# Simultaneous Equations

method # 1 (substitution)

$3x + 5y = 2$ *     same as * $\quad 6x + 10y = 4$

$-7x + 5y = 8$ **

* $3x = 2 - 5y \qquad x = \frac{2}{3} - \frac{5}{3}y$

** $-7\left(\frac{2}{3} - \frac{5}{3}y\right) + 5y = 8$

$$-\frac{14}{\cancel{3}} + \frac{35}{\cancel{3}}y + \frac{15}{\cancel{3}}y = \frac{24}{\cancel{3}}$$

$$-14 + 35y + 15y = 24$$

$$50y = 38$$

$$y = \frac{38}{50} = \frac{19}{25}$$

$$x = \frac{2}{3} - \frac{5}{3}y$$

$$= \frac{2}{3} - \frac{5}{3}\left(\frac{19}{25}\right)$$

$$= \frac{50}{75} - \frac{95}{75}$$

$$= \frac{-45}{75} = -\frac{3}{5}$$

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method # 2 (elimination)

$$3x + 5y = 2$$

$$-7x + 5y = 8$$

$$10x = -6 \qquad x = \frac{-6}{10} = \frac{-3}{5}$$

$$5y = 2 - 3x \qquad y = \frac{2}{5} - \frac{3}{5}x = \frac{2}{5} - \frac{3}{5}\left(-\frac{3}{5}\right)$$

$$= \frac{10}{25} + \frac{9}{25} = \frac{19}{25}$$

Method # 3 (Advanced)

Matrix methods

$3x + 5y = 2$

$-7x + 5y = 8$

$$\begin{pmatrix} 3 & 5 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

$$\bar{\bar{A}}\,\vec{x} = \vec{b}$$

$$\bar{\bar{A}} = \begin{pmatrix} 3 & 5 \\ -7 & 5 \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

$$\underline{\bar{\bar{A}}^{-1}\bar{\bar{A}}}\,\vec{x} = \bar{\bar{A}}^{-1}\vec{b} \qquad A^{-1}A = \underline{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\vec{x} = \bar{\bar{A}}^{-1}\vec{b}$$

$$\bar{\bar{A}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \bar{\bar{A}}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

2x2

$$\vec{x} = \bar{\bar{A}}^{-1}\vec{b}$$

See videos week 1

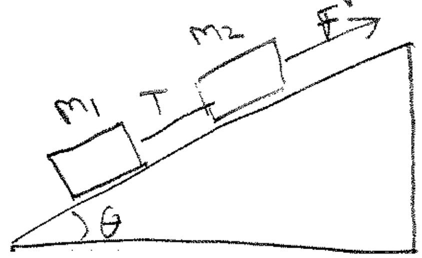
$$\vec{x} = A^{-1} b = \frac{1}{\underbrace{(15-(-35))}_{50}}\begin{pmatrix} 5 & -5 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

$$= \frac{1}{50}\begin{pmatrix} 5\times 2 - 5\times 8 \\ (+7)\times 2 + 3\times 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \frac{1}{50}\begin{pmatrix} -30 \\ 38 \end{pmatrix} = \begin{pmatrix} -3/5 \\ 19/25 \end{pmatrix}$$

# What is the point?

p111-1





$$T - m_1 g\sin\theta = m_1 a$$

$$F - T - m_2 g\sin\theta = m_2 a$$

What is a } given $F$, $m_1, m_2, \theta$
What is T }

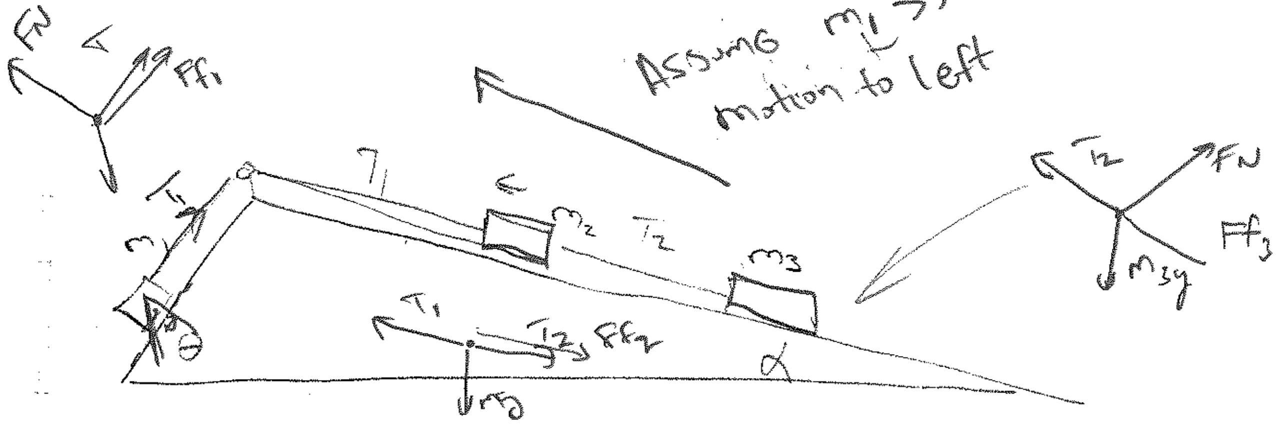
add eqns $\quad -m_1 g\sin\theta + F - m_2 g\sin\theta = (m_1 + m_2)a$

$$a = \frac{F - m_1 g\sin\theta - m_2 g\sin\theta}{m_1 + m_2}$$

What is T ?

$$T = m_1 g\sin\theta + m_1 a$$

Assume $m_1 >> m_2 + m_3$
motion to left




PIII-2

$m_1 a = m_1 g \sin\theta - T_1 - F_{f_1}$ $\mu_{k_1} F_{N_1} = \mu_{k_1} m_1 g \cos\theta$

$m_2 a = T_1 - T_2 - m_2 g \sin\alpha - F_{f_2}$ $a = ?$

$\mu_{k_2} F_{N_2} = \mu_{k_2} m_2 g \cos\alpha$ $T_1 = ?$

$T_2 = ?$

$m_3 a = T_2 - m_3 g \sin\alpha - F_{f_3}$

$\mu_{k_2} F_{N_3} = \mu_{k_2} m_3 g \cos\alpha$

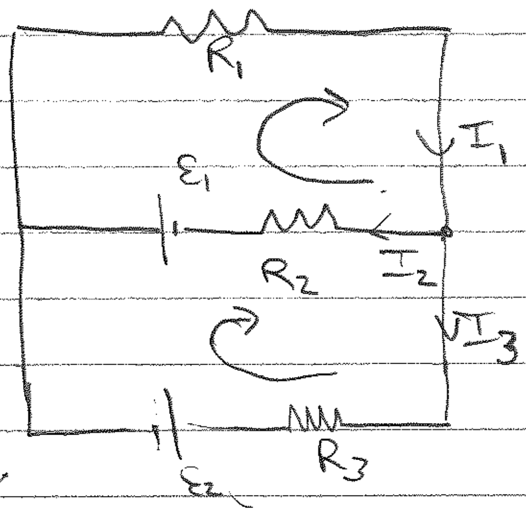
What is a? $(m_1 + m_2 + m_3) a = m_1 g \sin\theta - F_{f_1} - m_2 g \sin\alpha - F_{f_2} - m_3 g \sin\alpha - F_{f_3}$

$$a = \frac{1}{m_1 + m_2 + m_3}\left(m_1 g \sin\theta - F_{f_1} - m_2 g \sin\alpha - F_{f_2} - m_3 g \sin\alpha - F_{f_3}\right)$$

What is $T_1$ $T_1 = m_1 g \sin\theta - m_1 a - F_{f_1}$

What is $T_2$ $T_2 = m_3 a + m_3 g \sin\alpha + F_{f_3}$

Circuits




$I_1 = I_2 + I_3$

$$-R_2 I_2 + \mathcal{E}_1 - R_1 I_1 = 0$$

$$-I_3 R_3 + \mathcal{E}_2 - \mathcal{E}_1 + I_2 R_2 = 0$$

$$I_1 - I_2 - I_3 = 0 \qquad \Big\} \; I_1 = I_2 + I_3$$

$$I_1, I_2, I_3 = ?$$

$$* \quad R_1 I_1 + R_2 I_2 = \mathcal{E}_1$$

$$** \quad R_2 I_2 - R_3 I_3 = \mathcal{E}_1 + \mathcal{E}_2$$

$$*** \quad I_1 - I_2 - I_3 = 0$$

$$\begin{pmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & -R_3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_1 + \mathcal{E}_2 \\ 0 \end{pmatrix}$$

$$R_1(***) \quad R_1 I_1 - R_1 I_2 - R_1 I_3 = 0 \qquad I_1 = I_2 + I_3$$

$$*-R_1(***) \quad (R_1 + R_2) I_2 + R_1 I_3 = \mathcal{E}_1$$

$$R_2 I_2 - R_3 I_3 = \mathcal{E}_1 + \mathcal{E}_2$$

$$R_3 (R_1+R_2) I_2 + R_3 R_1 I_3 = R_3 \varepsilon_1$$

$$R_1 R_2 I_2 + R_1 R_3 I_3 = R_1(\varepsilon_1+\varepsilon_2)$$

$$\rightarrow \quad I_2 \left( R_3(R_1+R_2) + R_1 R_2 \right) = R_3 \varepsilon_1 + R_1(\varepsilon_1+\varepsilon_2)$$

$$I_2 = \frac{R_1 \varepsilon_1 + R_1(\varepsilon_1+\varepsilon_2)}{R_3R_1 + R_3R_2 + R_1R_2}$$

$$I_1 = (\varepsilon_1 - R_2 I_2)/R_1 \qquad I_3 = I_1 - I_2$$

$$R_2 (R_1+R_2) I_2 + R_2 R_1 I_3 = R_2 \varepsilon_1$$

$$(R_1+R_2) R_2 I_2 + (R_1+R_2) R_3 I_3 = (R_1+R_2)(\varepsilon_1+\varepsilon_2)$$

$$\left(R_2R_1 + (R_1+R_2)R_3\right) I_3 = R_2 \varepsilon_1 + (R_1+R_2)(\varepsilon_1+\varepsilon_2)$$

$$I_3 = \frac{R_2\varepsilon_1 + (R_1+R_2)(\varepsilon_1+\varepsilon_2)}{R_2R_1 + (R_1+R_2)R_3}$$

$$I_1 = I_2 + I_3$$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

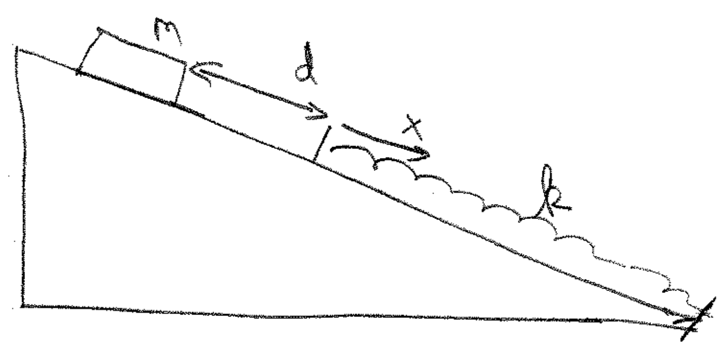
$x^2-3x-4=0$

P111-3

$$x=\frac{3\pm\sqrt{3^2-4(1)(-4)}}{2}=\frac{3\pm\sqrt{9+16}}{2}$$

$$=\frac{3\pm5}{2}\Rightarrow -1,\ 4$$

Symbolic




$\frac{1}{2}kx^2-(mg\sin\theta)x-mgd\sin\theta=0$ no friction

find X

$$x^2-\left(\frac{2mg\sin\theta}{k}\right)x-\left(\frac{mg}{k}2d\sin\theta\right)=0$$

$$x=\frac{\frac{2mg\sin\theta}{k}\pm\sqrt{\left(\frac{2mg\sin\theta}{k}\right)^2-4(1)\left(-\frac{mg\,2d\sin\theta}{k}\right)}}{2(1)}$$

**Quiz 1**

**Name**______________________________

(1) 1 mi is equivalent to 1609 m so 55 mph is:

A) 15 m/s B) 25 m/s C) 66 m/s D) 88 m/s (E) 1500 m/s

$$\left(55\ mi/hr\right)\left(\frac{1609\ m}{mi}\right)\left(\frac{1\ hr}{60\ min.}\right)\left(\frac{1\ min}{60\ sec}\right)$$

$$= 24.6\ m/s \approx 25\ m/s$$

(2) Suppose $A = B^nC^m$, where $A$ has dimensions LT, $B$ has dimensions $L^2T^{-1}$, and $C$ has dimensions $LT^2$. Then the exponents $n$ and $m$ have the values:

A) 2/3; 1/3 B) 2; 3 C) 4/2; -1/5 D) 1/5; 3/5 E) 1/2; 1/2

$$[L][T] = ([L]^2[T^{-1}])^n\ ([L][T^2])^m$$

$$= [L]^{2n+m}[T]^{-n+2m}$$

$1 = 2n+m$    $1 = 2n+m$    $3 = 5m$

$1 = -n+2m \xrightarrow{\times 2}$    $2 = -2n+4m$   add   $m = 3/5 \rightarrow$   $1 = 2n + 3/5$

$n = \frac{1-3/5}{2} = \frac{1}{5}$

(3) The coordinate of an object is given as a function of time by $x = 4t - 3t^2$, where $x$ is in meters and $t$ is in seconds. Its average velocity over the interval from $t = 0$ to $t = 2$ s is:

A) 0 m/s B) -2 m/s C) 2 m/s D) -14 m/s E) can not be computed

$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{X(2)-X(0)}{2-0} = \frac{-4\ m}{2\ s} = -2\ m/s$$

$$\left\{\begin{array}{l} X(2) = 4(2) - 3(2)^2 = 8-12 = -4 \\ X(0) = 4(0) - 3(0)^2 = 0 \end{array}\right.$$