### Chapter 2

## Motion Along a Straight Line

PowerPoint<sup>®</sup> Lectures for *University Physics, Thirteenth Edition* – Hugh D. Young and Roger A. Freedman

**Lectures by Wayne Anderson** 

Copyright © 2012 Pearson Education Inc.

#### **Goals for Chapter 2**

- To describe straight-line motion in terms of velocity and acceleration
- To distinguish between average and instantaneous velocity and average and instantaneous acceleration
- To interpret graphs of position versus time, velocity versus time, and acceleration versus time for straight-line motion
- To understand straight-line motion with constant acceleration
- To examine freely falling bodies
- To analyze straight-line motion when the acceleration is not constant

#### Introduction

- *Kinematics* is the study of motion.
- *Velocity* and *acceleration* are important physical quantities.
- A bungee jumper speeds up during the first part of his fall and then slows to a halt.



#### **Displacement, time, and average velocity**

- A particle moving along the *x*-axis has a coordinate *x*.
- The change in the particle's coordinate is  $\Delta x = x_2 x_1$ .
- The average x-velocity of the particle is  $v_{av-x} = \Delta x / \Delta t$ .



**Average Velocity** 

# The **average velocity** is rate at which the displacement occurs

$$V_{x,avg} \equiv \frac{\Delta X}{\Delta t} = \frac{X_f - X_i}{\Delta t}$$

- The x indicates motion along the x-axis
- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position time graph

#### **Negative velocity**

• The average *x*-velocity is *negative* during a time interval if the particle moves in the negative *x*-direction for that time interval.



#### A position-time graph—Figure 2.3

- A position-time graph (an *x*-*t* graph) shows the particle's position *x* as a function of time *t*.
- The average *x*-velocity is the slope of a line connecting the corresponding points on an *x*-*t* graph.



#### **Average Speed**

Speed is a scalar quantity • same units as velocity  $S_{avg} \equiv -\frac{1}{2}$ • total distance / total time: The speed has no direction and is always expressed as a positive number • The *instantaneous velocity* is the velocity at a specific instant of time or specific point along the path and is given by

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The *instantaneous velocity* can be positive, negative, or zero
- The instantaneous speed is the magnitude of the instantaneous velocity. The instantaneous speed has no direction associated with it.

#### Finding velocity on an *x*-*t* graph

• At any point on an x-t graph, the instantaneous xvelocity is equal to the slope of the tangent to the curve at that point.



160 m

t(s)

5

#### **Motion diagrams**

Figure below shows the *x*-*t* graph and the motion diagram for a moving particle.



The steeper the slope (positive or negative) of an object's *x*-*t* graph, the greater is the object's speed in the positive or negative *x*-direction.

#### **Average Acceleration**

Acceleration is the rate of change of the velocity

$$\boldsymbol{a}_{x,avg} \equiv \frac{\Delta \boldsymbol{V}_x}{\Delta t} = \frac{\boldsymbol{V}_{xf} - \boldsymbol{V}_{xi}}{t_f - t_i}$$

Dimensions are L/T<sup>2</sup>

SI units are m/s<sup>2</sup>

In one dimension, positive and negative can be used to indicate direction

Copyright © 2012 Pearson Education Inc.

The instantaneous acceleration is the limit of the average acceleration as  $\Delta t$  approaches 0

$$a_{x} = \lim_{\Delta t \to 0} \frac{\Delta V_{x}}{\Delta t} = \frac{dV_{x}}{dt} = \frac{d^{2}x}{dt^{2}}$$

The term acceleration will mean instantaneous acceleration

The slope of the velocitytime graph is the acceleration

The slope of the green line represents the instantaneous acceleration

The slope of the blue line is the average acceleration



© 2007 Thomson Higher Education

#### Finding acceleration on a $v_x$ -t graph

• As shown in Figure 2.12, the  $v_x$ -t graph may be used to find the instantaneous acceleration and the average acceleration.



#### **Graphical Representations of Motion**



© 2007 Thomson Higher Education

#### **Kinematic: Motion with Constant Acceleration**

For constant *a*, the velocity is a linear function of time

$$V_{xf} = V_{xi} + a_x t$$

Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration

• Assumes 
$$t_i = 0$$
 and  $t_f = t$ 

For constant acceleration, the average velocity can be expressed as the arithmetic mean of the initial and final velocities

$$V_{x,avg} = \frac{V_{xi} + V_{xf}}{2}$$

#### **Kinematic Equations, specific**

If 
$$x_i = 0$$
 then  $\Delta x = x$   $x = v_{avr}^* t$  and  

$$x = \frac{V_{xi} + V_{xf}}{2} t = \frac{V_{xi} + V_{xi}^* + at}{2} t = V_{xi} t + \frac{1}{2} at^2$$

By substituting the value of *t* from  $V_{xf} = V_{xi} + a_x t$ into  $v_{xi}t + \frac{1}{2}at^2$  we can obtain additional expression for displacement:  $V_{xi}^2 - V_{xi}^2$ 

$$\mathbf{x} = \frac{\mathbf{v}_{xf}^2 - \mathbf{v}_{xi}^2}{2a}$$

#### The equations of motion with constant acceleration

• The four equations shown to the right apply to any straightline motion with constant acceleration  $a_x$ .

 $v_{x} = v_{0x} + a_{x}t$   $x = x_{0} + v_{0x}t + \frac{1}{2}a_{x}t^{2}$   $v_{x}^{2} = v_{0x}^{2} + 2a_{x}(x - x_{0})$ 

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$$

#### **Graphical Motion with Constant Acceleration**

- (a) The position time graph is parabola.
- (b) The velocity-time graph is a straight line, the slope of which is the acceleration
- (c) The acceleration time graph is a straight line with a slope of zero



$$x(t) = x_i + v_i^* t + \frac{1}{2} a t^2$$

$$v(t) = v_i + at$$



A European sports car dealer claims that his car will accelerate at a constant rate from rest to 100 km/hr in 8.00 s. If so, what is the acceleration? (*Hint:* First convert speed to m/s.)

$$100 \frac{km}{h} x \frac{1000 m}{1 km} x \frac{1 h}{3600 s} = 27.78 m/s \quad v = 27.78 m/s \quad v_i = 0$$
  
t = 8.00s

$$(v - v_i)/t = a$$

Example

3.47 m/s<sup>2</sup>

 $x(t) = x_i + v_i^* t + \frac{1}{2} a t^2$ 

### Example



 A car travels 105 m with constant deceleration of 2.4 m/s<sup>2</sup> in 3 sec. What is the initial velocity of the car ?

$$\frac{x + \frac{1}{2}at^{2}}{t} = \frac{v_{i}t}{105 m + \frac{1}{2}x2.4m/s^{2}x(3s)^{2}}{3 s} = 38.6 \text{ m/s}$$

#### **Example:**Average and instantaneous velocities

A cheetah is crouched 20 m to the east of the observer. At time t=0 the cheetah begins to run due east toward an antelope that is 50 m from the observer. The cheetah's coordinate x varies with time according to the equation  $x = 20m + (5m/s^2)t^2$ . (a) Find the cheetah's displacement between 1 s and 2 s. (b) Find its average velocity during this interval. (c) Find its instaneous velocity at t = 1s. (a) The situation Cheetah  $V_{1x} = Q$ Vehicle Antelope starts (b) Our sketch  $\times$ (m) x1=?  $X_{2} = ?$  $x_0 = 20.0 \, \text{m}$ 50.0 m  $t_1 = 1.0 s$  $t_2 = 2.0 \text{ s}$ t = 0 $\Delta x = \hat{c}$ Vav-x = **2**) Place origin **1**) Point axis in (**3**) Mark initial (4) Mark positions (5) Add the known (c) Decisions for cheetah at 1 s direction cheetah runs. at vehicle. positions of cheetah and unknown

and antelope.

and 2 s.

quantities.

so that all values will

be positive.

#### **Example: A motorcycle with constant acceleration**

A motorcyclist heading east through a town accelerates at constant 4 m/s<sup>2</sup> after he leaves the city limits. At time t=0 he is 5 m east of the city-limits signpost, moving east at 15 m/s. (a) Find his position and velocity at t = 2 s. (b) Where is he when his velocity is 25 m/s



#### **Freely falling bodies**

- *Free fall* is the motion of an object under the influence of only gravity.
- In the figure, a strobe light flashes with equal time intervals between flashes.
- The velocity change is the same in each time interval, so the acceleration is constant.



We will neglect air resistance

Free fall motion is constantly accelerated motion in one dimension

Let upward be positive

Use the kinematic equations with  $a_v = -g =$ 

-9.80 m/s<sup>2</sup>

#### A freely falling coin

- Aristotle thought that heavy bodies fall faster than light ones, but Galileo showed that all bodies fall at the *same* rate.
- If there is no air resistance, the downward acceleration of any freely falling object is g = 9.8 $m/s^2 = 32$  ft/s<sup>2</sup>.



**Free Fall – equations: a = -g** 

$$v_f = v_i + at$$
  $v_f = v_i - g - t$ 

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
  $y_f = y_i + v_i \cdot t - \frac{1}{2} g t^2$ 

$$x_{f} = x_{i} + \frac{v_{f}^{2} - v_{i}^{2}}{2a}$$
  $y_{f} = y_{i} + \frac{v_{f}^{2} - v_{i}^{2}}{-2g}$ 

#### **Example: Up-and-down motion in free fall**

You throw a ball vertically upward from the roof of a 24 m tall building with a speed of 15 m/s. on its way back down, it just misses the railing. (a) Find the ball's position and velocity 1.00 and 4.00 s after leaving your hand. (b) At what time after being released has the ball fallen 5 m below the roof railing? © Find its velocity just before it hits the ground.



#### Free Fall -- object thrown upward

A ball is thrown upward from the ground ( $y_i = 0$ ) with initial velocity  $v_i$ . Find the maximum height reached by the ball.

Solution:

$$y = \frac{v_f^2 - v_i^2}{-2g}$$

The instantaneous velocity at the maximum height is zero

The maximum height is

$$y_{max} = \frac{v_i^2}{2g}$$



#### Is the acceleration zero at the highest point?—Figure 2.25

• The vertical velocity, but *not the acceleration*, is zero at the highest point.

(a) y-t graph (curvature is downward because  $a_y = -g$ is negative)



(b)  $v_v$ -t graph (straight line with

negative slope because  $a_v = -g$ 

is constant and negative)

#### **Velocity and position by integration**

- The acceleration of a car is not always constant.
- The motion may be integrated over many small time intervals to give  $v_x = v_{ox} + \int_0^t a_x dt$  and  $x = x_0 + \int_0^t v_x dt$ .





Total area under the x-t graph from  $t_1$  to  $t_2$ = Net change in x-velocity from  $t_1$  to  $t_2$ 

#### Motion with changing acceleration

- Follow Example 2.9.
- Figure 2.29 illustrates the motion graphically.



#### **Two bodies with different accelerations**

• Follow Example 2.5 in which the police officer and motorist have different accelerations.



(b)