Physics 121 - Electricity and Magnetism
Lecture 11 - Faraday's Law of Induction
Y&F Chapter 29, Sect. 1-5

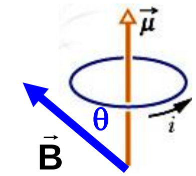
- **Magnetic Flux**
- **Motional EMF: moving wire in a B field**
- **Two Magnetic Induction Experiments**
- **Faraday's Law of Induction**
- **Lenz's Law**
- **Rotating Loops – Generator Principle**
- **Concentric Coils – Transformer Principle**
- **Induction and Energy Transfers**
- **Induced Electric Fields**
- **Summary**

Previously:

Magnetic fields produce forces and torques on charges and currents

$\vec{F}_{mag} = q\vec{v}\times\vec{B}$ $\vec{F}_m = i\vec{L}\times\vec{B}$ $\vec{\mu} \equiv NiA\hat{n}$ $\vec{\tau} = \vec{\mu}\times\vec{B}$ $U_m = -\vec{\mu}\circ\vec{B}$

Force on charge and wire carrying current torque and potential energy of a dipole

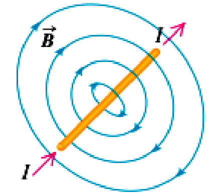


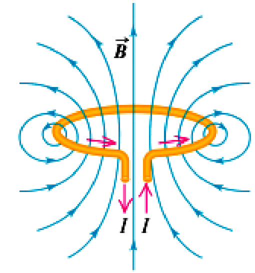
Currents (changing electric fields) produce magnetic fields

Biot-Savart Law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2}$ Ampere's Law $\oint \vec{B}\circ d\vec{s} = \mu_0 i_{enc}$ Current loops are elementary dipoles

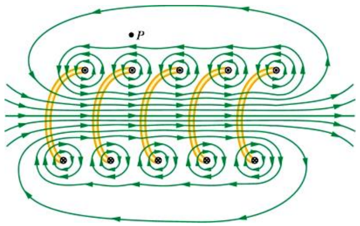
B due to long straight wire carrying a current i: $B = \frac{\mu_0 i}{2\pi r}$

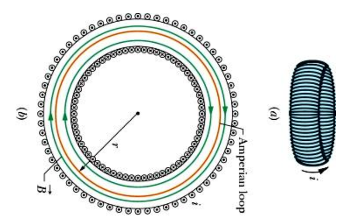
B due to circular loop carrying a current i : $B = \frac{\mu_0 i}{2R}$





B inside a solenoid: $B = \mu_0 in$ B inside a torus carrying a current i : $B = \frac{\mu_0 iN}{2\pi\, r}$





Now: Changing magnetic flux induces EMFs and currents in wires

Magnetic Flux: defined analogously to flux of electric field

Electrostatic Gauss Law

$$\int_S \vec{E} \circ d\vec{A} = q_{enc}/\varepsilon_0$$

Magnetic Gauss Law

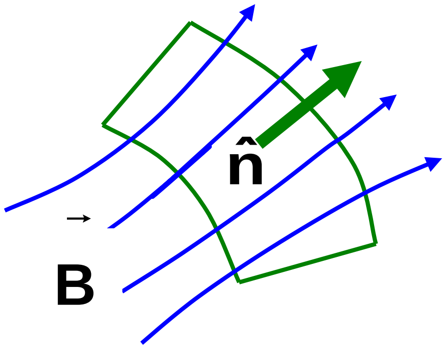
$$\int_S \vec{B} \circ d\vec{A} = 0$$

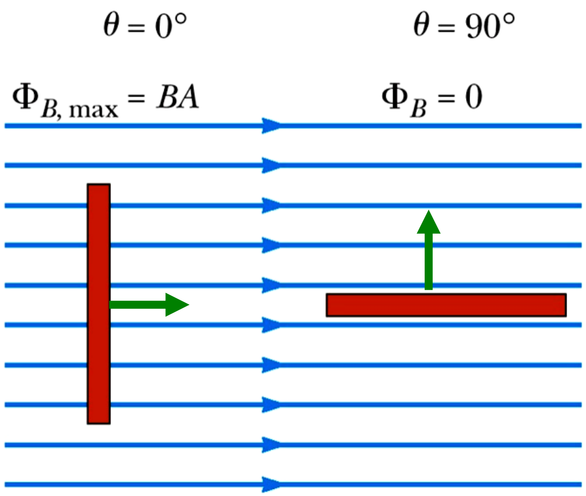
$$d\Phi_B \equiv \vec{B} \circ d\vec{A} = \vec{B} \circ \hat{n} dA$$

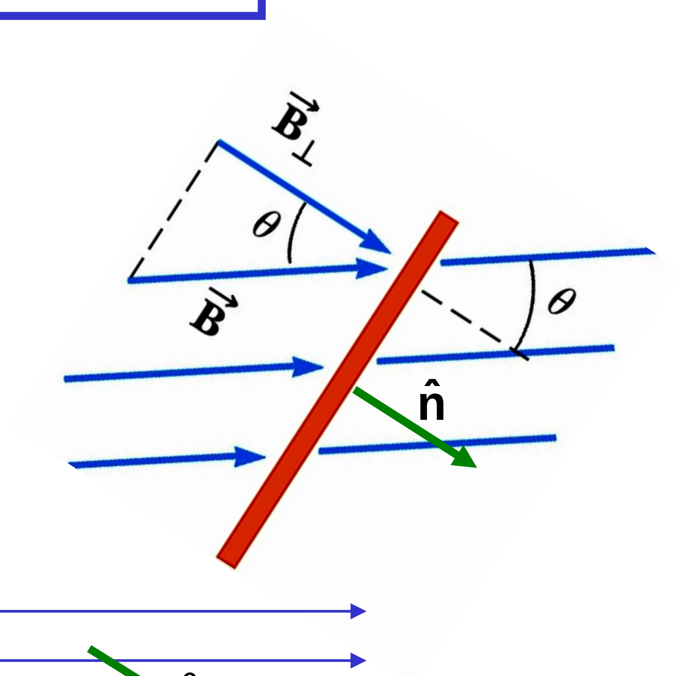
over surface (open or closed)

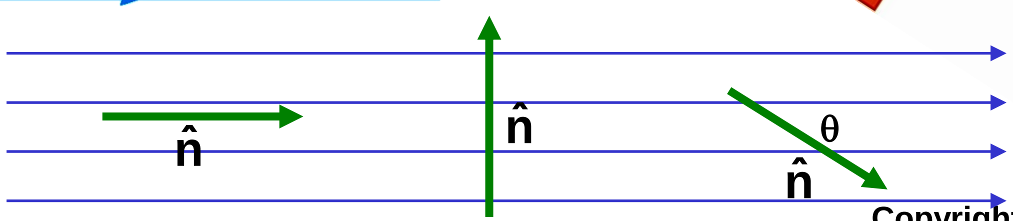
$$\Phi_B \equiv \int \vec{B} \circ d\vec{A}$$

Flux Unit: 1 Weber = 1 T.m ^{2}



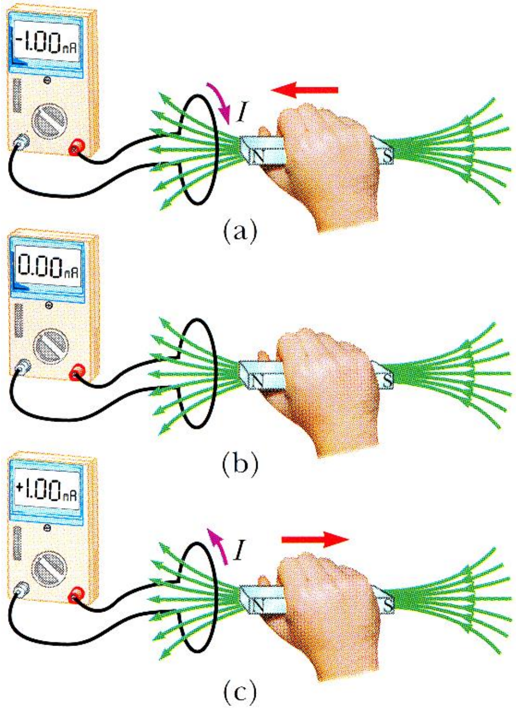






Induced EMF from changing magnetic flux

- Induced EMF/current appears only if there is relative **motion** between the loop and the magnet - the magnetic field inside the loop is **changing**
- Induced current stops when the relative motion stops (case b).
- Faster motion produces a larger current.
- Induced current direction reverses when magnet motion revereses direction (case c versus case a)
- Any relative motion that changes the flux works



EMF/current is induced in the loop whenever magnetic flux through the loop is changing.

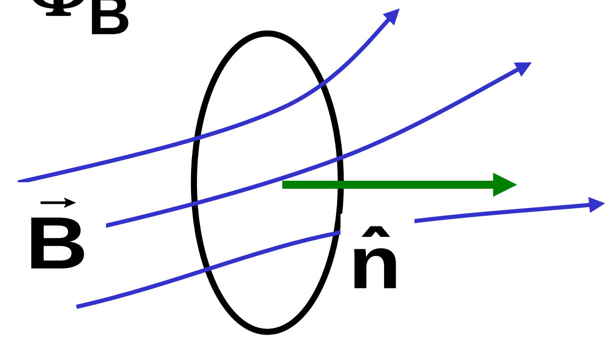
Induced current creates it's own induced B field, opposing the change in flux Φ_B (Lenz' Law)

CHANGING magnetic flux induces EMFs and currents in wires

Key Concepts:

- Magnetic flux Φ_B

$$\Delta\Phi_B \equiv \vec{B} \circ \Delta\vec{A}$$



- Faraday's Law of Induction

$$\mathcal{E} = \text{induced EMF} = -\frac{d\Phi_B}{dt}$$

- Lenz's Law

The induced EMF and current creates it's own magnetic field, opposing change in the existing flux

Generator principle:

- Loops rotating in B field generate EMF and current.
- Need to apply torque due to Lenz's law and energy conservation

How does induction work?

The force on moving charges can produce Induced EMFs

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\,\vec{E}$$

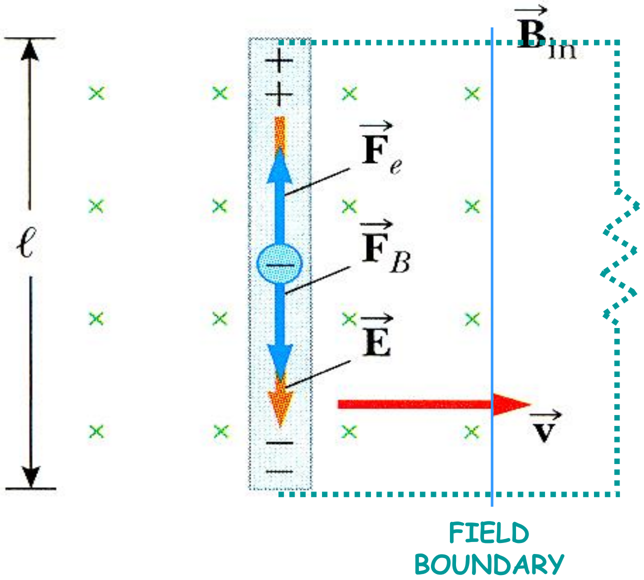
- Uniform magnetic field points into the slide.
- Wire of length ℓ moves with constant velocity v perpendicular to the field
- Electrons feel a magnetic force and migrate to the lower end of the wire. Upper end becomes positive.
- Result is an induced electric field E_{ind} inside wire
- Charges come to equilibrium when the forces on charges balance:

$$qE_{ind} = qvB \quad \text{or} \quad E_{ind} = vB$$

- Electric field E_{ind} in the wire corresponds to potential difference across the ends of wire:

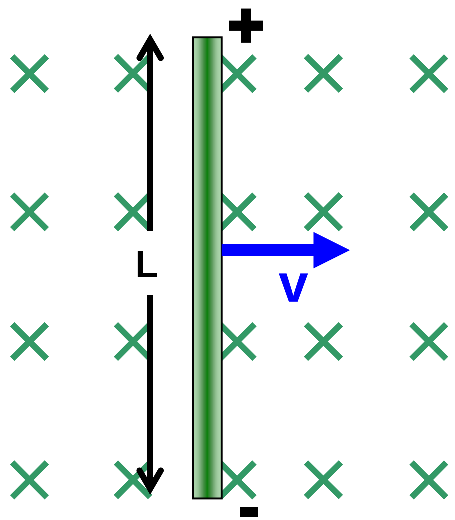
$$\mathcal{E}_{ind} = -E_{ind}\ell = -B\ell v$$

- Potential difference $\mathcal{E}_{ind}$ is maintained between the ends of the wire as long as the wire continues to move through the magnetic field.

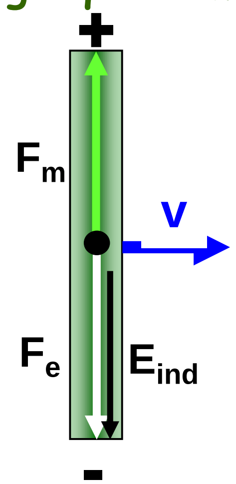


- ***EMF is induced*** **even through no batteries are present**
- **Current flows if there is a complete circuit.**
- **Such a current is an *induced current*.**

Induced EMF is proportional to the rate of change of flux



FBD of free charge q in a wire



- Separated charges induce field E_{ind} inside wire due to the motion.
- Charges come to equilibrium when $F_e = F_m$
- No current flows (yet) as circuit is not complete

$$F_m = qvB = F_e = qE_{ind}$$

The induced EMF in a moving wire is

$$\mathcal{E}_{ind} = -E_{ind}L = -BLv$$

Corresponding flux argument:

The rate of flux change = field B x the rate of sweeping out area

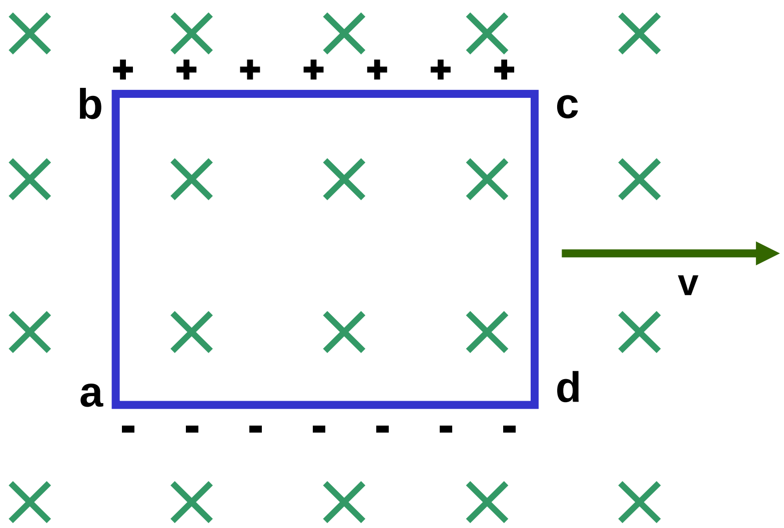
$$\text{Rate of sweeping out area} = dA/dt = Ldx/dt = Lv$$

$$\text{so} \quad \mathcal{E}_{ind} = -BLv = -B\frac{dA}{dt} = -\frac{d}{dt}(BA) = -\frac{d\Phi}{dt}$$

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

changing flux

Suppose a **loop** is moving in a uniform field



DOES CURRENT FLOW?

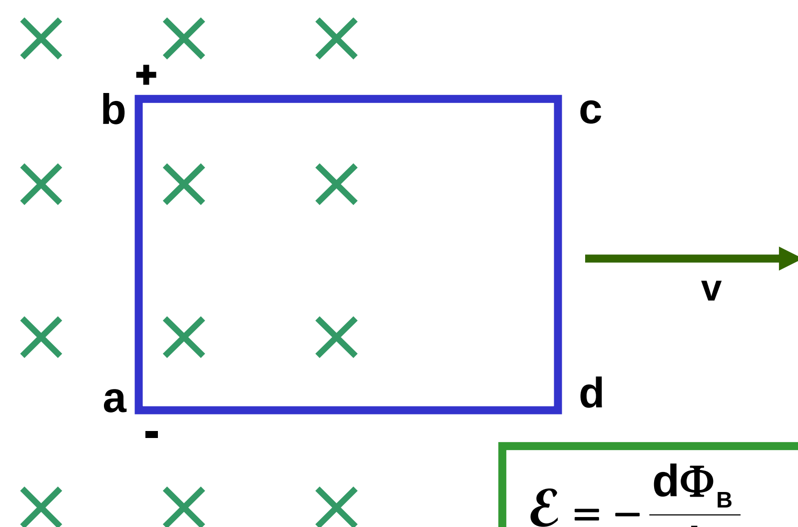
- If $\underline{B}$ field is uniform, segments a-b & d-c create equal but opposed EMFs in circuit
- No EMF from b-c & a-d
- or
- FLUX is constant
- No current flows

$$\frac{d\Phi_m}{dt} = 0$$

DOES CURRENT FLOW NOW?

- B field ends or is not uniform
- Segment c-d now creates NO EMF
- Segment a-b creates EMF as above
- Un-balanced EMF drives current just like a battery
- FLUX is DECREASING

$$\frac{d\Phi_m}{dt} < 0$$



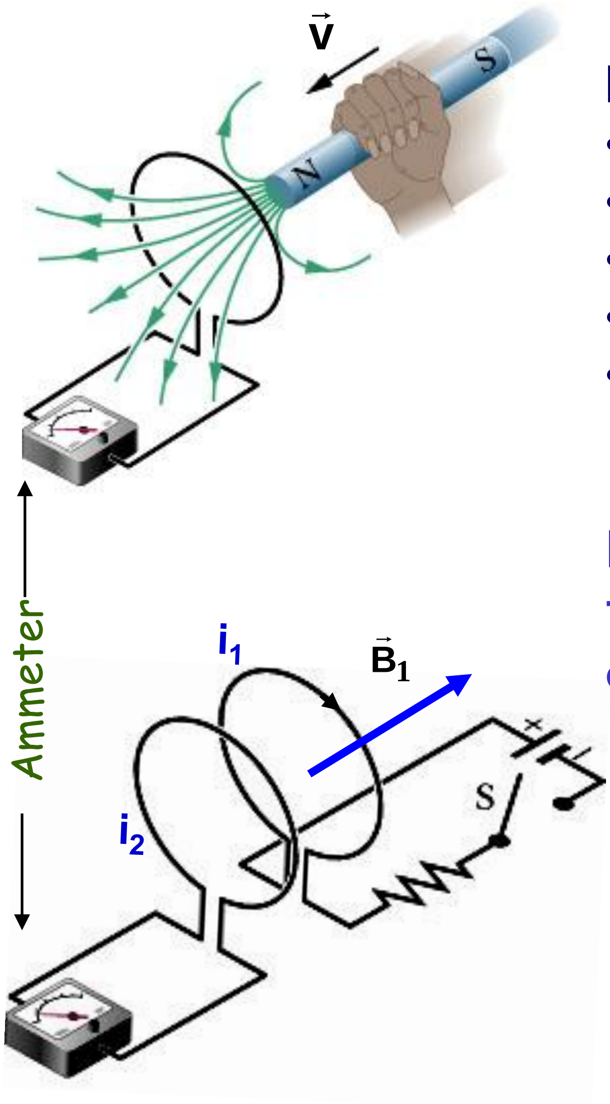
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

changing flux

Which way does current flow above?

What is different when loop is entering field?

Direction of induced fields and currents



Replace magnet with circuit below

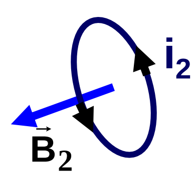
- Do not neet actual motion to cause induction
- Flux change is enough
- Let current i_1 be changing -> changing flux.
- Flux is constant if current is constant
- Current i_2 flows only while i_1 (flux Φ_1) is changing (after switch S closes or opens)

Induced current i_2 creates it's own induced field B_2 whose flux F_2 opposes the *change* in Φ_1 (Lenz' Law)

$F_1 = B_1A$ $F_2 = B_2A$

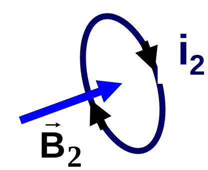
Close S

$$\frac{dB_1}{dt} > 0 \quad (B_1 \text{ growing})$$



Open S later

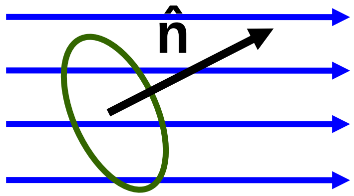
$$\frac{dB_1}{dt} < 0 \quad (B_1 \text{ decreasing})$$



Induced Dipoles

Faraday's Law

uniform B



$$\Phi_m = \vec{B} \circ \hat{n} A$$

"-" sign for Lenz's Law

Induced EMF (Volts)

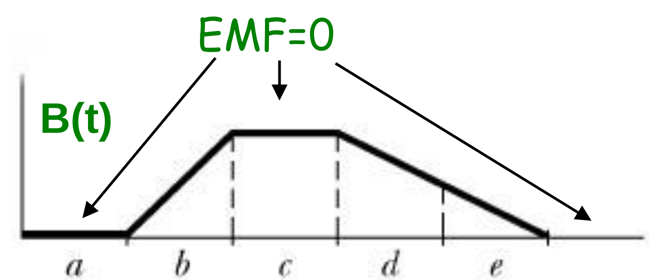
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

Rate of flux change (Webers/sec)

insert N for multiple turns in loop, loop of any shape

Change Flux by:

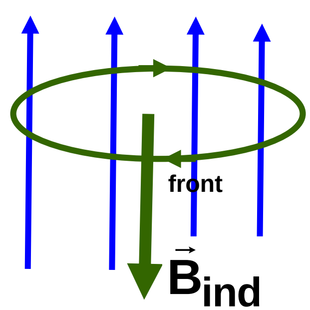
- changing |B| through a coil
- changing area of a coil or loop
- changing angle between B and coil
  e.g., rotating coils → generator effect



slope → EMF

Example: B through a loop increases by 0.1 Tesla in 1 second: Loop area $A = 10^{-3}\ m^2$. Find the induced EMF

uniform dB/dt



$$\mathcal{E}_{ind} = -\frac{\Delta\Phi_B}{\Delta t} = -A\frac{\Delta B}{\Delta t} = -\frac{0.1 \times 10^{-3}}{1}$$

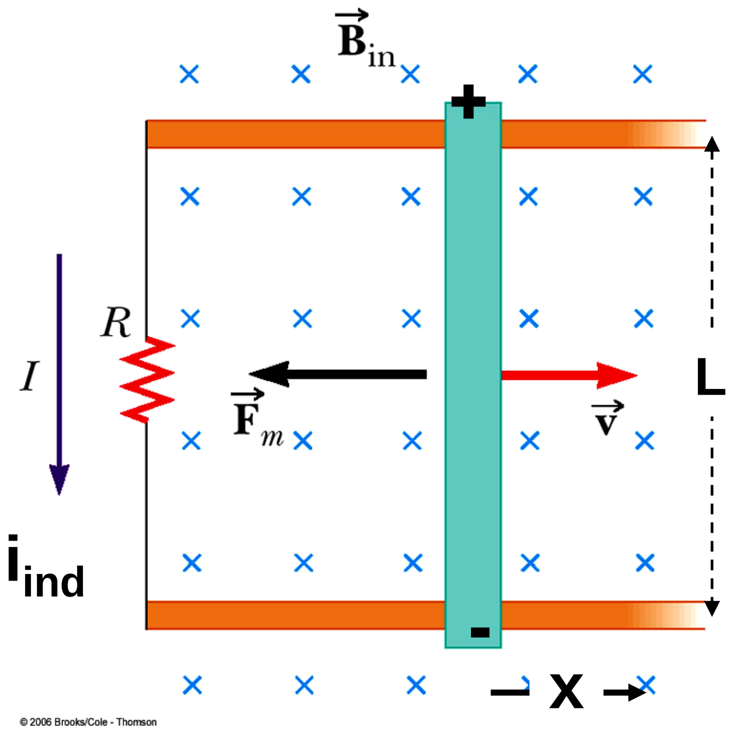
$$\mathcal{E}_{ind} = -10^{-4}\ \text{volts}$$

$$i_{ind} = \mathcal{E}_{ind} / R$$

Current i_{ind} creates field B_{ind} that opposes increase in B

Energy Transfer

Slider moves, increasing loop area (flux)



Induced EMF:

- Constant speed v to right
- Uniform field into the page
- Area & therefore FLUX are increasing

$$\mathcal{E}_{ind} = -\frac{d\Phi_m}{dt} = -B\frac{dA}{dt} = -BL\frac{dx}{dt} = -BLv$$

- Slider acts like a battery
- Lenz's Law says induced field B_{ind} is out of page.
- RH rule says current is CCW

$$i_{ind} = \mathcal{E}_{ind}/R \qquad R = \text{total resistance in circuit}$$

$$\vec{F}_m = i_{ind}\vec{L}\times\vec{B} \Rightarrow |F_m| = i_{ind}LB$$

F_M is a DRAG FORCE

- due to the induced current
- opposes the motion of the wire

F_{EXT} (an external force) is needed to keep wire from slowing down

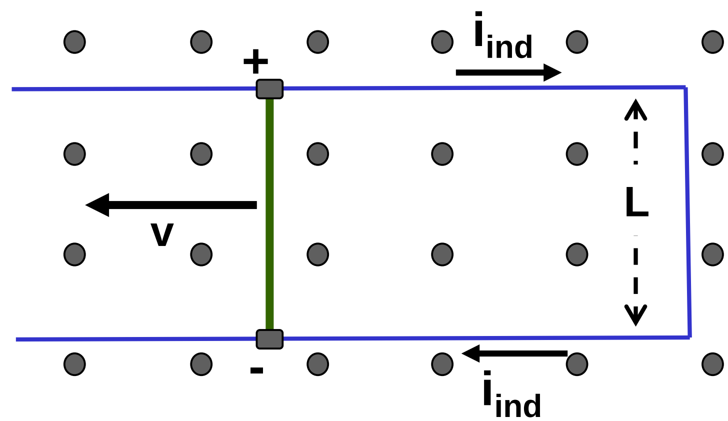
F_{EXT} needed is opposed to F_m

$$F_{ext} = F_m = i_{ind}LB = \frac{\mathcal{E}_{ind}}{R}LB = \frac{B^2L^2v}{R}$$

Power supplied & dissipated:

$$\text{Power} = F_{ext}v = \frac{B^2L^2v^2}{R} = \frac{\mathcal{E}_{ind}^{\,2}}{R}$$

Numerical Example



B = 0.35 T
L = 25 cm
v = 55 cm/s

a) Find the EMF generated:

$$\mathcal{E}_{ind} = -\frac{d\Phi_m}{dt} = -BLv = -(0.35)(.25)(.55) = -48 \text{ mV}$$

DIRECTION: B_{ind} is into slide, i_{ind} is clockwise

b) Find the induced current if R for the whole loop = 18 Ω:

$$i_{ind} = \frac{\mathcal{E}_{ind}}{R} = 48 \text{ mV} / 18\,\Omega = 2.67 \text{ mA} \quad \text{clockwise}$$

c) Find the thermal power dissipated:

$$P = \frac{\mathcal{E}_{ind}^2}{R} = \frac{(48 \times 10^3)^2}{18} = 1.28 \times 10^{-4} \text{ Watts}$$

$$P = i_{ind}^2 R = (2.67 \times 10^{-3})^2 \times 18 = 1.28 \times 10^{-4} \text{ Watts}$$

d) Find the power needed to move slider at constant speed

$$P_{mech} = Fv = iLBv = 2.67 \times 10^{-3} \times 0.25 \times 0.35 \times 0.55 = 1.28 \times 10^{-4} \text{ Watts}$$

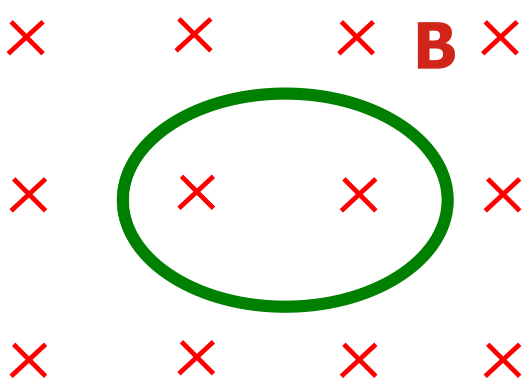
!!! Power dissipated via R = Mechanical power !!!

Induced Current and Emf

11 – 1: A circular loop of wire is in a uniform magnetic field covering the area shown. The plane of the loop is perpendicular to the field lines.

Which of the following will not cause a current to be induced in the loop?

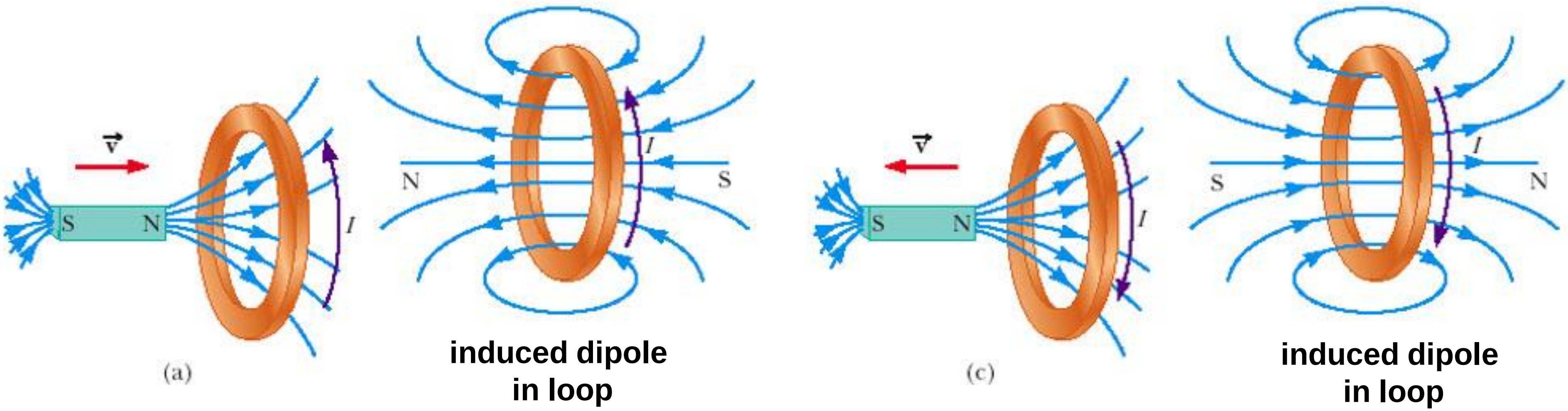
- **A. Sliding the loop into the field from the far left to right**
- **B. Rotating the loop about an axis perpendicular to the field lines.**
- **C. Keeping the orientation of the loop fixed and moving it along the field lines.**
- **D. Crushing the loop.**
- **E. Sliding the loop out of the field from left to right**



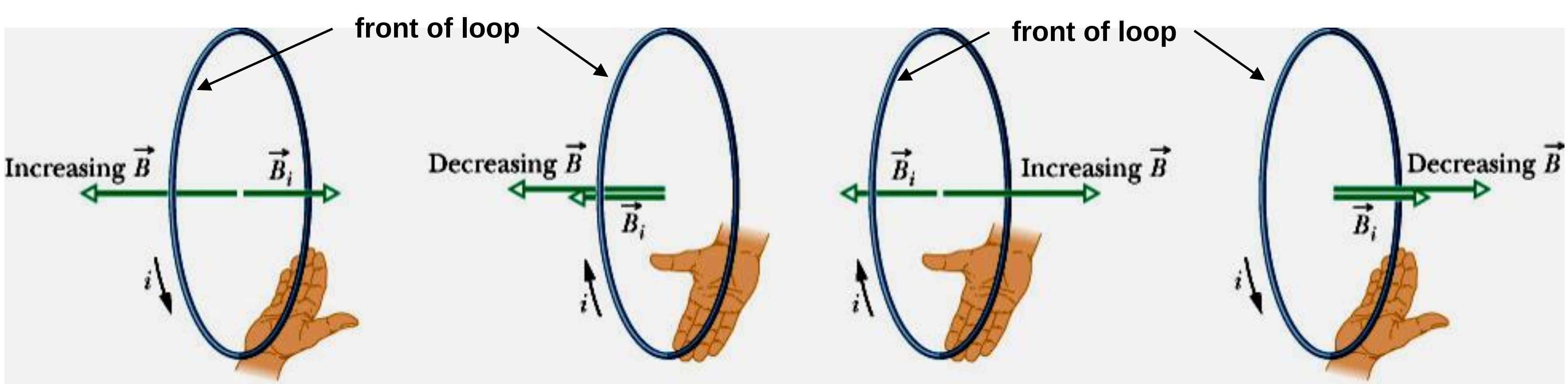
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\{BA\cos(\theta)\}$$

Lenz's Law

The induced current and EMF create induced magnetic flux that opposes the change in magnetic flux that created them



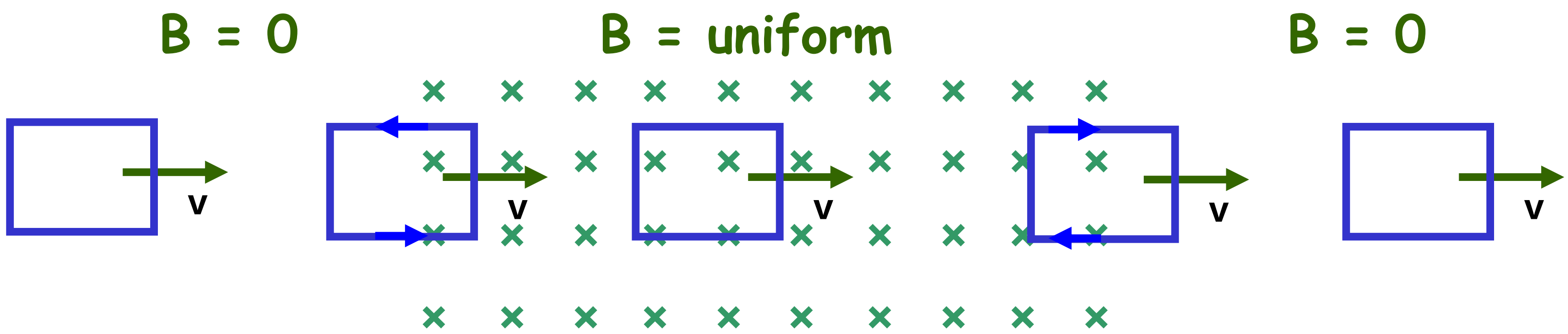
- **The induced current tends to keep the original magnetic flux through the loop from changing.**



Lenz's Law Example:

A loop crossing a region of uniform magnetic field

The induced current and EMF create induced magnetic flux that opposes the change in magnetic flux that created them



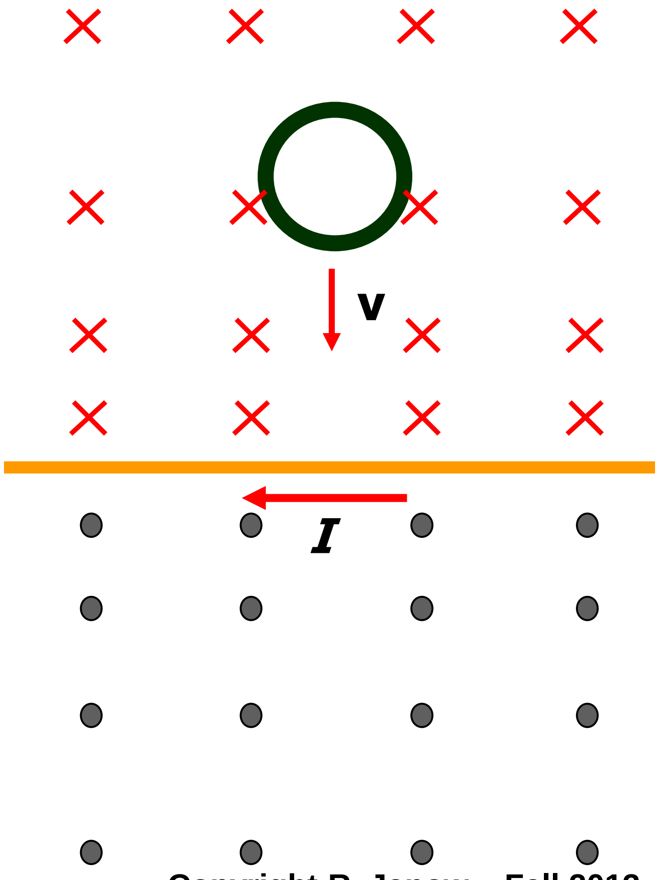
$\frac{d\Phi}{dt} = 0$	$\frac{d\Phi}{dt}$ is into slide	$\frac{d\Phi}{dt} = 0$	$\frac{d\Phi}{dt}$ is out of slide	$\frac{d\Phi}{dt} = 0$
$i_{ind} = 0$	B_{ind} is out	$i_{ind} = 0$	B_{ind} is down	$i_{ind} = 0$
	i_{ind} is CCW		i_{ind} is CW	

Direction of induced current

11-2: A circular loop of wire is falling toward a wire carrying a steady current to the left as shown

What is the direction of the induced current in the loop of wire?

A. Clockwise
B. Counterclockwise
C. Zero
D. Impossible to determine



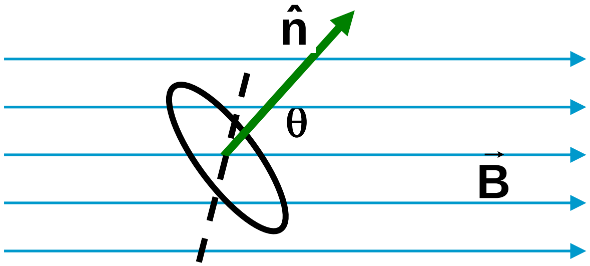
11-3: The loop continues falling until it is below the wire.

Now what is the direction of the induced current in the loop of wire?

A. Clockwise
B. Counterclockwise
C. Zero
D. Impossible to determine

Rotating Loop – Generator Effect (AC)

Loop rotating with angular velocity w = 2pf in B field:



Rotation axis is out of slide

$$\theta = \omega t \quad \text{i.e.,} \quad \hat{n} \text{ is along } \vec{B} \text{ when } t = 0$$

$$\Phi_B = \vec{B} \circ \hat{n} A = BA\cos(\theta) = \Phi_{max} \cos(\theta) = BA\cos(\omega t)$$

peak flux magnitude when ωt = 0, π, etc.

EMF induced is the time derivative of the flux

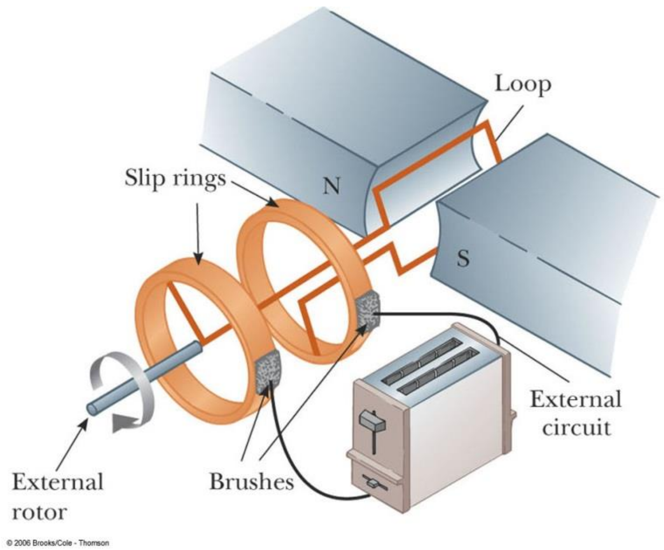
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = +BA\omega\sin(\omega t) \equiv \mathcal{E}_0 \sin(\omega t)$$

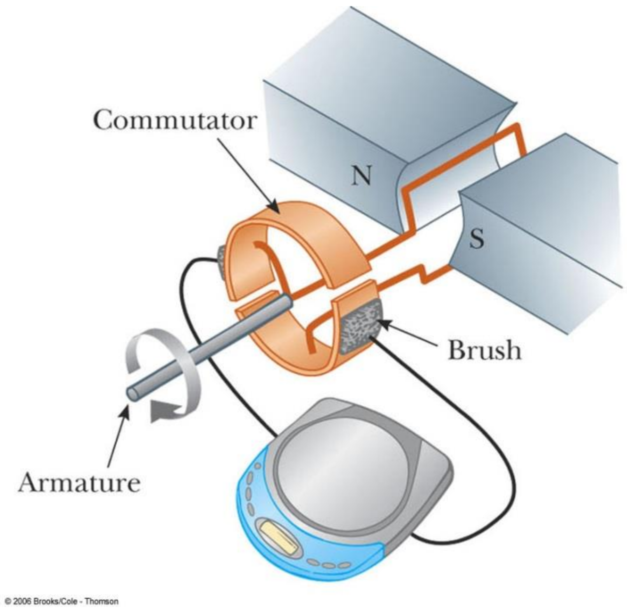
$\mathcal{E}_0 \equiv BA\omega$ is the peak value of the induced EMF

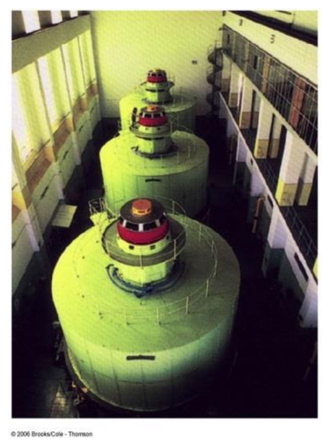
$\mathcal{E}_{ind}$ has sinusoidal behavior - alternating polarity
maxima when ωt = +/- $\pi/2$

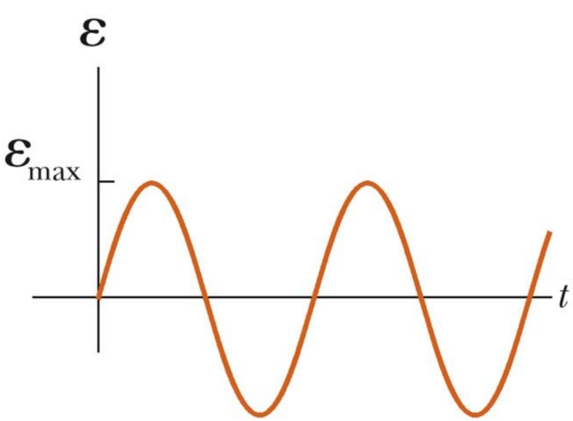
AC Generator

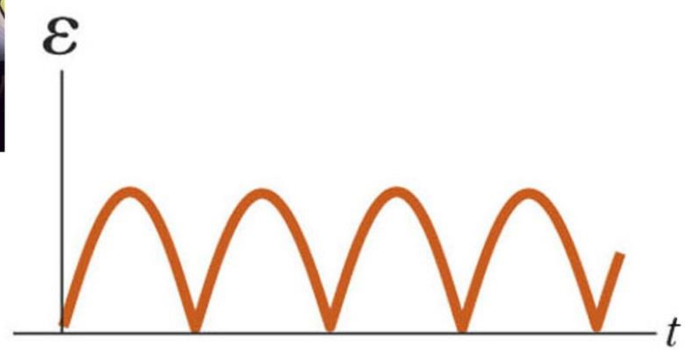
DC Generator







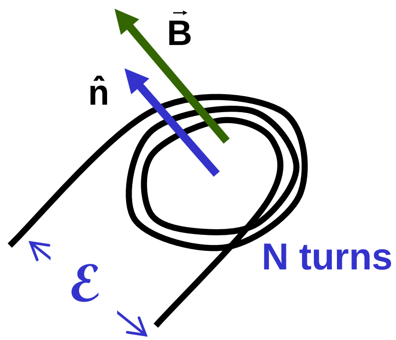




Back-torque $\underline{\mu} \times \underline{B}$ in rotating loop, $\mu \sim N.A.i_{ind}$

Numerical Example

Flat coil with N turns of wire



$$\mathcal{E}_{ind} = -N\frac{d\Phi_B}{dt}$$

Each turn increases the flux and induced EMF

- N = 1000 turns
- B through coil decreases from +1.0 T to -1.0 T in 1/120 s.
- Coil area A is 3 cm^2 (one turn)

Find the EMF induced in the coil

$$\frac{d\Phi_{tot}}{dt} = \frac{\Delta B}{\Delta t} \times \text{Area} \times \text{Number of turns} = \frac{-2.0}{1/120} \times 3\ cm^2 \times 10^{-4}\ m^2/cm^2 \times 1000$$

$$\frac{d\Phi_{tot}}{dt} = -72 \text{ Volts}$$

$$\boxed{\mathcal{E}_{ind} = +72 \text{ Volts}}$$

Flux change due to external B field produces induced EMF & B_{ind}

- Does INDUCED field/flux produce its own EMF? (Yes)
- A BACK EMF opposes current change – analogous to inertia

Transformer Principle

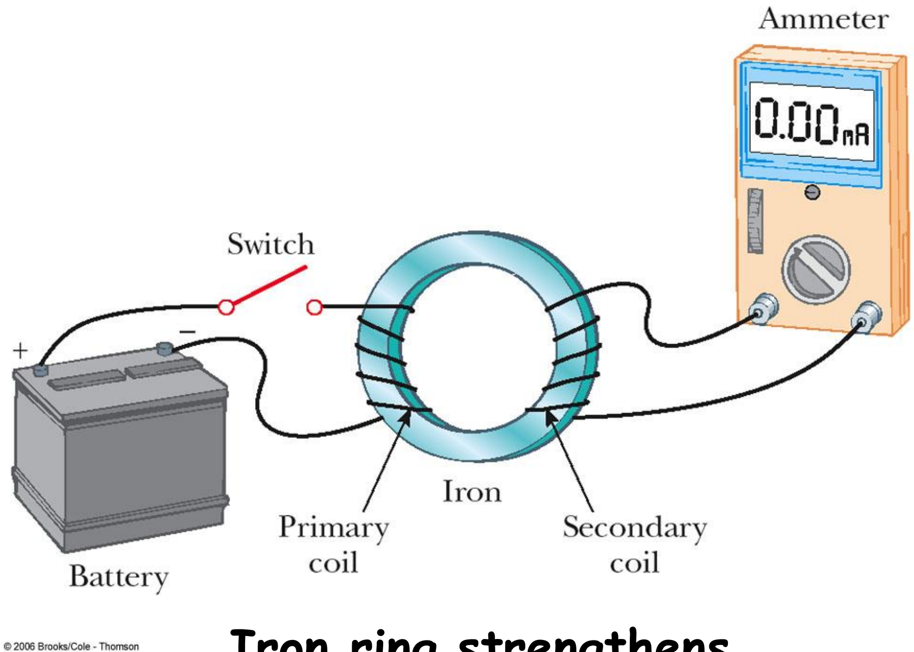
Changing primary current after switch closes

→ **Changing flux in primary coil**

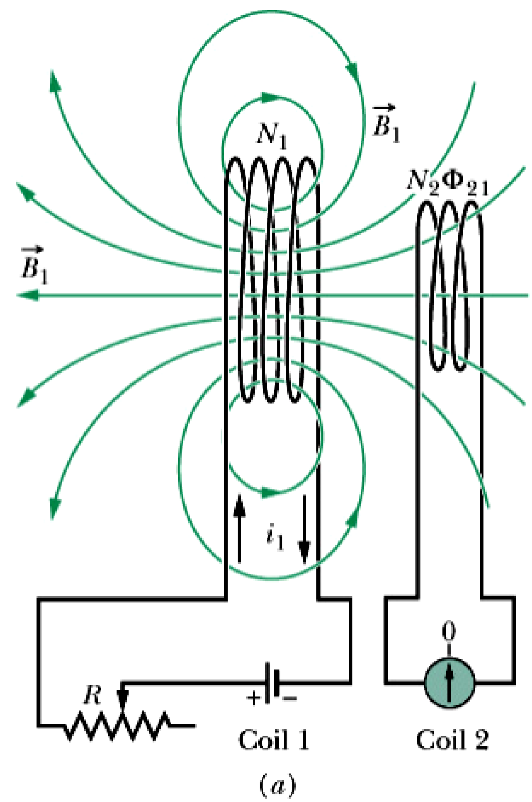
....which links to....

changing flux through secondary coil

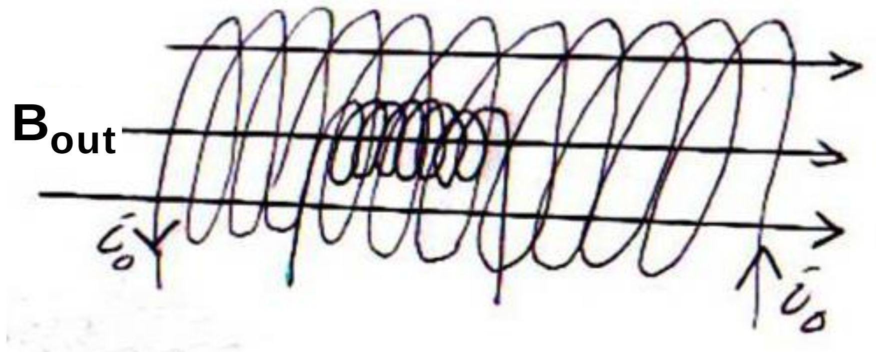
→ **changing secondary current**



Iron ring strengthens flux linkage



Air core Transformer Example: Concentric coils



OUTER COIL – IDEAL SOLENOID

- D = 3.2 cm = outer coil diameter
- n_{out} = 220 turns/cm = 22,000 turns/m
- Current i_{out} falls smoothly from 1.5 A to 0 in 25 ms
- Field within outer coil:

$$B_{out} = \mu_0 \, i_{out}(t) \, n_{out} \quad \text{as function of time} \qquad \mu_0 \equiv 4\pi \times 10^{-7} \qquad n_{out} = \frac{N_{out}}{L_{out}}$$

FIND INDUCED EMF IN INNER COIL DURING THIS PERIOD

- d = inner coil diameter = 2.1 cm = .021 m, $A_{in} = \pi d^2/4$, short length
- N_{in} = 130 turns = total number of turns in inner coil

$$\Delta\Phi_{in} \equiv \text{flux change through one turn of inner coil during } \Delta t$$

$$\mathcal{E}_{in} \equiv -N_{in}\frac{\Delta\Phi_{in}}{\Delta t} = -N_{in}\frac{\Delta B_{out} A_{in}}{\Delta t} = -N_{in} A_{in} \frac{\mu_0 n_{out} \Delta i_{out}}{\Delta t} \equiv -M\frac{\Delta i_{out}}{\Delta t}$$

$$\therefore \; \mathcal{E}_{in} \equiv -4\pi \times 10^{-7} \times 2.2 \times 10^{4} \times 130 \times \pi (.021/2)^2 \times \frac{-1.5}{2.5 \times 10^{-3}} = 75 \text{ mV}$$

Direction: Induced B is parallel to B_{outer} which is decreasing

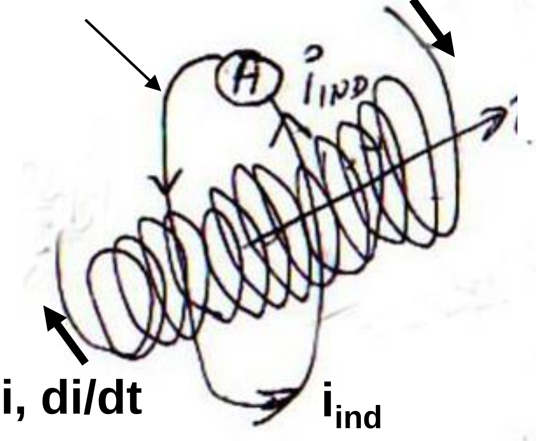
Would the transformer work if we reverse the role of the coils?

Changing magnetic flux induces electric fields: trivial transformer

A thin solenoid, cross section A, n turns/unit length

- zero field outside solenoid
- inside solenoid: $$B = \mu_0 i n$$

Flux through a conducting loop: $$\Phi = BA = \mu_0 i n A$$



Current i varies with time, so flux varies and an EMF is induced in loop "A":

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -\mu_0 n A \frac{di}{dt}$$

Current induced in the loop is: $i_{ind} = \dfrac{\mathcal{E}_{ind}}{R}$

If di/dt is positive, B is growing, then B_{ind} opposes change and i_{ind} is Counter-clockwise

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What makes the current i_{ind} flow?

- B = 0 there so it's not the Lorentz force
- An induced electric field E_{ind} along the loop causes current to flow
- It is caused directly by dF/dt
- Electric field lines are loops that don't terminate on charge.
- E-field is there even without the conductor (no current flowing)
- E-field is non-conservative (not electrostatic) as the line integral around a closed path is not zero

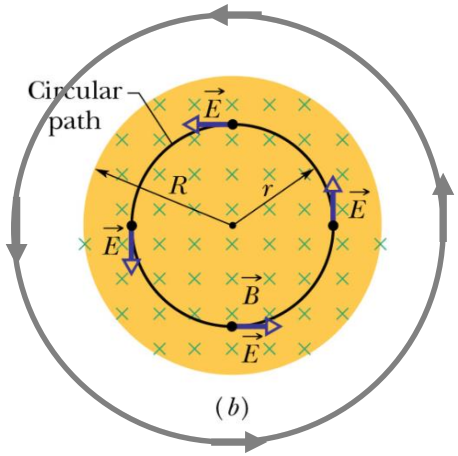
$$\therefore \ \mathcal{E}_{ind} = \oint_{loop} \vec{E}_{ind} \circ d\vec{s} = -\frac{d\Phi_B}{dt}$$

Generalized Faradays' Law
Path must be constant

Example: Find the induced electric field

$$\mathcal{E}_{ind} = \oint_{loop} E_{ind} \circ d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \circ d\vec{s} = \mu_0 i_{enc}$$

In the right figure, dB/dt = constant, find the expression for the magnitude E of the induced electric field at points within and outside the magnetic field.



Due to symmetry:

$$\oint \vec{E} \cdot \vec{ds} = \oint Eds = E\oint ds = E(2\pi r)$$

For r < R:

So

$$\Phi_B = BA = B(\pi r^2)$$

$$E(2\pi r) = \pi r^2 \frac{dB}{dt}$$

$$E = \frac{r}{2}\frac{dB}{dt}$$

For r > R:

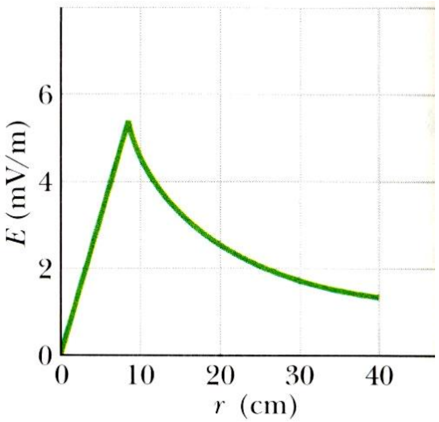
So

$$\Phi_B = BA = B(\pi R^2)$$

$$E(2\pi r) = \pi R^2 \frac{dB}{dt}$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}$$

The magnitude of electric field induced within the magnetic field grows linearly with r, then falls off as 1/r for r>R



Summary: Lecture 11 Chapter 29 – Induction I – Faraday's Law

CHAPTER 29 SUMMARY

Faraday's law: Faraday's law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop. This relationship is valid whether the flux change is caused by a changing magnetic field, motion of the loop, or both. (See Examples 29.1–29.6.)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad (29.3)$$

Lenz's law: Lenz's law states that an induced current or emf always tends to oppose or cancel out the change that caused it. Lenz's law can be derived from Faraday's law and is often easier to use. (See Examples 29.7 and 29.8.)

Motional emf: If a conductor moves in a magnetic field, a motional emf is induced. (See Examples 29.9 and 29.10.)

$$\mathcal{E} = vBL \qquad (29.6)$$

(conductor with length L moves in uniform $\vec{B}$ field, $\vec{L}$ and $\vec{v}$ both perpendicular to $\vec{B}$ and to each other)

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (29.7)$$

(all or part of a closed loop moves in a $\vec{B}$ field)