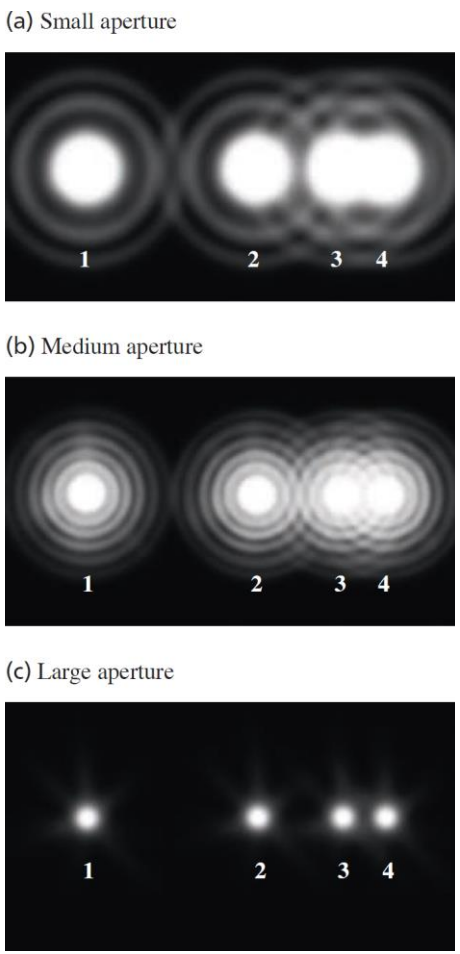
Circular apertures and resolving power: The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings. Equation (36.17) gives the angular radius θ_1 of the first dark ring, equal to the angular size of the Airy disk. Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. According to Rayleigh's criterion, two point objects are just barely resolved when their angular separation θ is given by Eq. (36.17). (See Example 36.6.)

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \qquad (36.17)$$



Diffraction and Image Formation

- Diffraction limits the resolution of optical equipment, such as telescopes.
- The larger the aperture, the better the resolution.
- A widely used criterion for resolution of two point objects, is called **Rayleigh's criterion**:
  - Two objects are just barely resolved (that is, distinguishable) if the center of one diffraction pattern coincides with the first minimum of the other.



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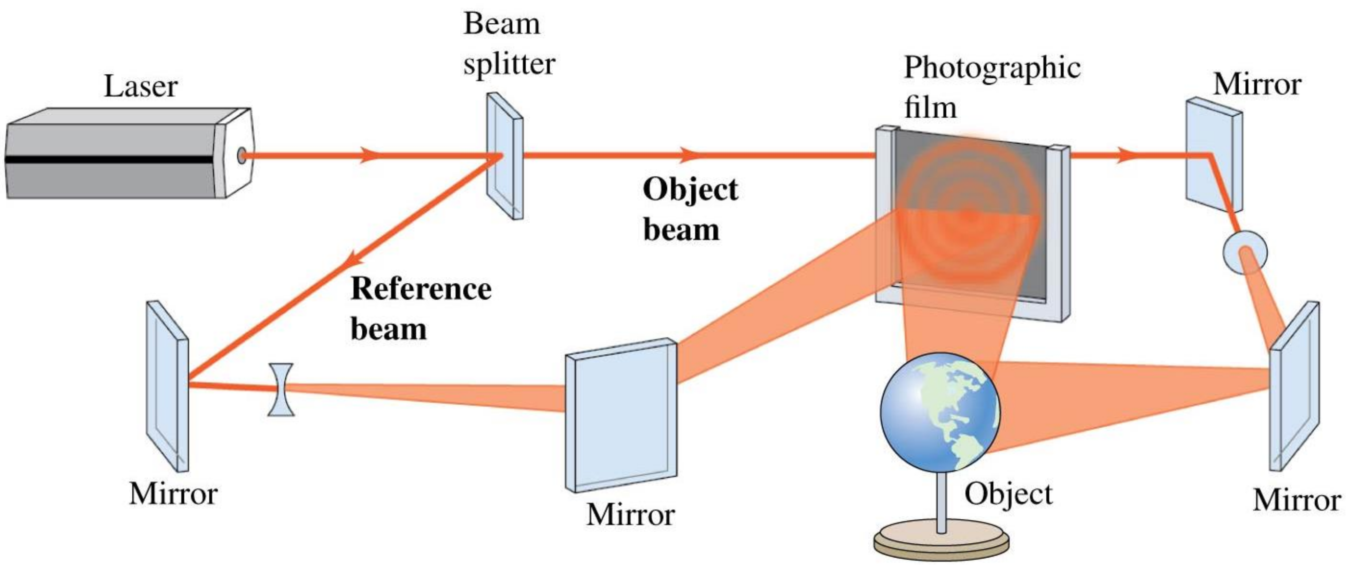
Bigger Telescope, Better Resolution

- Because of diffraction, large-diameter telescopes, such as the VLA radio telescope below, give sharper images than small ones.

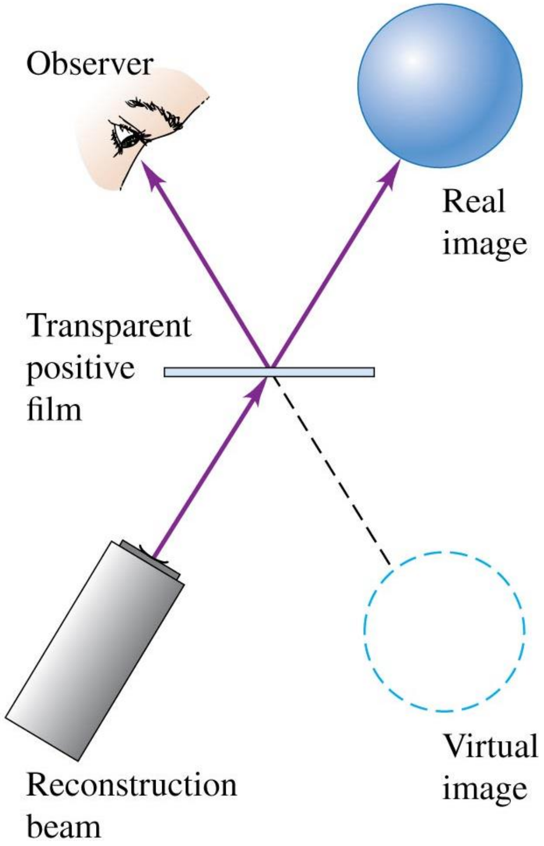
What Is Holography?

- By using a beam splitter and mirrors, coherent laser light illuminates an object from different perspectives.
- Interference effects provide the depth that makes a three-dimensional image from two-dimensional views.



Viewing the Hologram

- A hologram is the record on film of the interference pattern formed with light from the coherent source and light scattered from the object.
- Images are formed when light is projected through the hologram.
- The observer sees the virtual image formed behind the hologram.



An Example of Holography

- Shown below are photographs of a holographic image from two different angles, showing the changing perspective.