

Physics 121 - Electricity and Magnetism

Lecture 04 - Gauss' Law

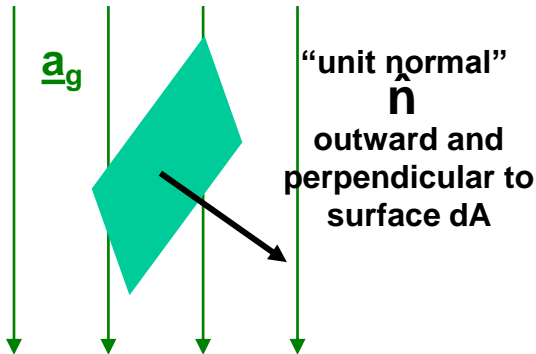
Y&F Chapter 22 Sec. 1 - 5

- **Flux Definition (gravitational example)**
- **Gaussian Surfaces**
- **Flux Examples**
- **Flux of an Electric Field**
- **Gauss' Law**
- **Gauss' Law Near a Dipole**
- **A Charged, Isolated Conductor**
- **Spherical Symmetry: Conducting Shell with Charge Inside**
- **Cylindrical Symmetry: Infinite Line of Charge**
- **Field near an infinite Non-Conducting Sheet of Charge**
- **Field near an infinite Conducting Sheet of Charge**
- **Conducting and Non-conducting Plate Examples**
- **Proof of Shell Theorem using Gauss Law**
- **Examples**
- **Summary**

Flux (symbol Φ) is basically a vector field magnitude \times area

Applies to flow of mass or fluid volume, **gravitational**, electric, magnetic field

Define: $d\Phi_g$ is differential flux of **gravitational** field \underline{a}_g crossing vector area $d\mathbf{A}$



$$\Delta\Phi_g \equiv \text{flux of } \vec{a}_g \text{ through } \Delta\vec{A}$$
$$= \vec{a}_g \circ \hat{n}\Delta A \text{ (a scalar)}$$

Flux through a closed or open surface S :

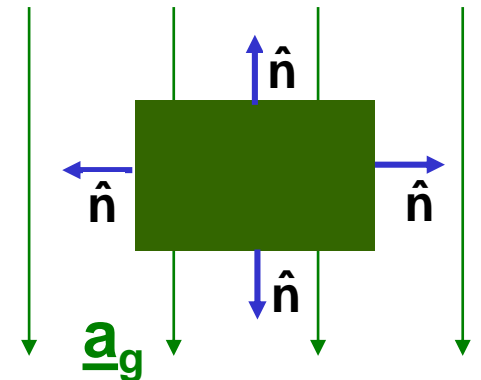
calculate “surface integral” of field over S

$$\Phi_S \equiv \int_S d\Phi = \int_S \vec{a}_g \circ \hat{n}dA$$

Evaluate integrand at all points on surface S

EXAMPLE : GRAVITATIONAL FLUX THROUGH A CLOSED IMAGINARY BOX (UNIFORM ACCELERATION FIELD)

- No mass inside the box
- $\Delta\Phi$ from each side = 0 since $\underline{a} \cdot \underline{n} = 0$, $\Delta\Phi$ from ends cancels
- TOTAL $\Phi = 0$
- Example could also apply to fluid flow



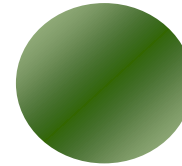
What if a mass (flux source) is in the box?

Can field be uniform? Can net flux be zero.

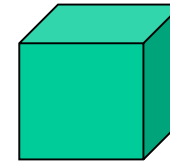
Gauss' Law (Carl Friedrich Gauss (1777-1855)) USES

Gaussian Surfaces: Closed 3D surfaces

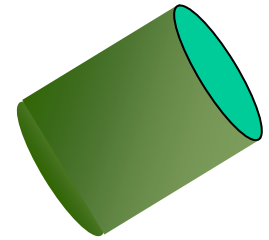
- Field lines cross a closed surface:
 - Once (or an odd number of times) for charges that are inside
 - Twice (or an even number of times) for charges that are outside
- Choose surface to match the field's symmetry where possible



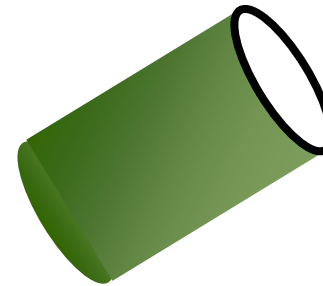
sphere



closed
box



cylinder with
end caps



no end caps
not closed
not a GS

The flux of electric field crossing a closed surface equals the net charge inside the surface (times a constant).

Simple example: charge at center of a spherical "Gaussian surface"

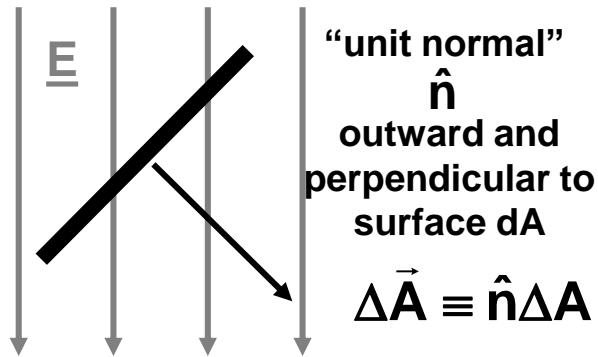
$$\text{Flux} \equiv \Phi = \text{Field} \times \text{Surface Area} = E \times S = \frac{Q}{4\pi\epsilon_0 R^2} \times 4\pi R^2 = Q / \epsilon_0$$

Flux is a SCALAR, Units: Nm^2/C .

Does this apply for non-point charges away from the center of the sphere?

Electric Flux: Integrate electric field over a surface

Definition: differential flux of \underline{E} crossing $d\underline{A}$ (area vector)



Divide up some surface S into tiny chunks of dA each and consider one of them

$$\begin{aligned}\Delta\Phi_E &\equiv \text{flux of } \vec{E} \text{ through } \Delta\vec{A} \\ &\equiv \vec{E} \circ \hat{n}\Delta A \quad (\text{a scalar})\end{aligned}$$

Flux through a closed or open surface S : Integrate on S

$$\Phi_E \equiv \oint_S d\Phi_E = \oint_S \vec{E} \circ \hat{n}dA$$

To do this: evaluate integrand at all points on surface S

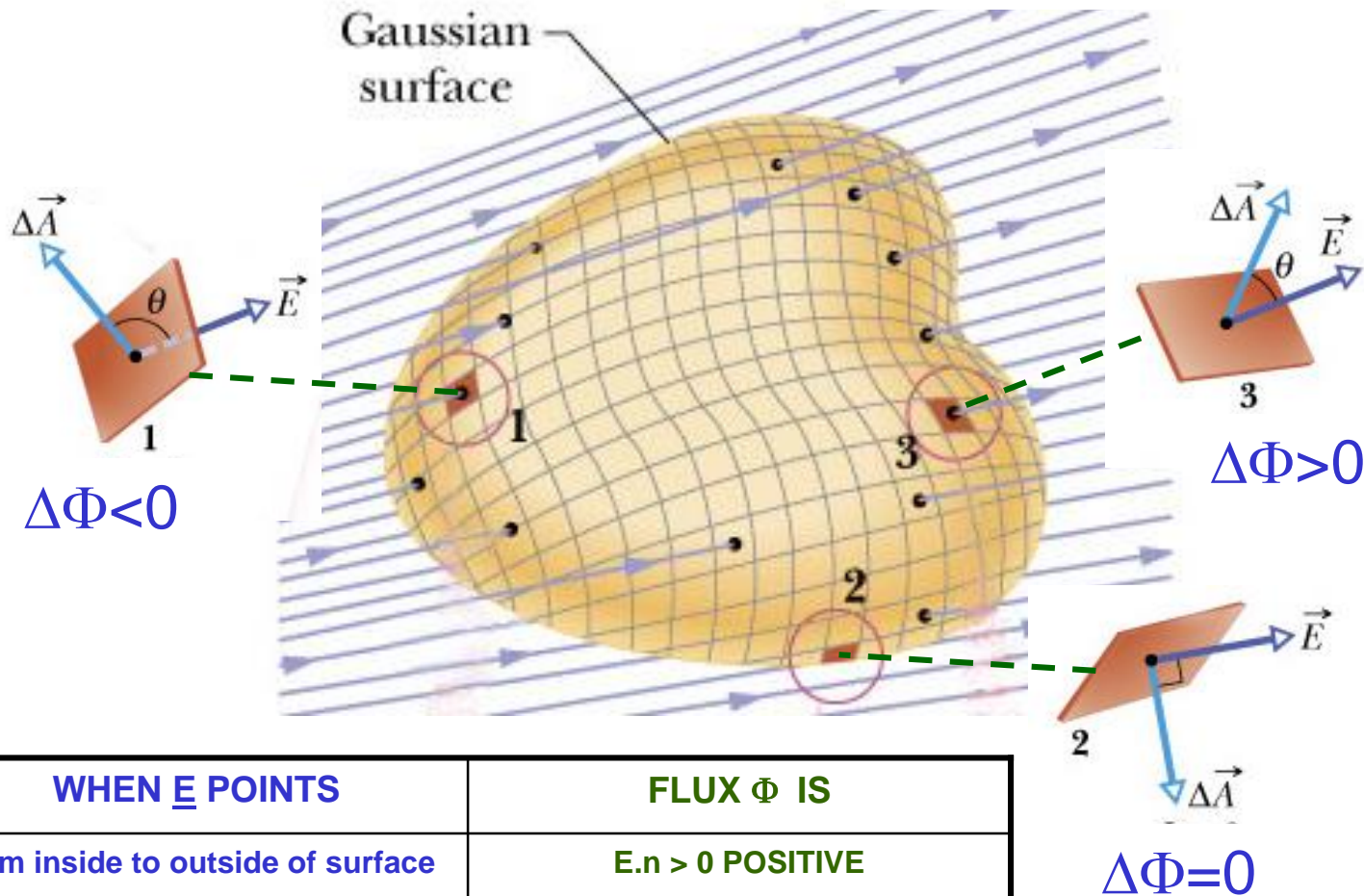
Gauss' Law: The flux through a closed surface S depends only on the net enclosed charge, not on the details of S or anything else.

$\Delta\Phi$ depends on the angle between the field and chunks of area

$$\Delta\Phi = \vec{E} \circ \hat{n}\Delta A = EA\cos(\theta)$$

$$\Delta\vec{A} = \hat{n}\Delta A$$

$\hat{n} \equiv$ outward unit vector



WHEN \underline{E} POINTS	FLUX Φ IS
from inside to outside of surface	$E \cdot n > 0$ POSITIVE
from outside to inside of surface	$E \cdot n < 0$ NEGATIVE
tangent to surface	$E \cdot n = 0$ ZERO

Evaluating flux through closed or open surfaces

Special case: field is constant across pieces of the surface

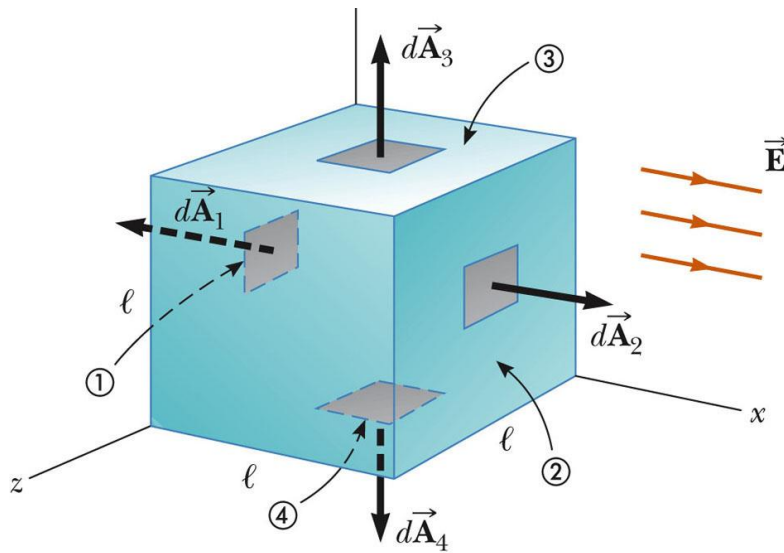
$$\Delta\Phi_E \equiv \vec{E} \circ \hat{n}\Delta A \text{ (a scalar)}$$

SUM $\Delta\Phi$ FROM SMALL
CHUNKS OF SURFACE ΔA

Units of Φ are : Nm^2/C

$$\Phi_E \equiv \sum_{\text{small areas}} \Delta\Phi_E = \sum_{\text{small areas}} \vec{E} \circ \hat{n}\Delta A$$

EXAMPLE: Flux through a cube
Assume:



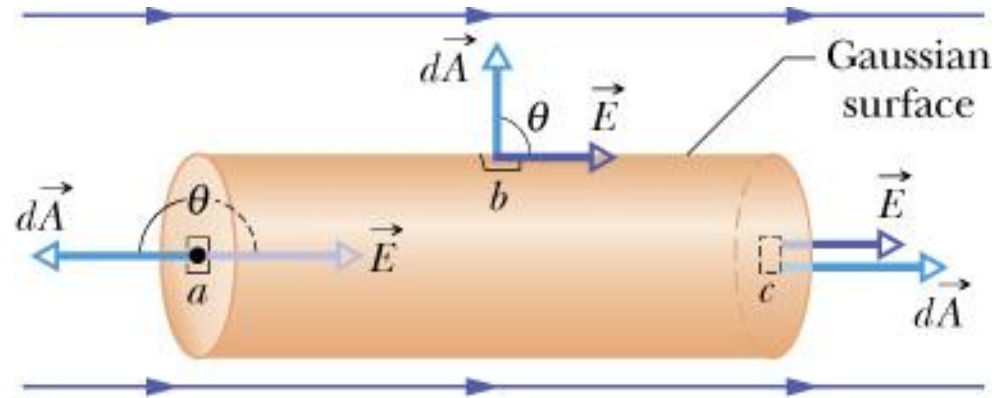
- Uniform E field everywhere
- Directed along x -axis
- Cube faces normal to axes
- Each side has area ℓ^2
- Field lines cut through two surface areas and are tangent to the other four surface areas
- For side 1, $\Phi = -E\ell^2$
- For side 2, $\Phi = +E\ell^2$
- For the other four sides, $\Phi = 0$
- Therefore, $\Phi_{\text{total}} = 0$

$$\therefore \Phi_E = \sum_{i=1,6} \Delta\Phi_i = 0$$

- What if the cube is oriented obliquely??
- How would flux differ if net charge is inside??

Flux of a uniform electric field through a cylinder

- Closed Gaussian surface
- Uniform \underline{E} means zero enclosed charge
- Calculate flux directly
- Symmetry axis along \underline{E}
- Break into areas a, b, c



$$\Phi_{\text{tot}} = \int_{a,b,c} \vec{E} \circ d\vec{A} = \Phi_a + \Phi_b + \Phi_c$$

Cap a: $\vec{E} \circ \hat{n} = -E$, $\cos(\theta) = -1$ $\therefore \Phi_a = -E \int_{\text{cap a}} dA = -E A_{\text{cap}}$

Cap c: $\vec{E} \circ \hat{n} = +E$, $\cos(\theta) = +1$ $\therefore \Phi_c = +E \int_{\text{cap c}} dA = +E A_{\text{cap}}$

Area b: $\vec{E} \circ \hat{n} = 0$, $\vec{E} \perp \vec{A}$ everywhere on b $\therefore \Phi_b = 0$

$$\therefore \Phi_{\text{tot}} = 0$$

What if E is not parallel to cylinder axis:

- Geometry is more complicated...but...
- $Q_{\text{inside}} = 0$ so $\Phi = 0$ still !!

Flux of an Electric Field

4-1: Which of the following figures correctly shows a positive electric flux out of a surface element?

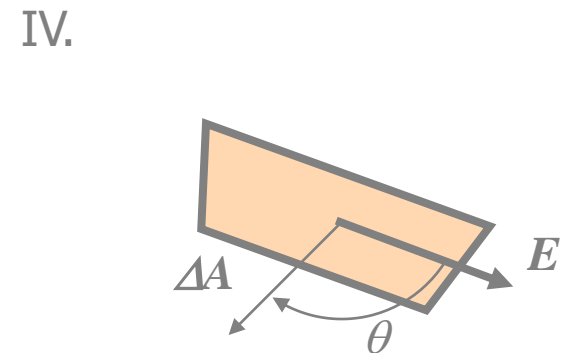
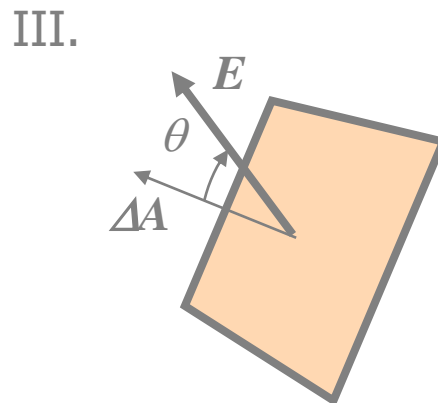
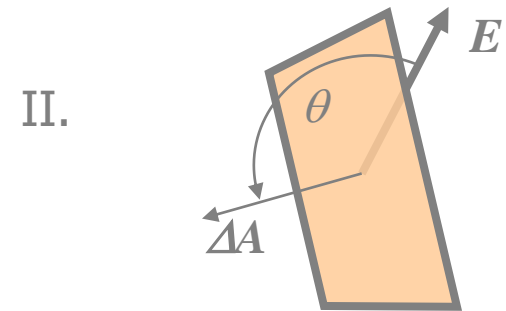
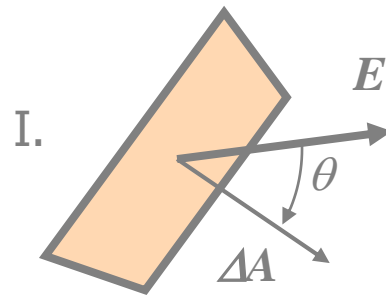
A.I.

B.II.

C.III.

D.IV.

E.I and III.



Statement of Gauss' Law

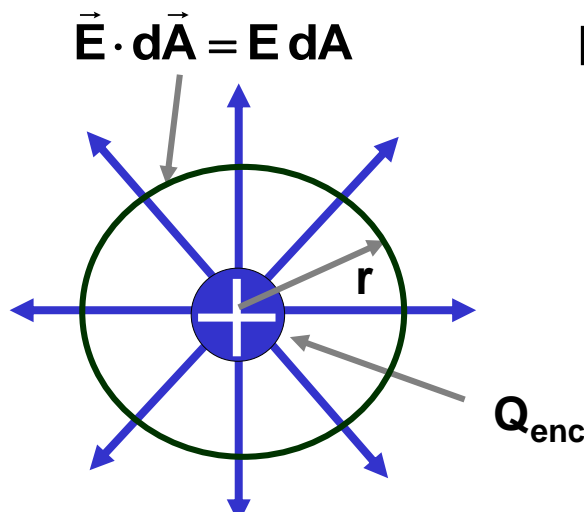
Let Q_{enc} be the NET charge enclosed by a (closed) Gaussian surface S . The net flux Φ through the surface is $Q_{\text{enc}}/\epsilon_0$

$$\Phi = \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad Q_{\text{enc}} = \epsilon_0 \Phi$$

- Does not depend on the shape of the surface.
- Charge outside the surface S can be ignored.
- Surface integral yields 0 if $\vec{E} = 0$ everywhere on surface

Example: Derive Coulomb's Law from Gauss' Law

Assume a point charge at center of a spherical Gaussian surface



$\vec{E} \cdot d\vec{A}$ is always just $E dA$ because \vec{E} and \hat{n} are always radial

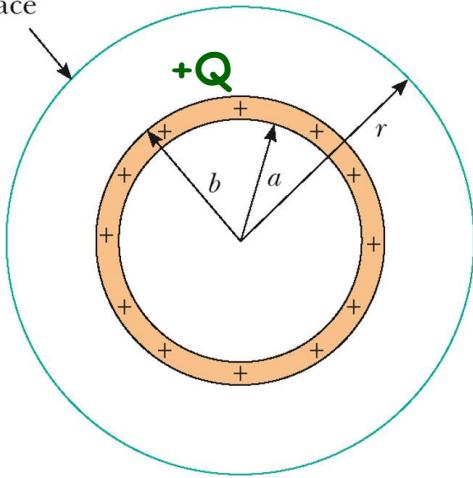
$$\Phi = \oint_S E dA = E \times 4\pi r^2 = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\therefore E(r) = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2}$$

Coulomb's Law

Spherical Shells of Charge

Gaussian surface

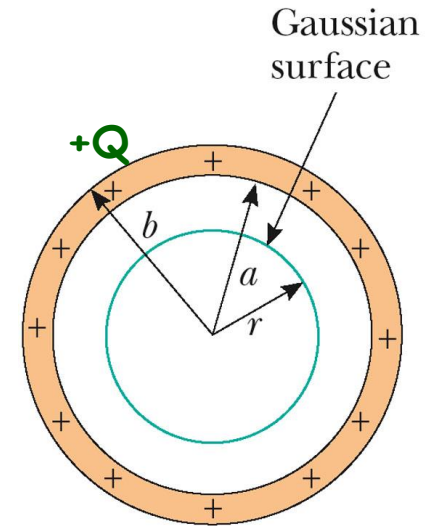


spherically symmetric

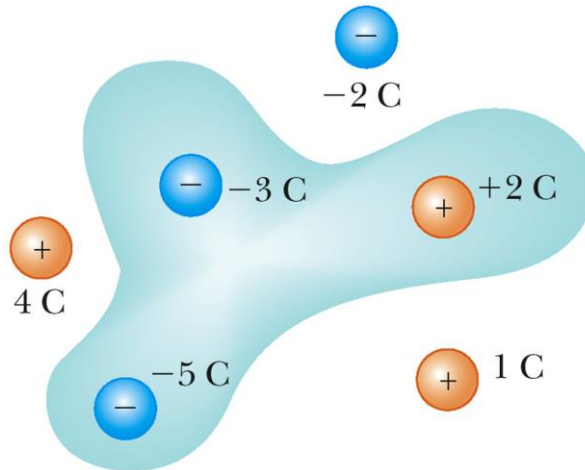
Shell Theorem

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

What are the fields on the Gaussian surfaces?



4-2: What is the flux through the Gaussian surface below?



- A. zero
- B. $-6\text{ C}/\epsilon_0$
- C. $-3\text{ C}/\epsilon_0$
- D. $+3\text{ C}/\epsilon_0$
- E. not enough info

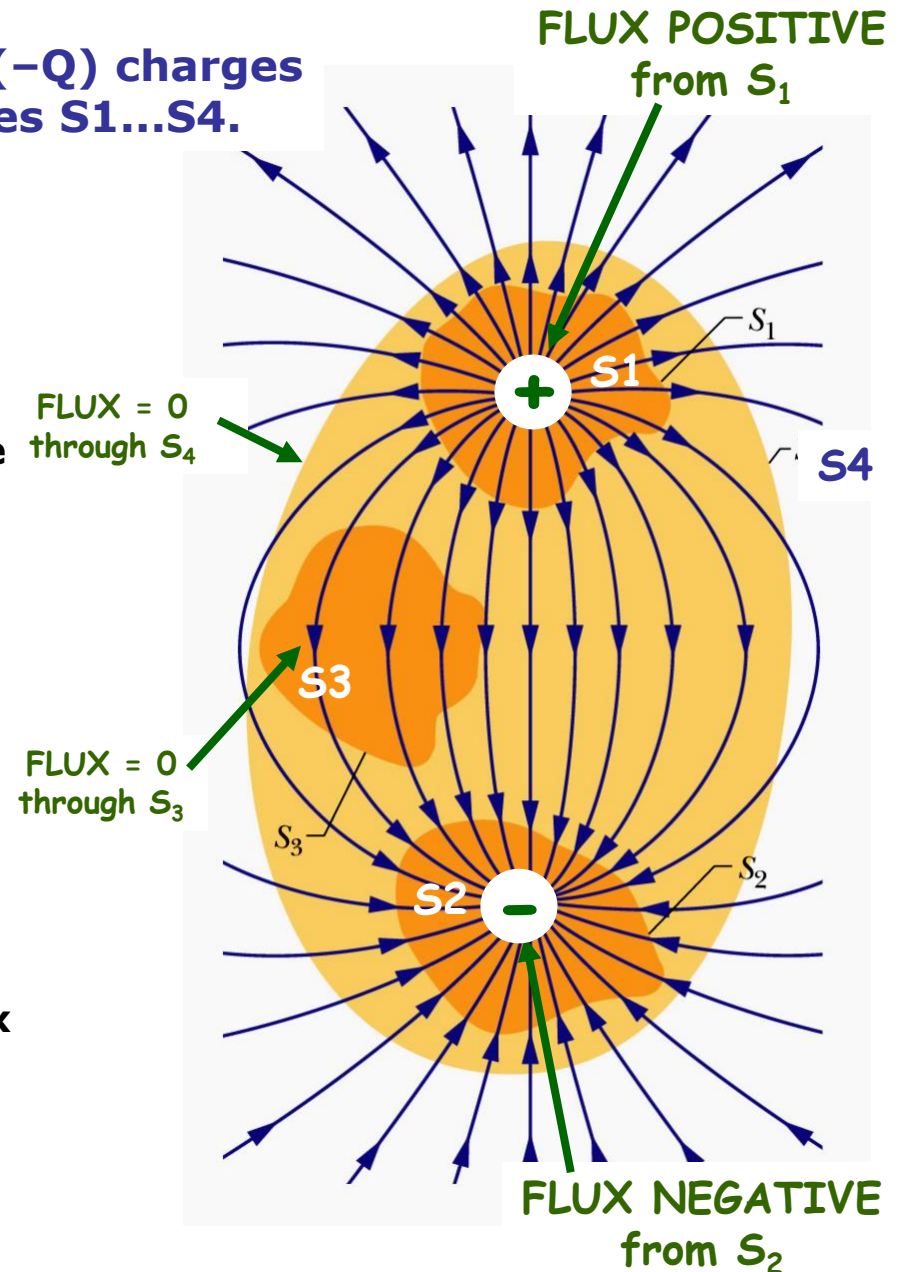
$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

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Flux through surfaces near a dipole

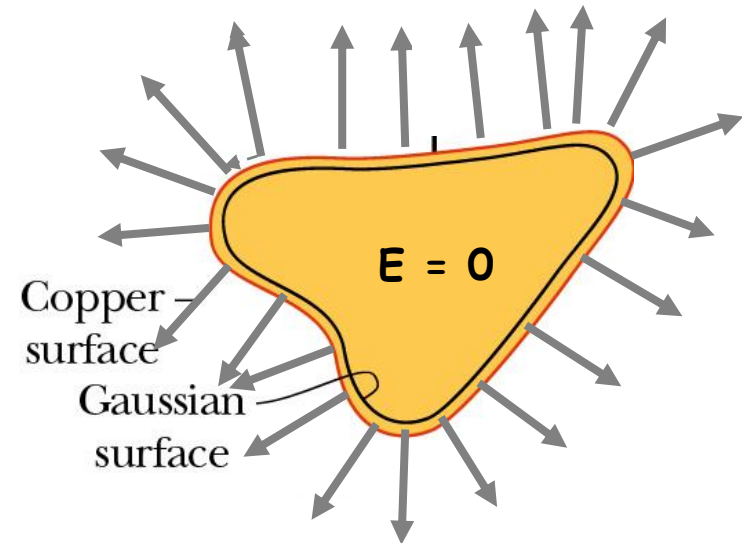
- Equal positive (+Q) and negative (-Q) charges
- Consider Gaussian (closed) surfaces S1...S4.

- **Surface S₁** encloses only the positive charge. The field is everywhere outward. Positive flux. $\Phi = +Q/\epsilon_0$
- **Surface S₂** encloses only the negative charge. The field is everywhere inward. Negative flux. $\Phi = -Q/\epsilon_0$
- **Surface S₃** encloses no charges. Net flux through the surface is zero. The flux is negative at the upper part, and positive at the lower part, but these cancel. $\Phi = 0$
- **Surface S₄** encloses both charges. Zero net charge enclosed, so equal flux enters and leaves, zero net flux through the surface. $\Phi = 0$



Where does net charge reside on an isolated conductor?

- Place net charge Q initially in the interior of a conductor
- Charges are free to move, but can not leak off $\vec{F} = Q\vec{E}$
- At equilibrium $E = 0$ everywhere inside a conductor
- Charge flows until $E = 0$ at every interior point



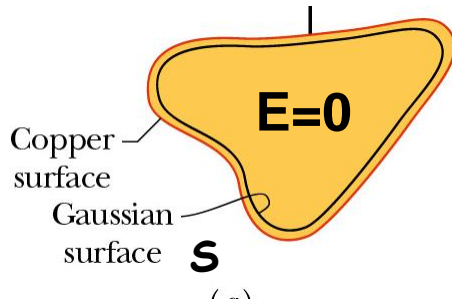
- Choose Gaussian surface as shown: $E = 0$ everywhere on it
- Use Gauss' Law!

$$\Phi = \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{\text{enc}}}{\epsilon_0} \Rightarrow Q_{\text{enc}} = 0$$

- The only place where un-screened charge can end up is on the outside surface of the conductor
- \underline{E} at the surface is everywhere normal to it; if \underline{E} had a component parallel to the surface, charges would move to screen it out.

Gauss's Law: net charge on an isolated conductor... ...moves to the outside surface

1: SOLID CONDUCTOR WITH EXCESS NET CHARGE ON IT



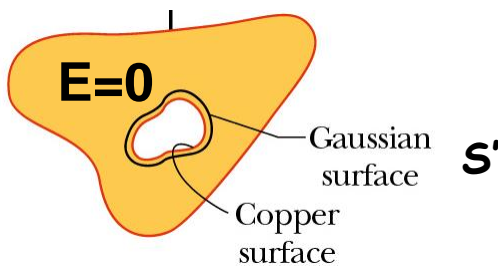
- Choose Gaussian surface S just inside the surface
- $E=0$ at every point in the metal, including on S
- Gauss' Law says:

$$\Phi = \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\Phi_S = 0 = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \therefore Q_{\text{enc}} = 0$$

All net charge on a conductor moves to the outer surface

2. HOLLOW CONDUCTOR WITH A NET CHARGE, NO CHARGE IN CAVITY



- Choose Gaussian surface S' just outside the cavity
- $E=0$ everywhere within the metal, including S'
- Gauss' Law says:

$$\Phi_{S'} = 0 = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \therefore Q_{\text{enc}} = 0$$

There is zero net charge on the inner surface: all net charge is on outer surface

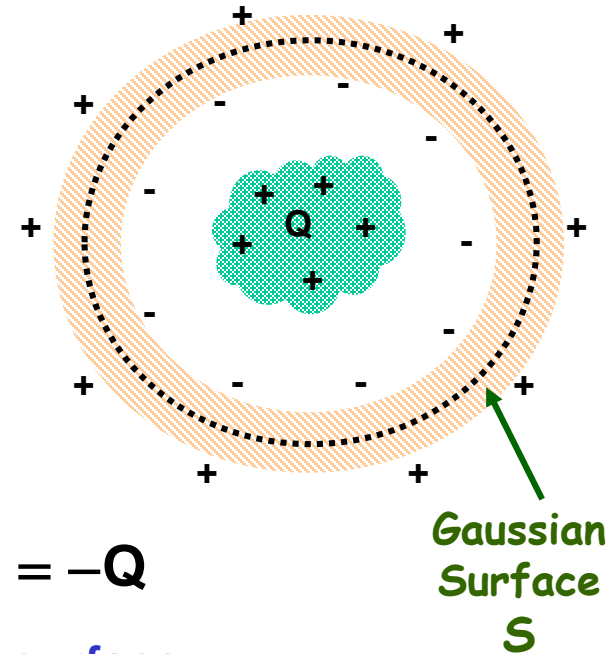
Does the charge density have to be zero at each point on the inner surface?

Charge inside a conducting spherical shell

- Electrically **neutral** shell
- Arbitrary charge distribution $+Q$ in cavity
- Choose Gaussian surface S completely within the conductor

$$\Phi_S = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

- $\vec{E} = 0$ everywhere on S , so $F_S = 0$ and $q_{\text{enc}} = 0$
- Negative charge Q_{inner} is induced on inner surface, distributed so that $E = 0$ in the metal, hence...



$$q_{\text{enc}} = 0 = +Q + Q_{\text{inner}} \quad \longrightarrow \quad Q_{\text{inner}} = -Q$$

- The shell is neutral, so $+Q$ must appear on the outer surface

OUTSIDE:

- Choose another spherical Gaussian surface S' outside the shell
- Gauss Law:

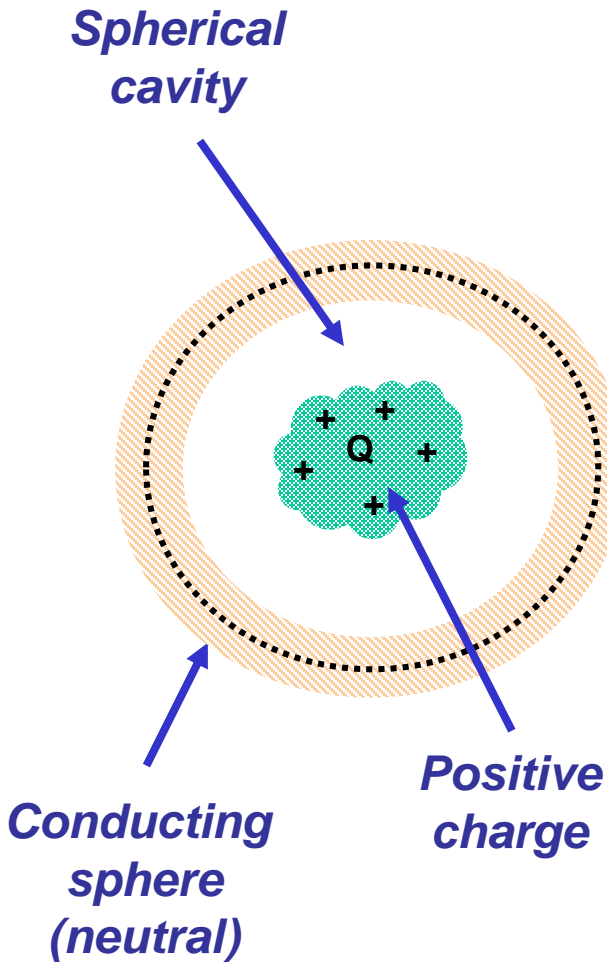
$$\Phi = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{+Q}{\epsilon_0} = 4\pi r^2 E(r) \quad \longrightarrow \quad E(r) = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{radially outward}$$

Whatever the inside distribution may be, outside the shell it is shielded and the field looks like that of a point charge at the center

For a spherical shell, $+Q$ outside is uniform, o/w E inside could not = 0

Conducting spherical shell with charge inside

4-3: Place a charge inside a cavity in an isolated conductor. Which statement is true?



- A. E field is still zero in the cavity.
- B. E field is not zero in the cavity, but it is zero in the conductor.
- C. E field is zero outside the conducting sphere.
- D. E field is the same as if the conductor were not there (i.e. radial outward everywhere).
- E. E field is zero in the conductor, and negative (radially inward) outside the conducting sphere.

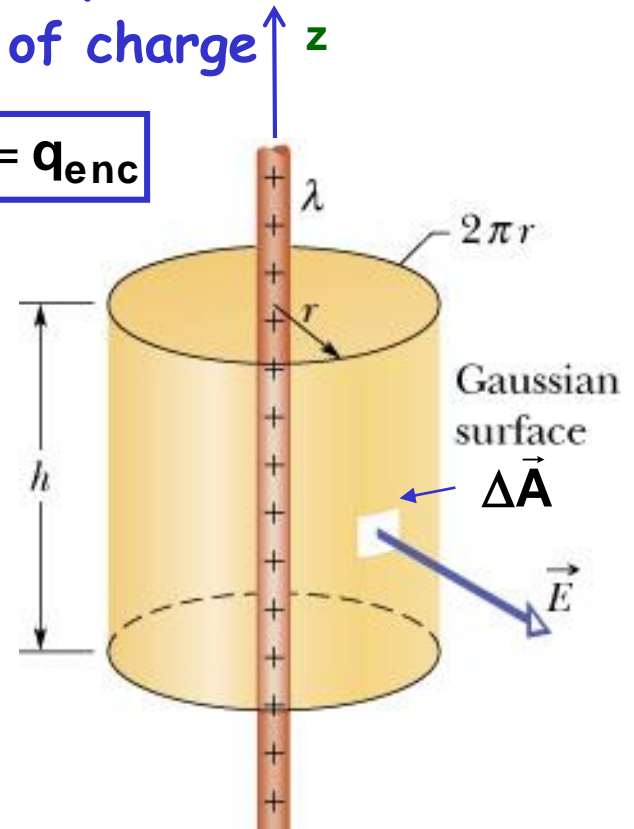
$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$



Example: Use Gauss' Law to find the electric field at a distance r from an infinitely long (thin) line of charge z

- Cylindrical symmetry around z -axis
- Uniform charge per unit length λ
- Every point on the infinite line has the identical surroundings, so....
 - E is radial, by symmetry
 - E has the same value everywhere on any concentric cylindrical surface
 - Flux through end caps = 0 as \underline{E} is perpendicular to $D\underline{A}$
- \underline{E} on cylinder is parallel to unit vector for $D\underline{A}$

$$\lambda h = q_{\text{enc}}$$



$$\text{Cylinder area} = 2\pi r h$$

$$\therefore \Phi = \oint_S \vec{E} \cdot d\vec{A} = 2\pi r h E = \frac{\lambda h}{\epsilon_0}$$

$$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

radially outward

- No integration needed now due to Gauss Law
- Good approximation for finite line of charge when $r \ll L$, far from the ends.

Field Lines and Conductors

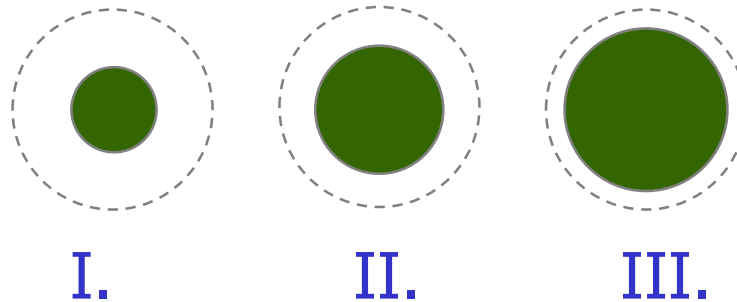
4-4: The drawing shows three cylinders in cross-section, each with the same total charge. Each has the same size cylindrical gaussian surface (again shown in cross-section). Rank the three according to the electric field at the gaussian surface, greatest first.

A. I, II, III

B. III, II, I

C. All tie.

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$



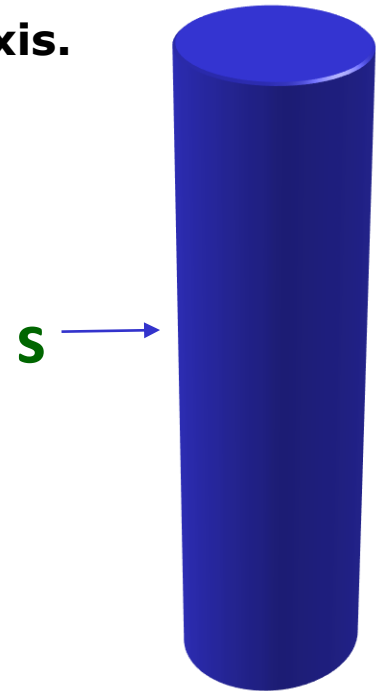
Use symmetry arguments where applicable

4-5: Outside a sphere of charge the electric field is just like that of a point charge of the same total charge, located at its center.

Outside of an infinitely long, uniformly charged conducting cylinder, which statement describes the electric field?

- A. Like that of a point charge at the center of the cylinder.
- B. Like a circular ring of charge at its center.
- C. Like an infinite line of charge along the cylinder axis.
- D. Cannot tell from the information given.
- E. The field equals zero

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$



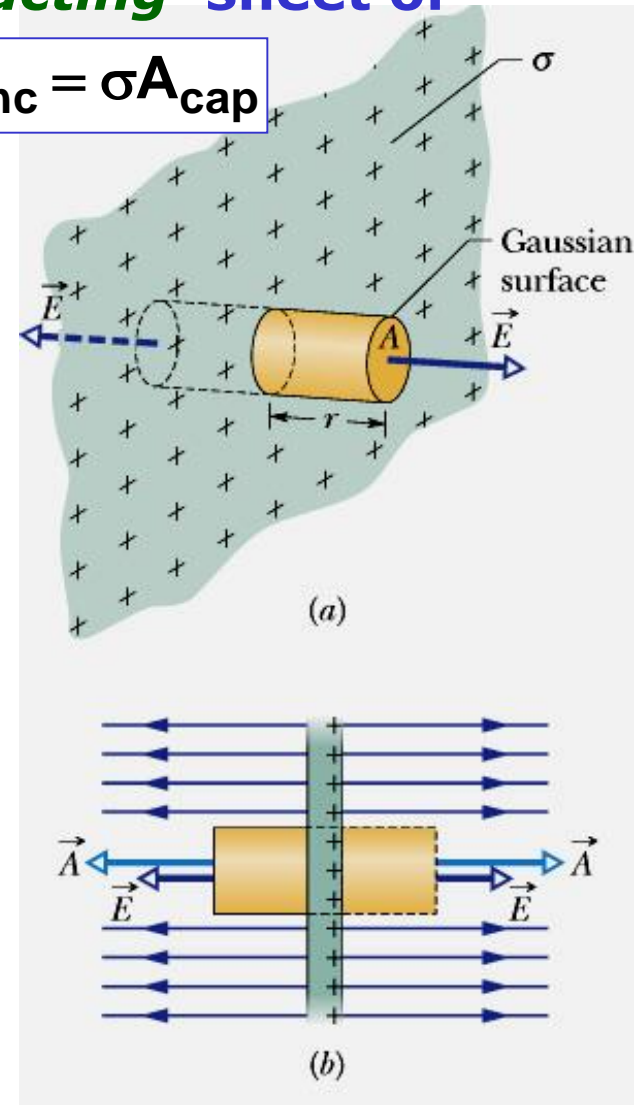
Electric field near an infinite *non-conducting* sheet of charge – using Gauss Law

- Cylindrical Gaussian Surface penetrating sheet (rectangular OK too)
- Uniform positive charge per unit area σ
- \mathbf{E} has same value when chosen point moves parallel to the surface
- \mathbf{E} points radially away from the sheet (both sides),
- \mathbf{E} is perpendicular to cylindrical part of surface
Flux through it = 0
- On both end caps \mathbf{E} is parallel to \mathbf{A}
so $F = +EA_{\text{cap}}$ on each

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2EA_{\text{cap}} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{\sigma A_{\text{cap}}}{\epsilon_0}$$

$$\therefore \mathbf{E} = \frac{\sigma}{2\epsilon_0}$$

$$q_{\text{enc}} = \sigma A_{\text{cap}}$$

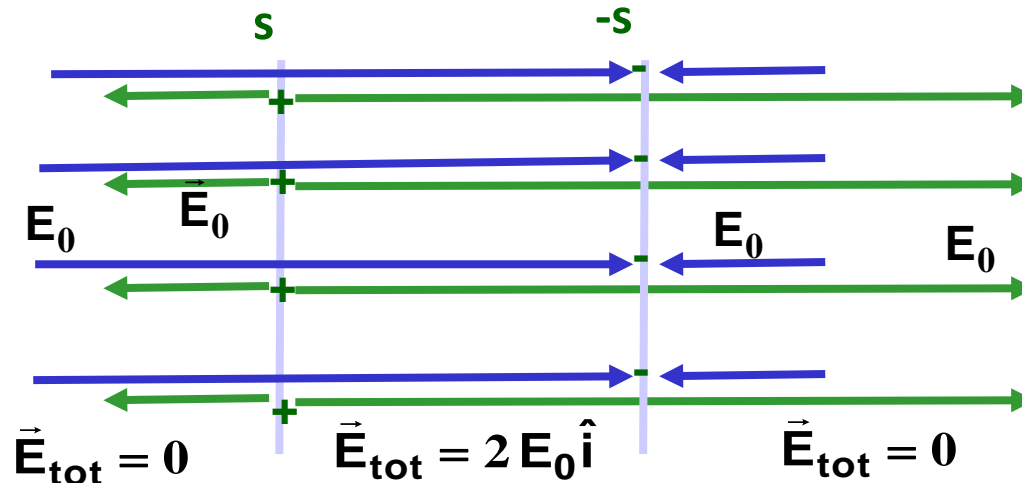


- Uniform field - independent of distance from sheet
- Same as earlier result but no integration needed now
- Good approximation for finite sheet when $r \ll L$, far from edges.

Example: Fields near parallel nonconducting sheets - 1

- Bring two “large” **nonconducting** sheets of charge close to each other,
- Approximate as infinite sheets
- The charge cannot move, use superposition.
- There is no screening, as there would be in a conductor.
- Each sheet produces a uniform field of magnitude: $E_0 = \frac{\sigma}{2\epsilon_0}$

Oppositely Charged Plates, same $|s|$

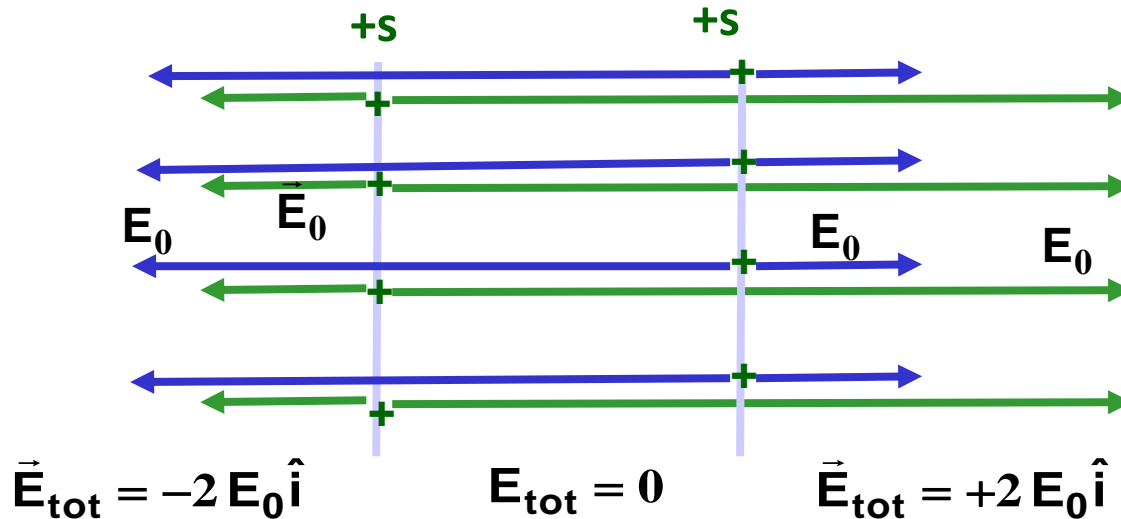


- Left region: Field due to the positively charged sheet is canceled by the field due to the negatively charged sheet. E_{tot} is zero.
- Right region: Same argument. E_{tot} is zero.
- Between plates: Fields reinforce. E_{tot} and is twice E_0 and to the right.

Example: Fields near parallel nonconducting sheets - 2

Positively Charged Plates, same s

$$E_0 = \frac{\sigma}{2\epsilon_0}$$



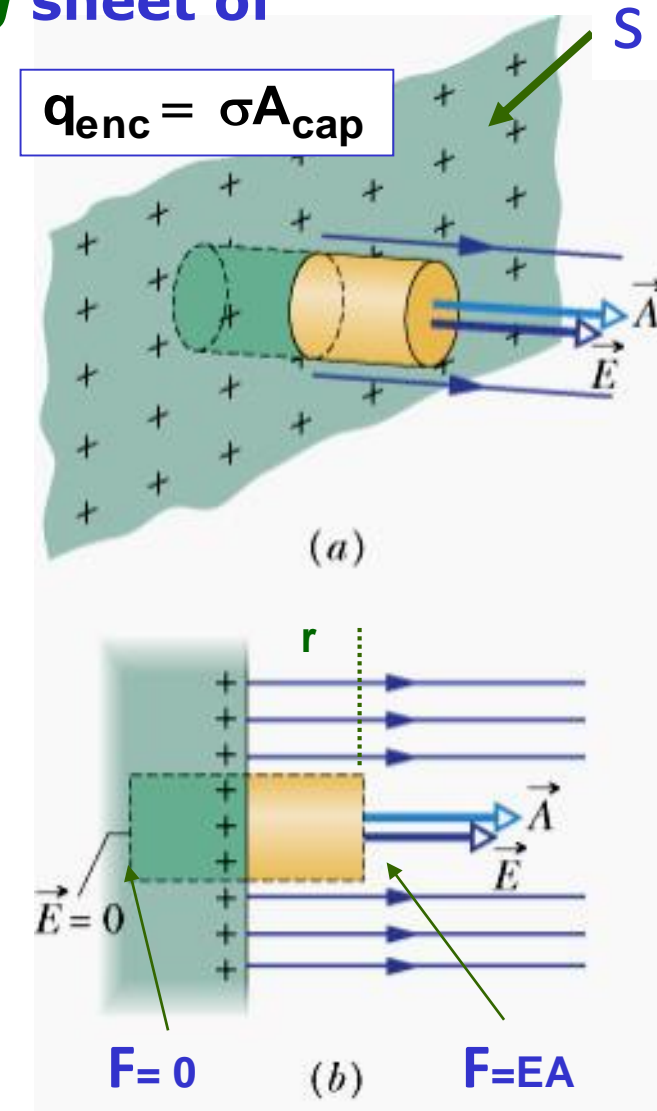
- Now, the fields reinforce to the left and to the right of both plates.
- Between plates, the fields cancel.
- Signs are reversed for a pair of negatively charged plates

Electric field near an *infinite conducting sheet* of charge – using Gauss' Law

- Uniform charge per unit area s on one face
- Use cylindrical or rectangular Gaussian surface
- End caps just outside and just inside ($E = 0$)
- \underline{E} points radially away from sheet outside otherwise current flows (!!)
- Flux through the cylindrical tube = 0
 \underline{E} normal to surface
- On left cap (inside conductor) $E = 0$ so $F = 0$
- On right cap \underline{E} is parallel to $D\underline{A}$ so $F = EA_{\text{cap}}$

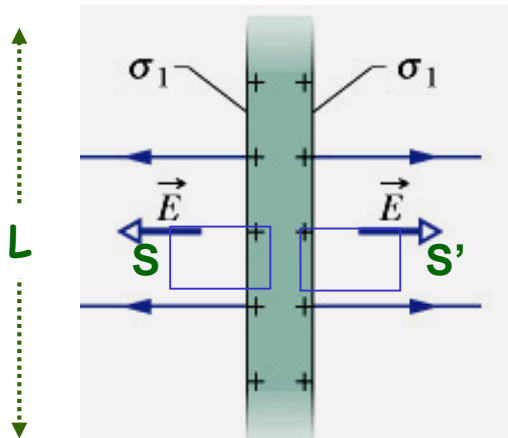
$$\therefore \Phi = EA_{\text{cap}} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{\sigma A_{\text{cap}}}{\epsilon_0}$$

$$\therefore \underline{E} = \frac{\sigma}{\epsilon_0}$$



- Field is twice that for a non-conducting sheet with same s
- Same enclosed charge, same total flux now "squeezed" out the right hand cap, not both
- Otherwise like previous result: uniform, no r dependence, etc.

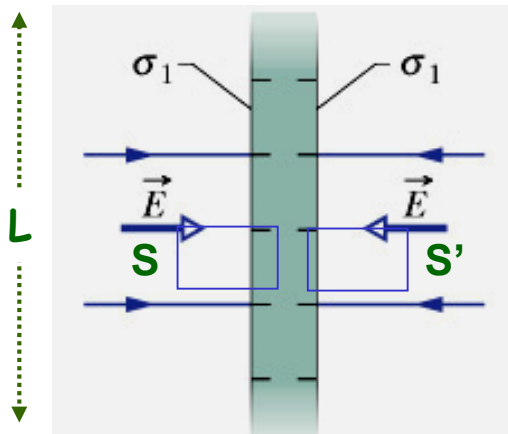
Charge on a finite sized conducting plate in isolation



- $E = 0$ inside the conductor
- Charge density s_1 is the same on both faces, o/w charges will move to make $E = 0$ inside
- Cylindrical Gaussian surfaces S, S'
- $S \rightarrow$ charges distribute on surfaces

or

$$\mathbf{E} = \pm \frac{\sigma_1}{\epsilon_0}$$



- Same field magnitude on opposite sides, opposite directions
- Same field by replacing each conductor by 2 charged non-conducting sheets alone:
 - cancellation inside conductor
 - reinforcement outside

Electric field near oppositely charged *conducting* plates (large, but not infinite)

Initially: charge density $+/- \sigma_1$ on both faces of each plate (neutral)

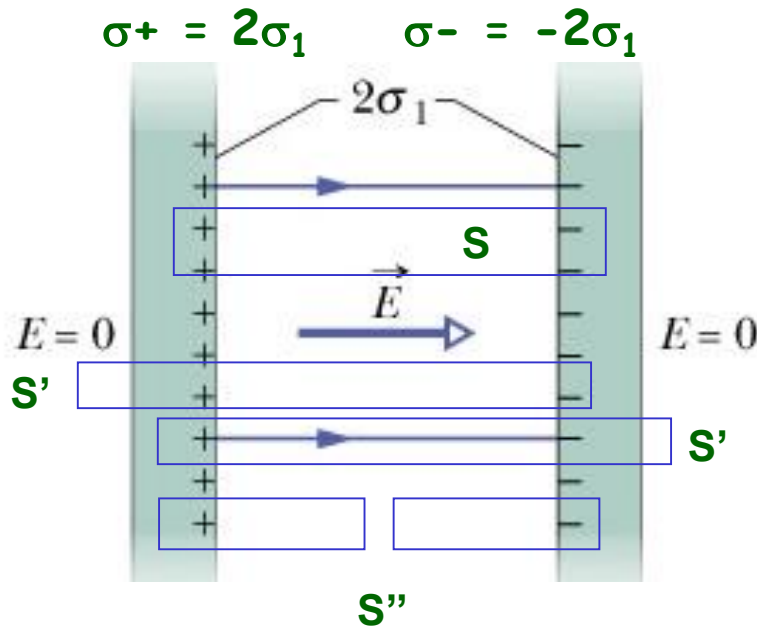
Then bring plates close to each other.

Free charge moves to make $E = 0$ within the metal.

All surface charge density ends up on inner faces, zero on outer.

(valid for infinite sheet)

$$\sigma^{+/-} = +/- 2\sigma_1$$



- Charge moves by induction to keep $E = 0$ inside each conductor
- Field between plates remains uniform
- Flux through Gaussian surface $S = 0$,
so $\sigma^- = -\sigma^+$
- Using either S' , $q_{enc} = 0$ so $E = 0$ outside
- So the charge density = 0 on the outer faces,
- All the net charge density must move to the inner surfaces
- Using either surface S'' :

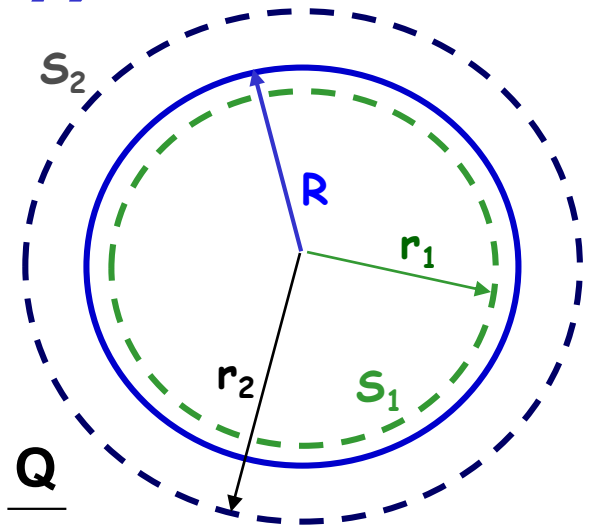
$$E_{\text{inside}} = \frac{\sigma^{+/-}}{\epsilon_0} = \pm \frac{2\sigma_1}{\epsilon_0}$$

- This already counts effects of both plates
- Fields everywhere would be the same if charge distributions were there w/o the conductors.

Parallel plate
capacitor

Proof of the Shell Theorem using Gauss' Law (spherical symmetry)

- Hollow shell of charge, net charge Q
- Spherically symmetric surface charge density σ , radius R
- Two spherical Gaussian surfaces:
 - S_1 is just inside shell, $r_1 < R$
 - S_2 is outside shell, $r_2 > R$
- q_{enc} means charge enclosed by S_1 or S_2



OUTSIDE: (on S_2)
$$\Phi_2 = \oint_{S_2} \vec{E} \cdot d\vec{A} = 4\pi r_2^2 E_2 = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\therefore E_2 = \frac{Q}{4\pi\epsilon_0 r_2^2} \quad \text{for } r_2 > R$$

Shell of charge acts like a point charge at the center of the sphere

INSIDE: (on S_1)
$$\Phi_1 = \oint_{S_1} \vec{E} \cdot d\vec{A} = 4\pi r_1^2 E_1 = \frac{q_{\text{enc}}}{\epsilon_0} = 0$$

$$\therefore E_1 = 0 \quad \text{for } r_1 < R$$

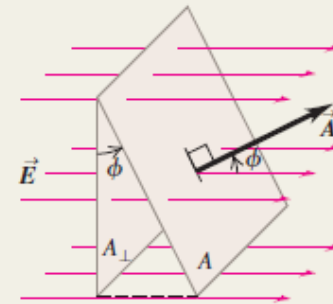
Shell of charge creates zero field inside (anywhere)

At point r inside a spherically symmetric volume (radius R) of charge,

- only the shells of charge with radii smaller than r contribute as point charges
- shells with radius between r and R produce zero field inside.

Electric flux: Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of an area element and the perpendicular component of \vec{E} , integrated over a surface. (See Examples 22.1–22.3.)

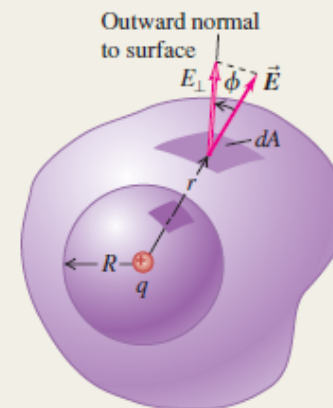
$$\begin{aligned}\Phi_E &= \int E \cos \phi \, dA \\ &= \int E_{\perp} \, dA = \int \vec{E} \cdot d\vec{A} \quad (22.5)\end{aligned}$$



Gauss's law: Gauss's law states that the total electric flux through a closed surface, which can be written as the surface integral of the component of \vec{E} normal to the surface, equals a constant times the total charge Q_{encl} enclosed by the surface. Gauss's law is logically equivalent to Coulomb's law, but its use greatly simplifies problems with a high degree of symmetry. (See Examples 22.4–22.10.)

When excess charge is placed on a conductor and is at rest, it resides entirely on the surface, and $\vec{E} = \mathbf{0}$ everywhere in the material of the conductor. (See Examples 22.11–22.13.)

$$\begin{aligned}\Phi_E &= \oint E \cos \phi \, dA \\ &= \oint E_{\perp} \, dA = \oint \vec{E} \cdot d\vec{A} \\ &= \frac{Q_{\text{encl}}}{\epsilon_0} \quad (22.8), (22.9)\end{aligned}$$



Electric field of various symmetric charge distributions: The following table lists electric fields caused by several symmetric charge distributions. In the table, q , Q , λ , and σ refer to the *magnitudes* of the quantities.

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Single point charge q	Distance r from q	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Charge q on surface of conducting sphere with radius R	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
	Inside sphere, $r < R$	$E = 0$
Infinite wire, charge per unit length λ	Distance r from wire	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$
Infinite conducting cylinder with radius R , charge per unit length λ	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$
	Inside cylinder, $r < R$	$E = 0$
Solid insulating sphere with radius R , charge Q distributed uniformly throughout volume	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
	Inside sphere, $r < R$	$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3}$
Infinite sheet of charge with uniform charge per unit area σ	Any point	$E = \frac{\sigma}{2\epsilon_0}$
Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$	Any point between plates	$E = \frac{\sigma}{\epsilon_0}$
Charged conductor	Just outside the conductor	$E = \frac{\sigma}{\epsilon_0}$