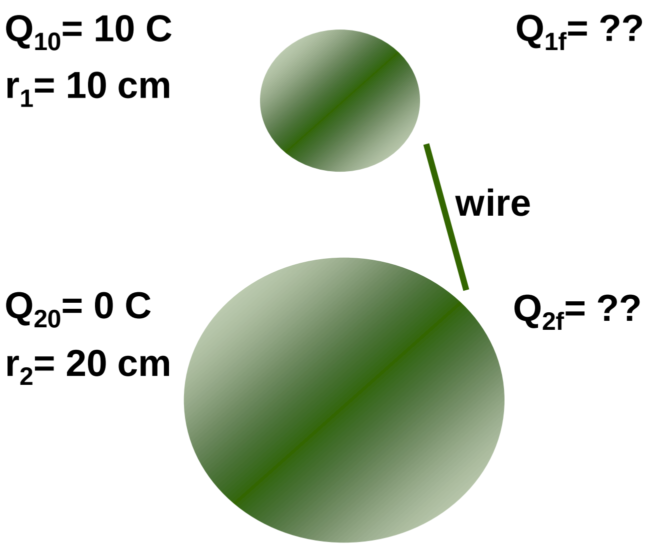
Physics 121 - Electricity and Magnetism

Lecture 05 -Electric Potential

Y&F Chapter 23 Sect. 1-5

- **Electric Potential Energy versus Electric Potential**
- **Calculating the Potential from the Field**
- **Potential due to a Point Charge**
- **Equipotential Surfaces**
- **Calculating the Field from the Potential**
- **Potentials on, within, and near Conductors**
- **Potential due to a Group of Point Charges**
- **Potential due to a Continuous Charge Distribution**
- **Summary**

Electrostatics: Two spheres, different radii, one with charge

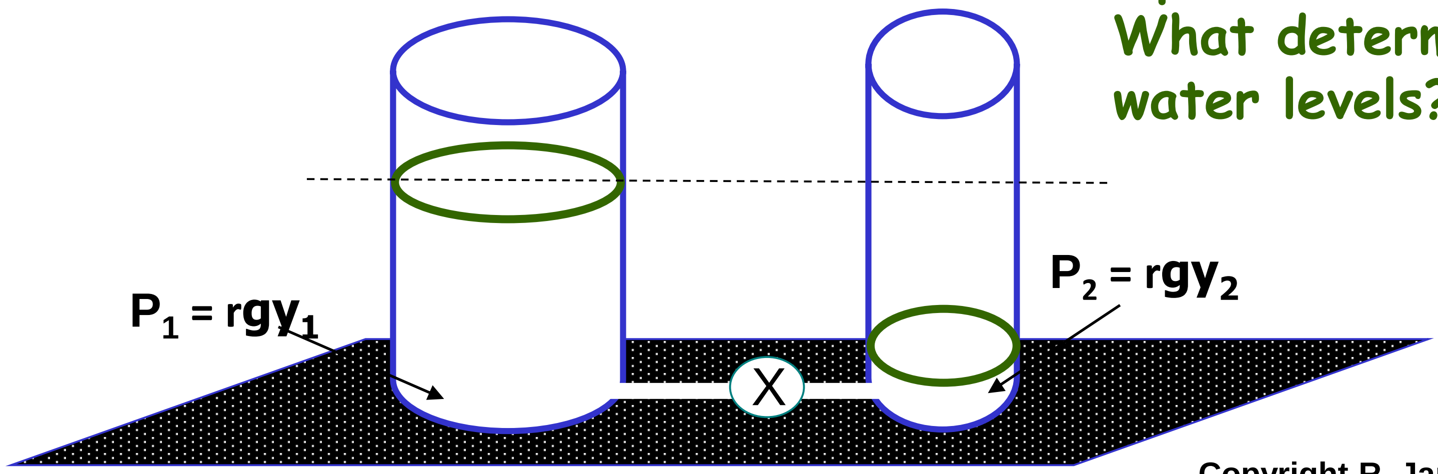


Connect wire between spheres, then disconnect it

Are final charges equal?
What determines how charge redistributes itself?

A mechanical analogy: Water pressure

Open valve, water flows
What determines final water levels?



ELECTRIC POTENTIAL $V(\vec{r})$

DEFINITION:
Electrostatic Potential = **Potential Energy per unit test charge due to an electric field**

- Related to Electrostatic Potential Energy……but……
- Summarizes effect of charge on a distant point without specifying a test charge there (Like field, unlike PE)
- ΔPE: ~ work done (= force x displacement)
- ΔV: ~ work done/unit charge (= field x displacement)
- Scalar field → Easier to use than E (vector)
- Both ΔPE and ΔV imply a reference level
- Both PE and V are conservative forces/fields, like gravity
- Can determine motion of charged particles using:
  - Second Law, F = qE
  - or PE, Work-KE theorem &/or mechanical energy conservation

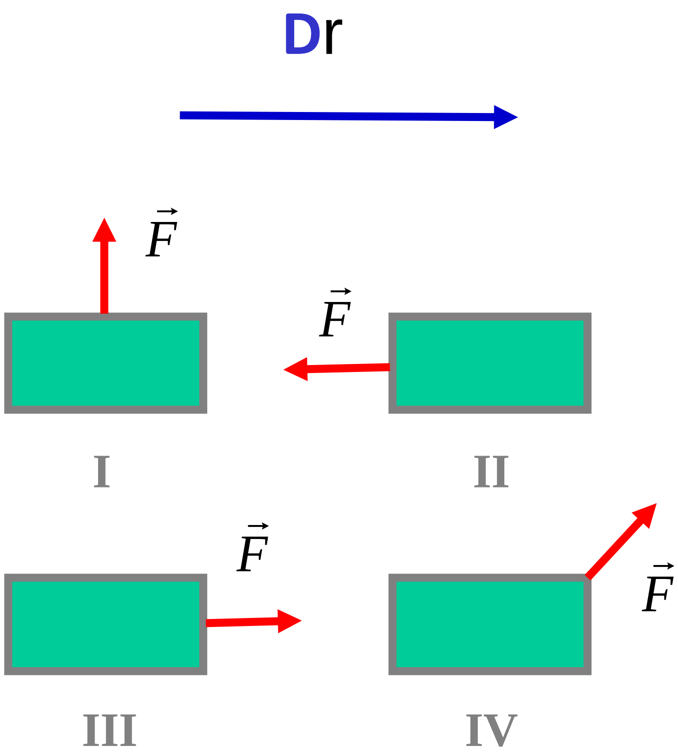
Units, Dimensions:

- Potential Energy: [U] are Joules
- Potential: [V] are [U]/[q] Joules/C. = VOLTS
- Synonyms: [V], [F][d]/[q], and [q][E][d]/[q] = N.m / C.
- Units of field [E] are [V]/[d] = Volts / meter – same as N/C.

Reminder: Work Done by a Constant Force

5-1: The figure shows four examples of force F is applied to an object. In all four cases, the force has the same magnitude and the displacement of the object is to the right and has the same magnitude.

Rank the cases in order of the work done by the force on the object, from most positive to the most negative.



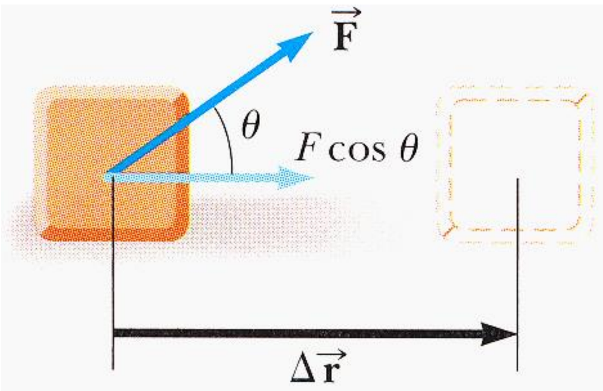
A. I, IV, III, II
B. II, I, IV, III
C. III, II, IV, I
D. I, IV, II, III
E. III, IV, I, II

Work Done by a Constant Force (a reminder)

The work ΔW done on a system by a constant external force on it is the product of:

- **the magnitude F of the force**
- **the magnitude Δs of the displacement of the point of application of the force**
- **and cos(θ), where θ is the angle between force and displacement vectors:**

$$\Delta W \equiv \vec{F}\cdot\Delta\vec{s} = F\Delta s\cos\theta$$



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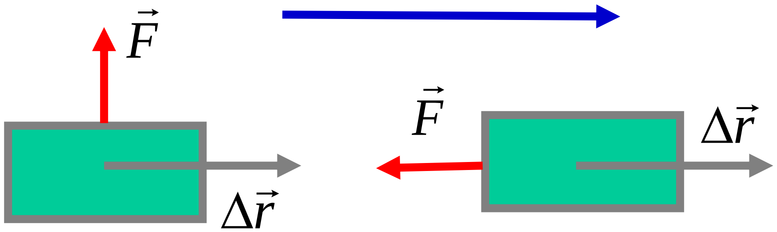
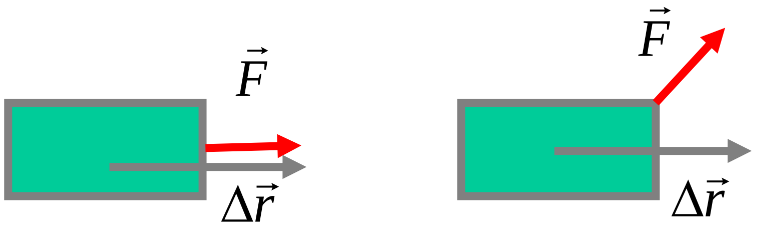


I *II*

$W_I = 0$ $W_{II} = -F\Delta r$



III *IV*

$W_{III} = F\Delta r$ $W_{IV} = F\Delta r\cos\theta$

If the force varies in direction and/or magnitude along the path:

$$\Delta W \equiv \int_i^f \vec{F}\circ d\vec{s}$$

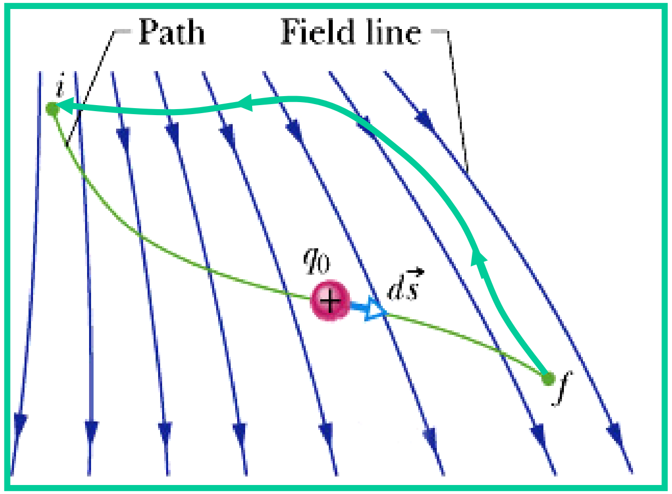
"Path Integral"

Electrostatic Potential Energy versus Potential

Conservative fields are associated with potential energy functions

- Work done around any closed path equals zero.
- Work done by the field on a test charge moving from i to f does not depend on the path taken.

$$\Delta U = -\Delta W = -\vec{F}_e \circ \Delta\vec{s} \qquad \text{(basic definition)}$$



$$\vec{F}_e = q_0\vec{E} \qquad \Delta U = q_0 \Delta V$$

POTENTIAL ENERGY DIFFERENCE:

Charge q_0 moves from i to f along ANY path

$$U_f - U_i \equiv \Delta U \equiv -\Delta W \equiv -\int_i^f \vec{F}_e \circ d\vec{s} = -q_0 \int_i^f \vec{E} \circ d\vec{s}$$

POTENTIAL DIFFERENCE:

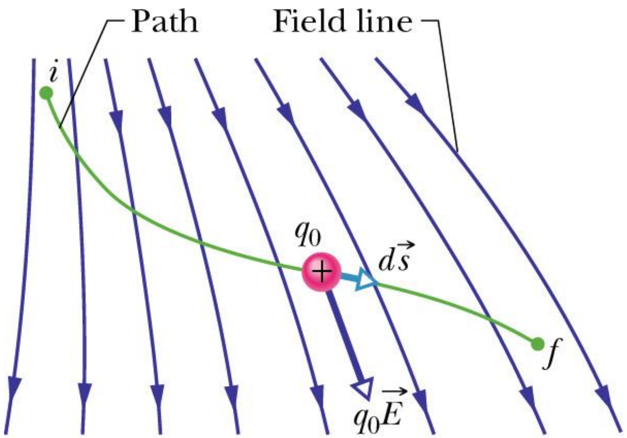
$$\Delta V \equiv -\frac{\Delta W}{q_0} = -\vec{E} \circ \Delta\vec{s} \qquad \text{(from basic definition)}$$

Potential is potential energy per unit charge

$$V_f - V_i \equiv \Delta V \equiv -\Delta W / q_0 = -\int_i^f \vec{E} \circ d\vec{s}$$

(Evaluate integrals on ANY path from i to f)

Some distinctions and details



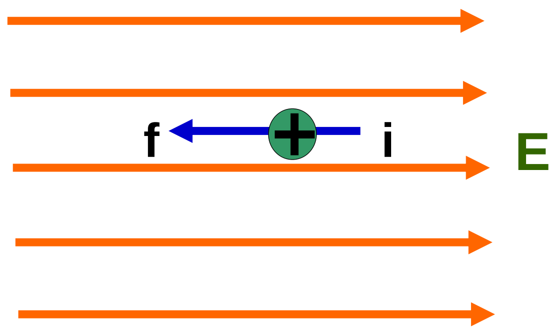
- The field depends on a charge distribution elsewhere).
- A test charge q_0 moved between i and f gains or loses potential energy DU.
- ΔU does not depend on path
- ΔV is also path-independent and also does not depend on $|q_0|$ (test charge).
- Use Work-KE theorem to link potential differences to motion

$$\Delta U = q_0 \Delta V$$

- Only differences in electric potential and PE are meaningful:
  - Relative reference: Choose arbitrary zero reference level for ΔU or ΔV.
  - Absolute reference: Set $U_i = 0$ with all charges infinitely far apart
  - Volt (V) = SI Unit of electric potential
  - 1 volt = 1 joule per coulomb = 1 J/C
  - 1 J = 1 VC and 1 J = 1 N m
- Electric field units – new name:
  - 1 N/C = (1 N/C)(1 VC/1 Nm) = 1 V/m
- Electrostatic energy: electron volt
  - 1 eV = work done moving charge e through a 1 volt potential difference = $(1.60\times10^{-19}$ C)(1 J/C) = 1.60×10^{-19} J

Work and PE : Who/what does positive or negative work?

5-2: In the figure, suppose we exert a force and move the proton from point *i* to point *f* in a uniform electric field directed as shown. Which statement of the following is true?



A. **Electric field does positive work on the proton.**
Electric potential energy of the proton increases.

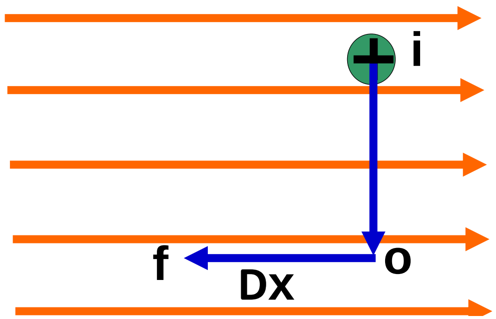
B. **Electric field does negative work on the proton.**
Electric potential energy of the proton decreases.

C. **Our force does positive work on the proton.**
Electric potential energy of the proton increases.

D. **Our force does positive work on the proton.**
Electric potential energy of the proton decreases.

E. **The changes cannot be determined.**

EXAMPLE: Find change in potential as test charge $+q_0$ moves from point i to f in a uniform field



DU and DV depend only on the endpoints

ANY PATH from i to f gives same results

To convert potential to/from PE just multiply/divide by q_0

$$\Delta V_{fi} = -\int_{path} \vec{E} \circ d\vec{s}$$

$$\vec{F}_e = q_0 \vec{E}$$

$$\Delta U_{fi} \equiv -\Delta W_{fi} = -\int_{path} \vec{F} \circ d\vec{s}$$

$$\Delta V_{fi} \equiv \Delta U_{fi} / q_0 \qquad \Delta U_{fi} = q_0 \Delta V_{fi}$$

EXAMPLE: CHOOSE A SIMPLE PATH THROUGH POINT "O"

$$\Delta V_{f,i} = \Delta V_{o,i} + \Delta V_{f,o}$$

$\Delta V_{o,i} = 0$ Displacement i → o is normal to field (path along equipotential)

$$\therefore \ \Delta V_{f,i} = \Delta V_{f,o} = -\vec{E} \circ \Delta\vec{x} = +E\,|\Delta x|$$

- External agent must do positive work on positive test charge to move it from o → f
  - units of E can be volts/meter
- E field does negative work

What are signs of DU and ΔV if test charge is negative?

Potential Function for a Point Charge

- For charges infinitely far apart choose $V_{infinity} = 0$ (reference level)
- $\Delta U =$ work done on a test charge as it moves to final location
- $\Delta U = q_0 \Delta V$
- Field is conservative → may choose most convenient path = radial

Find potential V(R) a distance R from a point charge q :

$$V(R) \equiv V_\infty - V_R = -\int_R^\infty \vec{E} \circ d\vec{s} \quad \text{along radial path from } r = R \text{ to } \infty$$

$$\vec{E}(r) = k\frac{q}{r^2}\hat{r} \;\Rightarrow\; -\int_R^\infty \vec{E} \circ d\vec{s} = -V(R) = -kq\int_R^\infty \frac{dr}{r^2} = kq\left.\frac{1}{r}\right|_R^\infty = -k\frac{q}{R}$$

$$\therefore \; V(R) = +k\frac{q}{R}$$

- Positive for q > 0, Negative for q<0
- Inversely proportional to r^1 NOT r^2

Similarly, for potential ENERGY: (use same method but integrate force)

$$U(r) \equiv q_0 V(R) = k\frac{q \cdot q_0}{R}$$

- Shared PE between q and q_0
- Overall sign depends on both signs

Example: $q = 1\ \mu C$. $R = 1$ m → $V_R = +9000$ Volts
If a test charge $q_0 = +/- 3\ \mu C$ then $U_R = +/- .027$ Joules

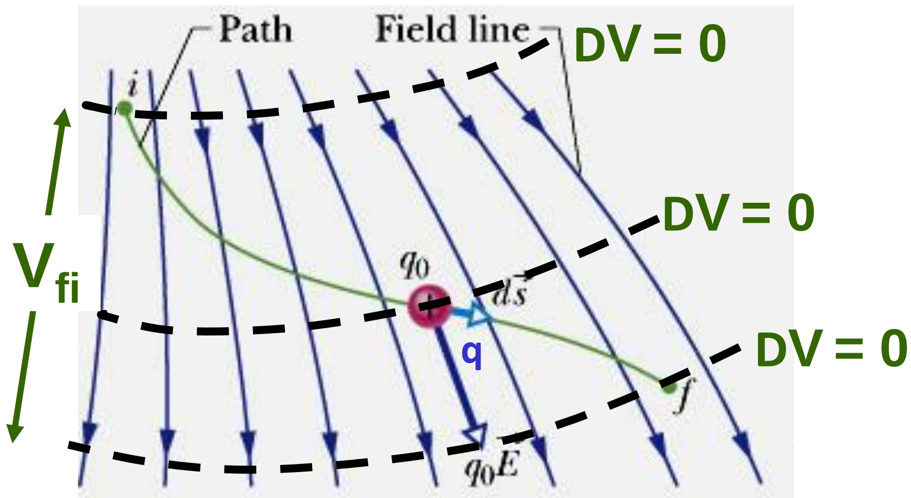
On equi-potential surfaces:

- Voltage and potential energy are constant i.e. $\Delta V=0$, $\Delta U=0$
- Zero work is done moving charges along an equi-potential
- No change in potential energy on an equi-potential
- Electric field must be perpendicular to displacement along surface

$$\Delta V \equiv -\vec{E} \circ \Delta\vec{s} = -E\,\Delta s\cos(\theta) = 0 \text{ on surface}$$

and $$\Delta U = -\Delta W = -\vec{F}_e \circ \Delta\vec{s} = 0$$

$V_i > V_f$



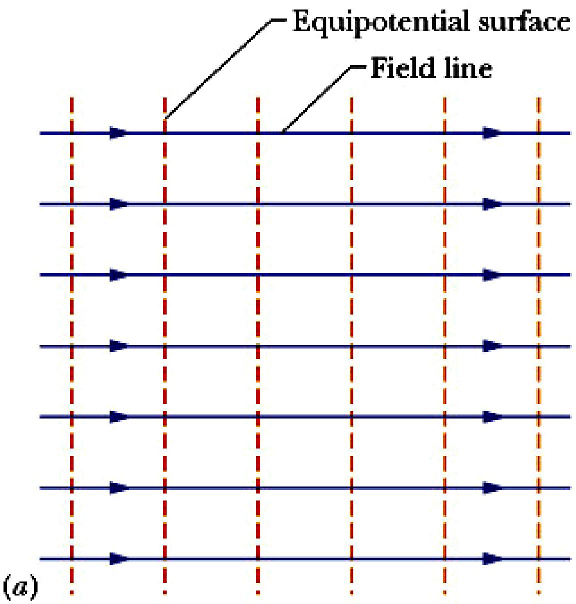
• Equipotentials are perpendicular to the electric field lines

CONDUCTORS ARE ALWAYS EQUIPOTENTIALS
- Charge on conductors moves to make $E_{in} = 0$
- E_{surf} is perpendicular to surface

⇒ so DV = 0 along any path on or in a conductor

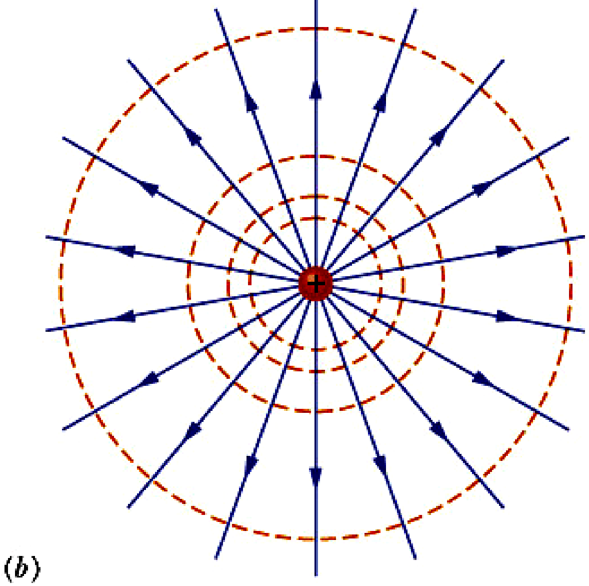
Examples of equipotential surfaces

Uniform Field



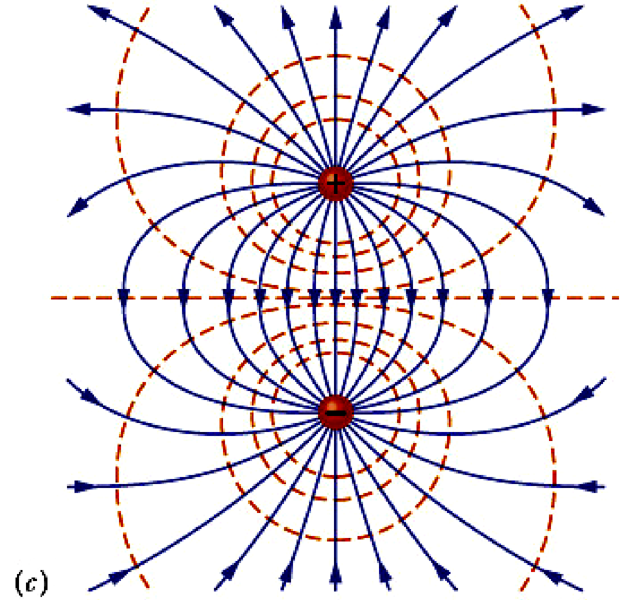
Equipotentials are planes

Point charge or outside sphere of charge



Equipotentials are spheres

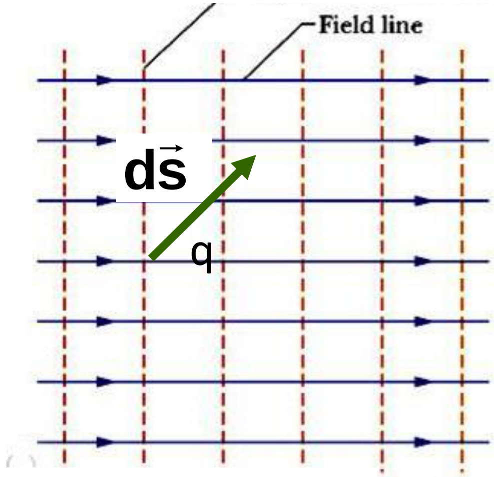
Dipole Field



Equipotentials are not simple shapes

The field E(r) is the gradient of the potential

$$dV \equiv -\vec{E} \circ d\vec{s} = -E\, ds \cos(\theta)$$



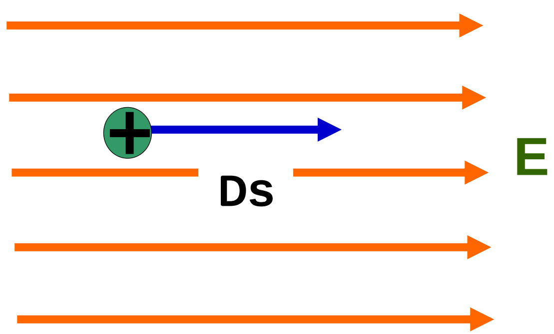
- Equipotentials are perpendicular to the field
- For path along equipotential, DV = 0
- Component of d$\underline{s}$ on $\underline{E}$ produces potential change

- **Gradient = spatial rate of change**

$$\therefore\ \vec{E} = -\frac{dV}{d\vec{s}} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \qquad d\vec{s} \text{ is } \perp \text{ to equipotential}$$

$$\text{Math note: } \frac{\partial f(x,y,z)}{\partial x} \text{ is a "partial" derivative}$$

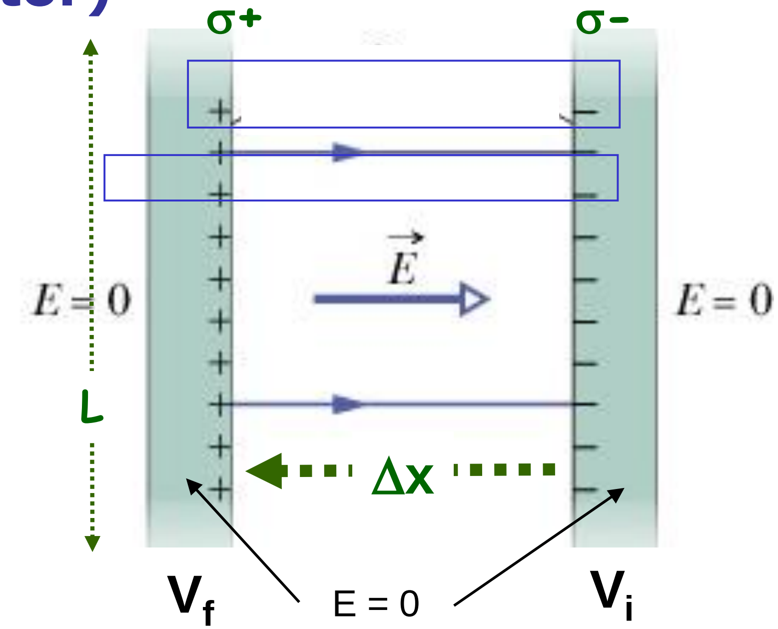
EXAMPLE: UNIFORM FIELD $\underline{E}$



$$\Delta V \equiv -\vec{E} \circ \Delta\vec{s} = -E\,\Delta s$$

$$\Delta U = q_0 \Delta V = -q_0 E \Delta s = -F \Delta s$$

Potential difference between oppositely charged conductors (parallel plate capacitor)



- Equal and opposite charge densities
- All charge on inner surfaces

$$\Delta x << L$$

$$\sigma^+ = -\sigma^- \quad |\sigma^+| \equiv \sigma \quad E = \frac{\sigma}{\varepsilon_0}$$

$$\Delta V \equiv V_f - V_i = -\vec{E} \circ \Delta \vec{x}$$

Example:

Find the potential difference ΔV across the capacitor, assuming:

- σ = 1 nanoCoulomb/m²
- Δx = 1 cm & points from plate "i" to plate "f"
- Uniform field E

$$\Delta V = -\vec{E} \circ \Delta \vec{x} = -E\,\Delta x = \frac{1 \times 10^{-9}}{\varepsilon_0} \times 10^{-2}$$

$$\Delta V = +1.13 \text{ volts}$$

A test charge +q loses potential energy ΔU = qΔV as it moves from + plate to – plate along any path (including external circuit)

Comparison of point charge and mass formulas

VECTORS	FORCE	FIELD	
Gravitation	$\vec{F}(\vec{r}) = G\frac{mM}{r^2}\hat{r}$	$\vec{g}(\vec{r}) = G\frac{M}{r^2}\hat{r}$	force/unit mass (acceleration)
Electrostatics	$\vec{F}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$	$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$	force/unit charge (n/C)

SCALARS	POTENTIAL ENERGY	POTENTIAL	
Gravitation	$U_g(r) = -G\frac{mM}{r}$	$V_g(r) = -G\frac{M}{r}$	PE/unit mass (not used often)
Electrostatics	$U_e(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$	$V_e(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	PE/unit charge

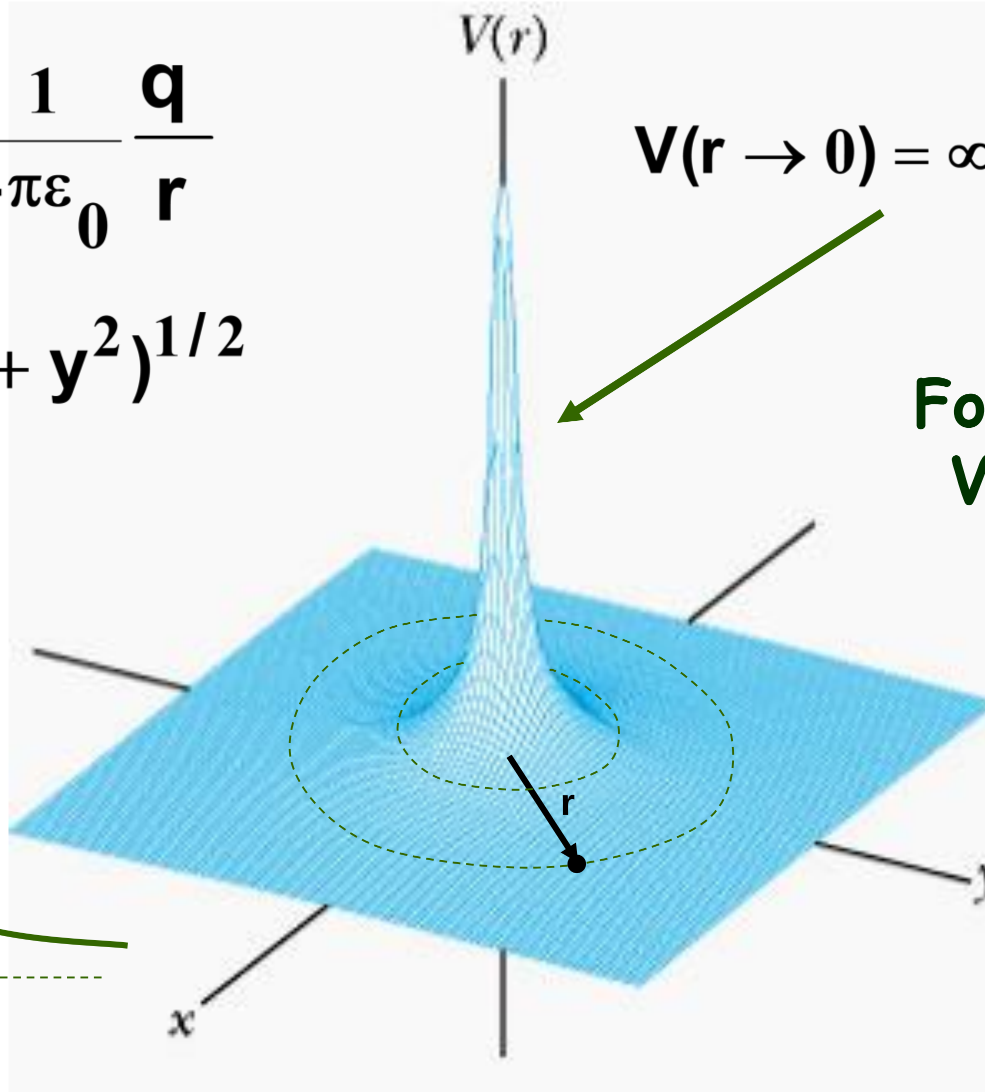
Fields and forces ~ $1/R^2$ but Potentials and PEs ~ $1/R^1$

Visualizing the potential function V(r) for a positive point charge (2 D)

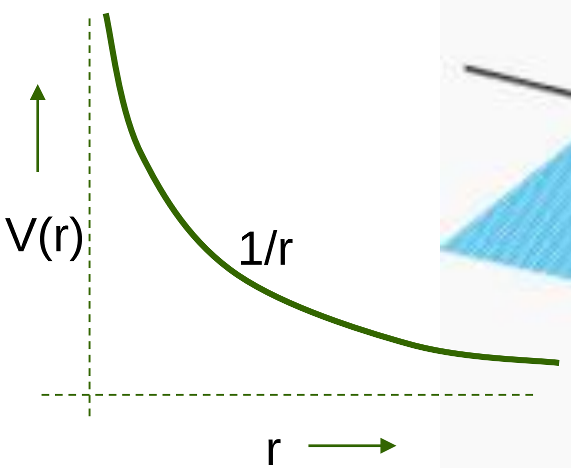
$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$r = (x^2 + y^2)^{1/2}$$

$V(r \to 0) = \infty$ "pole"



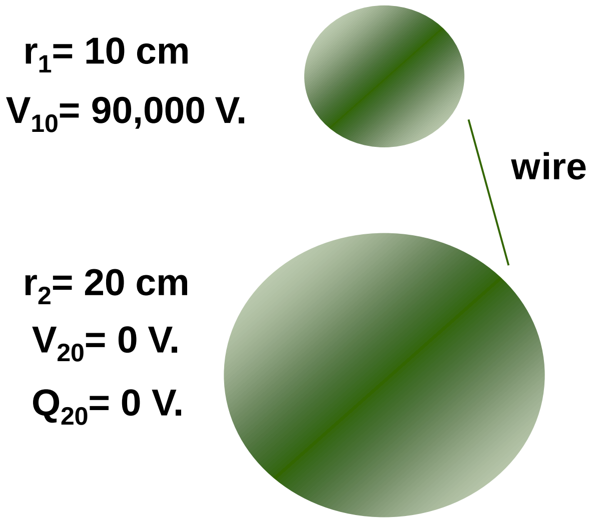
For q negative
V is negative
(funnel)



Conductors are always equipotentials

Example: Two spheres, different radii, one charged to 90,000 V.
Connect wire between spheres – charge moves

Conductors come to same potential
Charge redistributes to make it so



$$V_{1f} = V_{2f} \qquad Q_{1f} + Q_{2f} = Q_{10}$$

Initially:

$$V_{10} = 9\times 10^4 \text{ Volts} = \frac{kQ_{10}}{r_1} \Rightarrow Q_{10} = 1.0\ \mu\text{C.}$$

Find the final charges:

$$V_{1f} = \frac{kQ_{1f}}{r_1} = V_{2f} = \frac{k[Q_{10} - Q_{1f}]}{r_2}$$

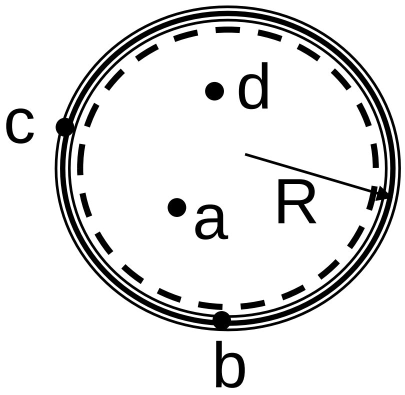
$$Q_{1f} = Q_{10}\left(1 + \frac{r_2}{r_1}\right)^{-1} = 0.33\ \mu\text{C.}$$

$$Q_{2f} = Q_{10} - Q_{1f} = 0.67\ \mu\text{C.}$$

Find the final potential(s):

$$V_{1f} = \frac{kQ_{1f}}{r_1} = \frac{9\times 10^9 \times 0.33 \times \text{x}10^{-6}}{0.1} = 30{,}000 \text{ Volts} = V_{2f}$$

Potential inside a hollow conducting shell



$V_c = V_b$ (shell is an equipotential)
$V_b = V_c = 18{,}000$ Volts on surface
R = 10 cm
Shell can be any closed surface (sphere or not)

Find potential V_a at point "a" inside shell

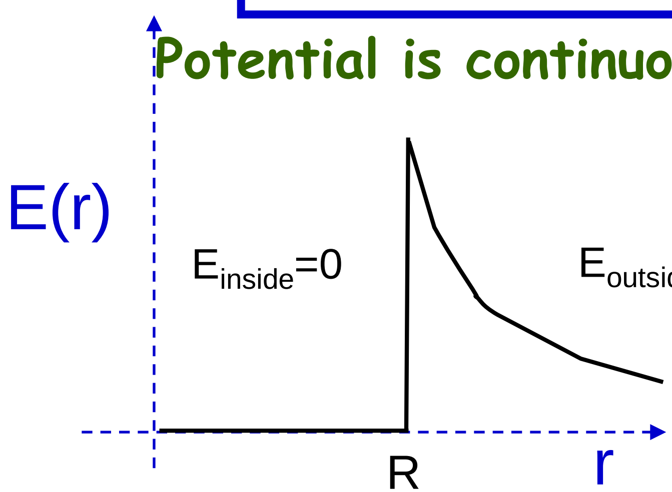
Definition: $$\Delta V_{ab} \equiv V_a - V_b = -\int_b^a \vec{E} \circ d\vec{s}$$

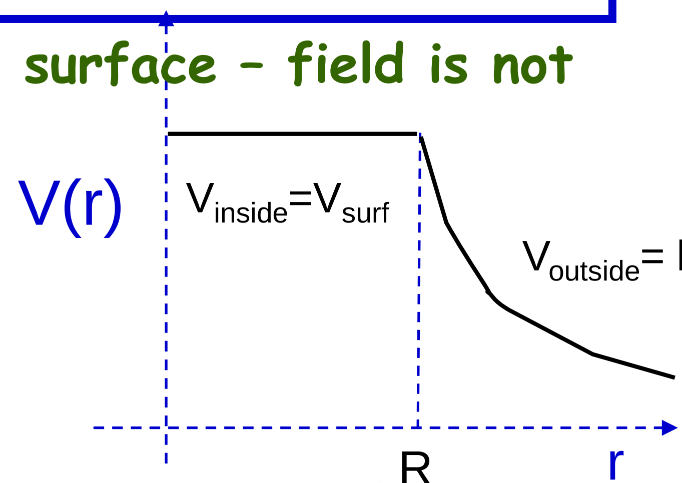
Apply Gauss' Law: choose GS just inside shell:

$$q_{enc} = 0 \;\Rightarrow\; E = 0 \text{ everywhere inside } \Rightarrow \Delta V = 0$$

$$\therefore\; V_a = V_{surface} = V_b = V_c = V_d = 18{,}000 \text{ Volts}$$

Potential is continuous across surface - field is not

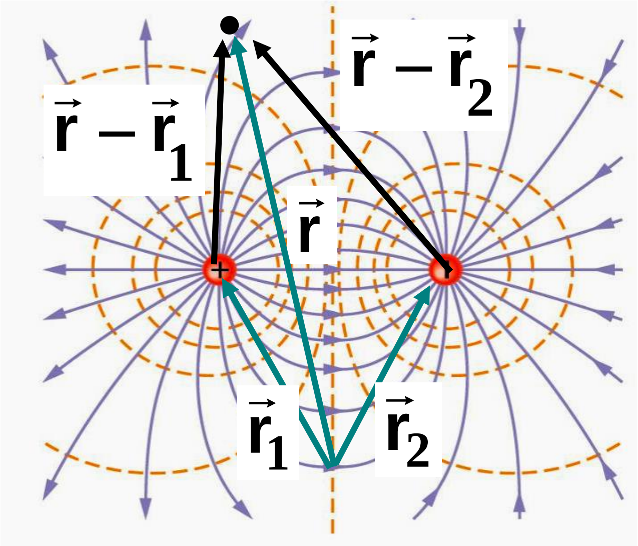




Potential due to a group of point charges

- Use superposition for n point charges

$$V(\vec{r}) = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|}$$



- The sum is an algebraic sum, not a vector sum.

- Reminder: For the electric field, by superposition, for n point charges

$$\vec{E}(\vec{r}) = \sum_{i=1}^{n} \vec{E}_i \equiv \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|^2} \hat{r}_i$$

- E may be zero where V does not equal to zero.
- V may be zero where E does not equal to zero.

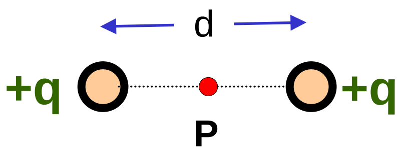
Examples: potential due to point charges

Use Superposition

Note: E may be zero where V does not = 0

V may be zero where E does not = 0

TWO EQUAL CHARGES – Point P at the midpoint between them



$$E_P = 0 \quad \text{by symmetry}$$

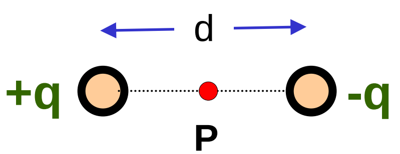
$$V_P = \frac{kq}{d/2} + \frac{kq}{d/2} = 4\frac{kq}{d} \quad \text{obviously not zero}$$

F and E are zero at P but work would have to be done to move a test charge to P from infinity.

Let q = 1 nC, d = 2 m:

$$V_P = 4\frac{9 \times 10^{9} \times 10^{-9}}{2} = 18 \text{ Volts}$$

DIPOLE – Otherwise positioned as above



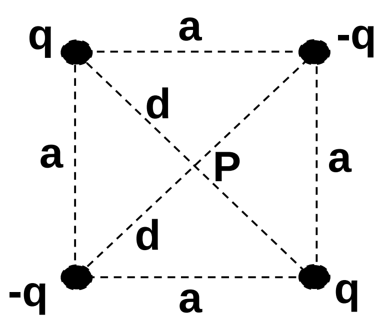
$$E_P \neq 0 \quad \text{obviously} \qquad E_P = 2\frac{kq}{d^2/4} = 8\frac{kq}{d^2}$$

$$\text{but } V_P = \frac{kq}{d/2} - \frac{kq}{d/2} = 0$$

Let q = 1 nC, d = 2 m:

$$E_P = \; = 8\frac{9 \times 10^{9} \times 10^{-9}}{4} = 18 \text{ V/m (or N/C)}$$

Another example: square with charges on corners



Find E & V at center point P

$$d = a\sqrt{2}/2$$

$$E_P = 0 \quad \text{by symmetry}$$

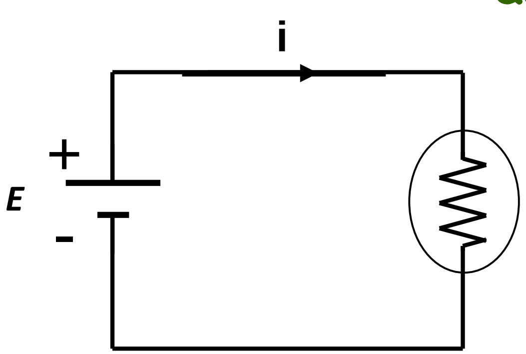
$$V_P = \sum_i \frac{kq_i}{r_i} = \frac{k}{d}\sum_i q_i = \frac{k}{d}[q - q + q - q] \Rightarrow V_P = 0$$

Another example: same as above with all charges positive

$$E_P = 0 \quad \text{by symmetry, again}$$

$$V_P = \sum_i \frac{kq_i}{r_i} = \frac{k}{d}\sum_i q_i = \frac{4kq}{a\sqrt{2}/2} = \frac{8kq}{a\sqrt{2}} = 510 \text{ Volts}$$

Another example: find work done by 12 volt battery in 1 minute as 1 ampere current flows to light lamp



$$\Delta W \equiv \text{work done} = -\Delta U = -Q\Delta V$$

$$Q = \text{charge moved from + to - by current}$$

$$= i\,\Delta t = 1\,\text{amp} \times 60\,\text{sec} = 60\,\text{C.}$$

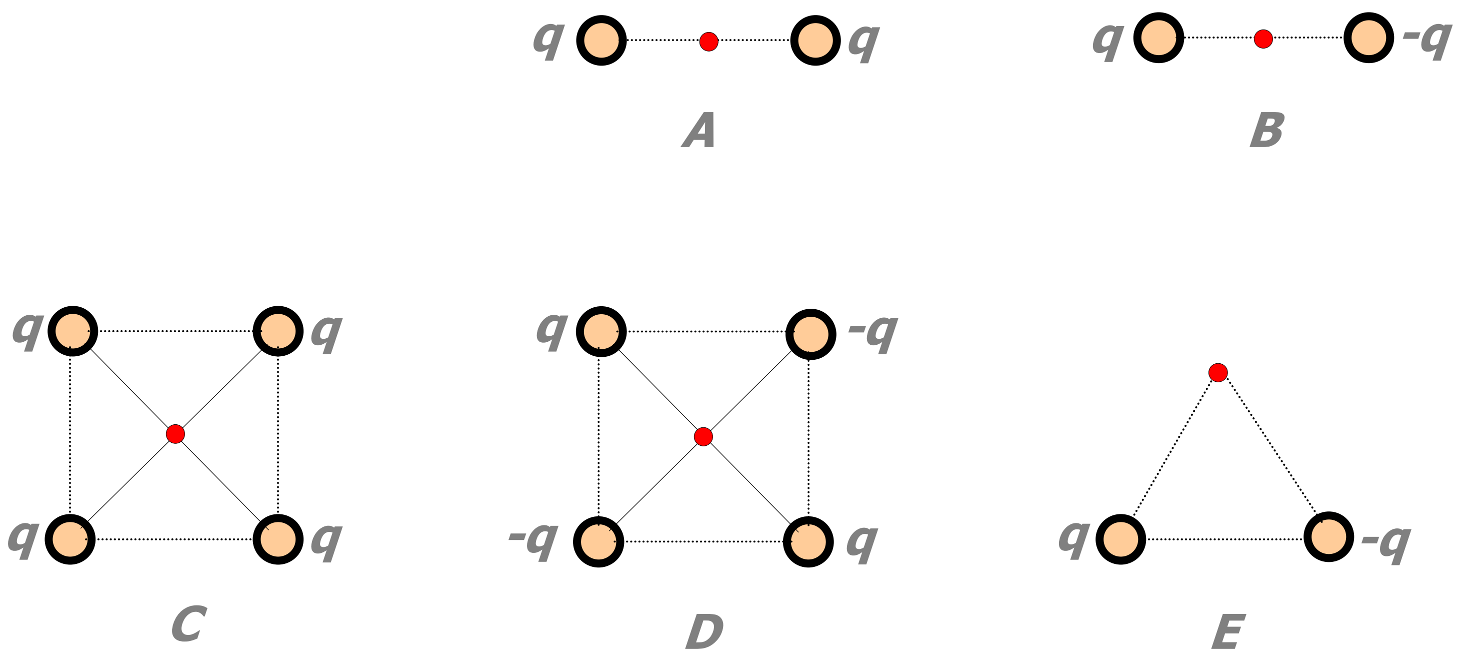
$$\Delta U = Q\Delta V = 60 \times \Delta V \qquad \Delta V = -12\,\text{Volts}$$

$$\therefore \Delta U = -720\,\text{Joules}$$

$$\Delta W = -\Delta U = +720\,\text{Joules (from battery)}$$

Electric Field and Electric Potential

5-3: Which of the following figures have V=0 and E=0 at the red point?



Method for finding potential function V at a point P due to a continuous charge distribution

1. Assume V = 0 infinitely far away from charge distribution (finite size)

2. Find an expression for dq, the charge in a "small" chunk of the distribution, in terms of λ, σ, or ρ

$$dq = \begin{Bmatrix} \lambda dl \quad \text{for a linear distribution} \\ \sigma d^2A \quad \text{for a surface distribution} \\ \rho d^3V \quad \text{for a volume distribution} \end{Bmatrix}$$

Typical challenge: express above in terms of chosen coordinates

3. At point P, dV is the differential contribution to the potential due to a point-like charge **dq** located in the distribution. Use symmetry.

$$dV = \frac{dq}{4\pi\varepsilon_0 r} \qquad \text{scalar}, \quad r = \text{distance from dq to P}$$

4. Use "superposition". Add up (integrate) the contributions over the whole distribution, varying the displacement r as needed. **Scalar V_P.**

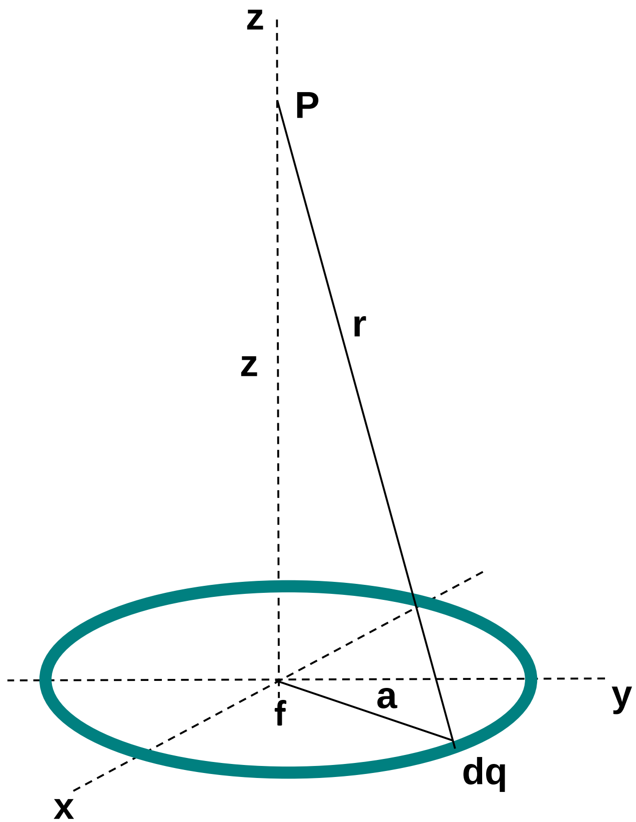
$$V_P = \int_{\text{dist}} dV_P = \frac{1}{4\pi\varepsilon_0} \int_{\text{dist}} \frac{dq}{r} \quad \text{(line, surface, or volume integral)}$$

5. Field E can be gotten from potential by taking the "gradient":

$$dV \equiv -\vec{E} \circ d\vec{s} \quad \Rightarrow \quad \vec{E} = -\frac{\partial V}{\partial \vec{s}} \equiv -\vec{\nabla} V$$

Rate of potential change perpendicular to equipotential

Example: Potential along Z-axis of a ring of charge



Q = charge on the ring

l = uniform linear charge density = Q/2pa

r = distance from dq to "P" = $[a^2 + z^2]^{1/2}$

ds = arc length = $a d\phi$

$$dq = \lambda ds = \lambda a d\phi$$

$$dV = k\frac{dq}{r}$$

All scalars - no need to worry about direction

$$V = \int_{ring} dV = \frac{ka\lambda}{r}\int_0^{2\pi} d\phi = \frac{kQ}{r}$$

$$\therefore\ V = \frac{kQ}{[z^2 + a^2]^{1/2}}$$

- *As z → 0, V → kQ/a*
- *As a → 0 or z → inf, V → point charge*

FIND ELECTRIC FIELD USING GRADIENT (along z by symmetry)

$$E_z = -\frac{\partial V}{\partial z}\hat{k} = \frac{kQ}{2r^3}\frac{\partial(z^2)}{\partial z}\hat{k} = \frac{kQz}{[z^2 + a^2]^{3/2}}\hat{k}$$

As Before

- E → 0 as z → 0 (for "a" finite)
- E → point charge formula for z >> a

Example: Potential Due to a Charged Rod

- A rod of length L located parallel to the x axis has a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance d from the origin.
- Start with

$$r \equiv [x^2 + d^2]^{1/2}$$

$$dq = \lambda dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+d^2)^{1/2}}$$

- Integrate over the charge distribution

$$V = \int dV = \int_0^L \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{(x^2+d^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + (x^2+d^2)^{1/2}\right)\right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + (L^2+d^2)^{1/2}\right) - \ln(d)\right]$$

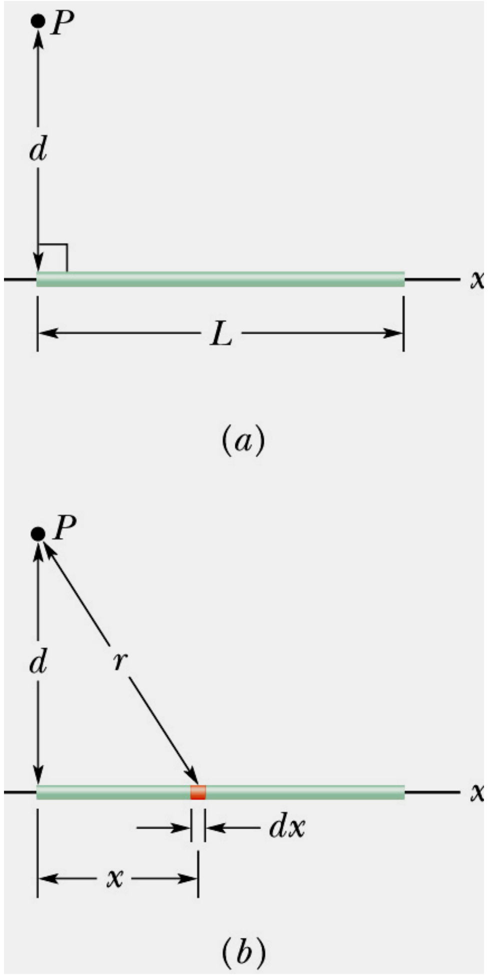
- Check by differentiating

$$\frac{d}{dx}\log(x+r) \quad \text{for } r = [x^2+d^2]^{1/2}$$

$$\frac{d}{dx}\log(x+r) = \frac{1}{x+r}\frac{d(x+r)}{dx} = \frac{1}{x+r}(1+\frac{dr}{dx}) = \frac{1}{x+r}(1+\frac{x}{r}) = \frac{1}{x+r}(\frac{r+x}{r}) = \frac{1}{r}$$

- Result

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$

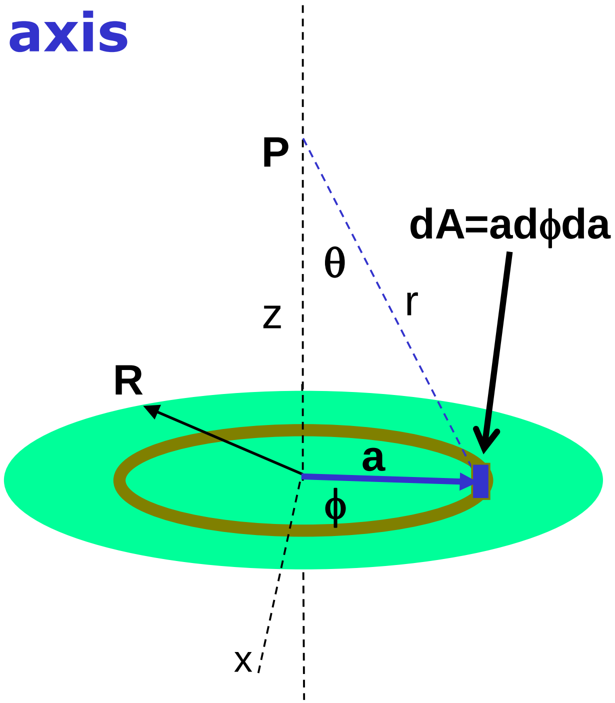


Example: Potential on the symmetry axis of a disk of charge

- Q = charge on disk whose radius = R.
- Uniform surface charge density $\sigma = Q/4\pi R^2$
- Disc is a set of rings, each of them da wide in radius
- For one of the rings:

$$r^2 = a^2 + z^2 \qquad \cos(\theta) = z/r \qquad dA = a\,d\phi\,da$$

$$dq \equiv \sigma\, dA = \sigma\, a\, da\, d\phi \qquad dV_{P,z} = \frac{k\,dq}{r}$$



$$V_{P,z} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^R \frac{\sigma\, a\, da\, d\Phi}{[a^2 + z^2]^{1/2}}$$

Double integral

- Integrate twice: first on azimuthal angle f from 0 to 2p which yields a factor of 2p then on ring radius a from 0 to R

$$V_{P,z} = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{a\, da}{[a^2 + z^2]^{1/2}}$$

Use Anti-derivative: $\frac{a}{[a^2 + z^2]^{1/2}} = \frac{d}{da}[a^2 + z^2]^{1/2}$

$$V_{disk} = \frac{\sigma}{2\varepsilon_0}\left[\left[z^2 + R^2\right]^{1/2} - |z|\right]$$

(note: $(1 \pm x)^{1/2} \approx 1 \pm \frac{1}{2}x - \frac{1}{2}\frac{1}{4}x^2 ...$ for $x^2 << 1$)

"Far field" (z>>R): disc looks like point charge

$$V_{disk} \approx \frac{\sigma}{2\varepsilon_0}\left[z + \frac{1}{2}\frac{R^2}{z} - |z|\right] = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z}$$

"Near field" (z<<R): disc looks like infinite sheet of charge

$$V_{disk} \approx \frac{\sigma R}{2\varepsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q}{2\pi\varepsilon_0}\frac{1}{R}\left(1 - \frac{z}{R}\right) \Rightarrow E \equiv -\frac{dV}{dz} = \frac{\sigma}{2\varepsilon_0}$$

CHAPTER 23 SUMMARY

Electric potential energy: The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by the change in a potential-energy function U.

The electric potential energy for two point charges q and q_0 depends on their separation r. The electric potential energy for a charge q_0 in the presence of a collection of charges q_1, q_2, q_3 depends on the distance from q_0 to each of these other charges. (See Examples 23.1 and 23.2.)

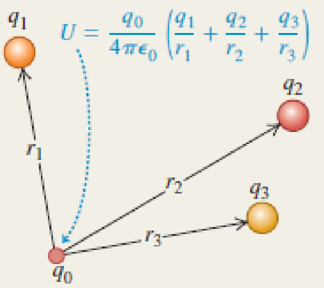
$$W_{a \to b} = U_a - U_b \quad (23.2)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \quad (23.9)$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad (23.10)$$

(q_0 in presence of other point charges)



Electric potential: Potential, denoted by V, is potential energy per unit charge. The potential difference between two points equals the amount of work that would be required to move a unit positive test charge between those points. The potential V due to a quantity of charge can be calculated by summing (if the charge is a collection of point charges) or by integrating (if the charge is a distribution). (See Examples 23.3, 23.4, 23.5, 23.7, 23.11, and 23.12.)

The potential difference between two points a and b, also called the potential of a with respect to b, is given by the line integral of $\vec{E}$. The potential at a given point can be found by first finding $\vec{E}$ and then carrying out this integral. (See Examples 23.6, 23.8, 23.9, and 23.10.)

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad (23.14)$$

(due to a point charge)

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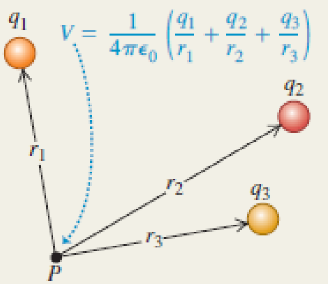
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad (23.16)$$

(due to a charge distribution)

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\cos\phi \, dl \quad (23.17)$$



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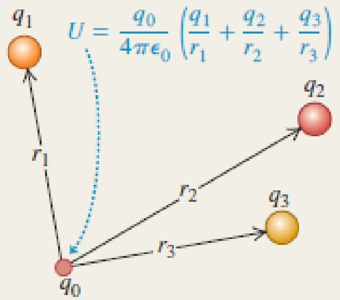

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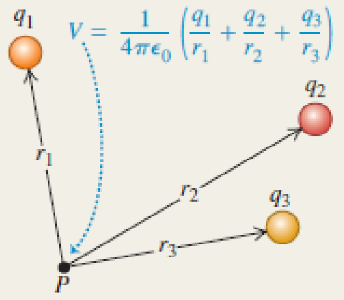
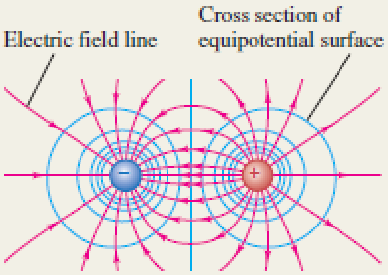


Equipotential surfaces: An equipotential surface is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface and all points in the interior of a conductor are at the same potential. When a cavity within a conductor contains no charge, the entire cavity is an equipotential region and there is no surface charge anywhere on the surface of the cavity.



Finding electric field from electric potential: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field $\vec{E}$ at any point are given by partial derivatives of V. (See Examples 23.13 and 23.14.)

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \tag{23.19}$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \tag{23.20}$$

(vector form)