Electricity and Magnetism

Lecture 07 - Physics 121

Current, Resistance, DC Circuits: Y&F Chapter 25 Sect. 1-5

Kirchhoff's Laws: Y&F Chapter 26 Sect. 1

- **Circuits and Currents**
- **Electric Current i**
- **Current Density J**
- **Drift Speed**
- **Resistance, Resistivity, Conductivity**
- **Ohm's Law**
- **Power in Electric Circuits**
- **Examples**
- **Kirchhoff's Rules applied to Circuits**
- **EMF's - "Pumping" Charges**
- **Work, Energy, and EMF**
- **Simple Single Loop and Multi-Loop Circuits**
- **Summary**

Electric Current: Net charge crossing a surface per unit time

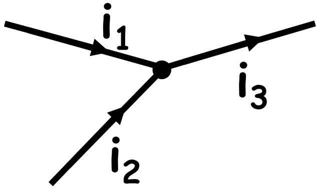
$$i \equiv \frac{dq}{dt} \quad \text{or} \quad dq \equiv i\,dt \quad \therefore \quad q(t) = \int_0^t i(t')dt' = i \times t \;\text{(if } i \text{ is constant)}$$

Units: 1 Ampere = 1 Coulomb per second

Convention: flow is from + to – as if free charges are +

Charge / current is conserved - charge does not pile up or vanish

At any junction $\sum i_{in} = \sum i_{out}$

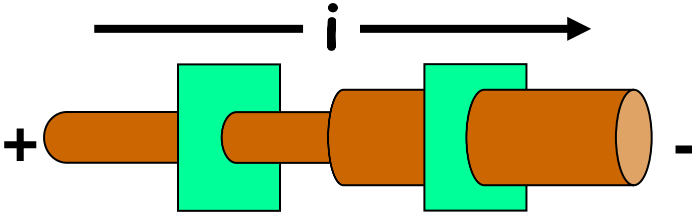


$i_1 + i_2 = i_3$

kirchhoff's Rules: (summary)

- Junction Rule: Σ currents in = Σ currents out at any junction
- Voltage Rule: Σ ΔV's = 0 for any closed path

Current is the same across each cross-section of a wire



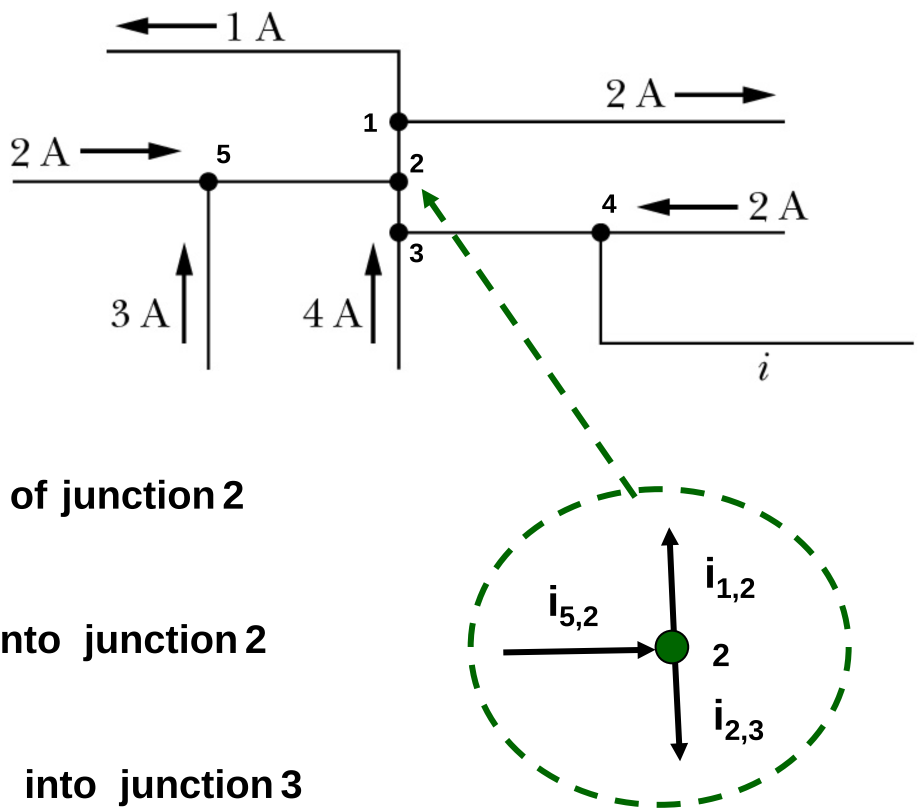
Current density J may vary
[J] = current/area

Energy in a circuit:

- EMFs provide energy (electro-motive force)
- Resistances dissipate energy as heat
- Capacitances store energy in E field
- Inductances store energy in B field

CURRENT CONSERVATION EXAMPLE:

Find the unknown current i



Name the junctions
Name the links by the junctions they connect

Apply the junction rule: $\boxed{\sum i_{in} = \sum i_{out}}$

Using Junction 1:

$i_{1,2} = 2\text{ A} + 1\text{ A} = 3\text{ A}$ into junction 1, out of junction 2

Using Junction 5:

$i_{5,2} = 2\text{ A} + 3\text{ A} = 5\text{ A}$ out of junction 5, into junction 2

Using Junction 2:

$i_{2,3} + i_{1,2} = i_{5,2} = 2\text{ A}$ out of junction 2, into junction 3

Using Junction 3:

$i_{3,4} = 4\text{ A} + 2\text{ A} = 6\text{ A}$ out of junction 3, into junction 4

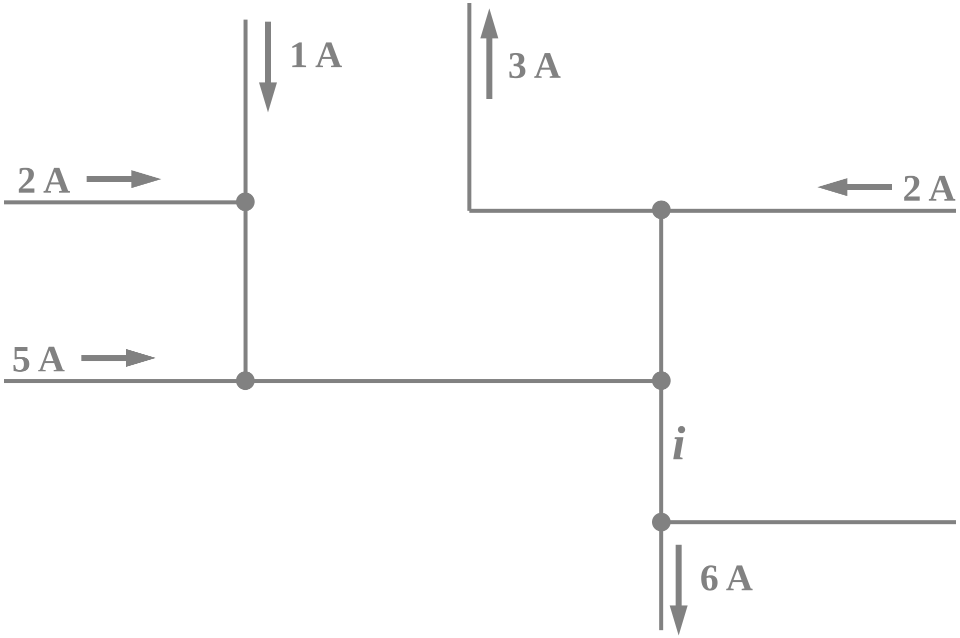
Using Junction 4:

$i = 6\text{ A} + 2\text{ A} = 8\text{ A}$ out of junction 4, to the right

Try this one yourself

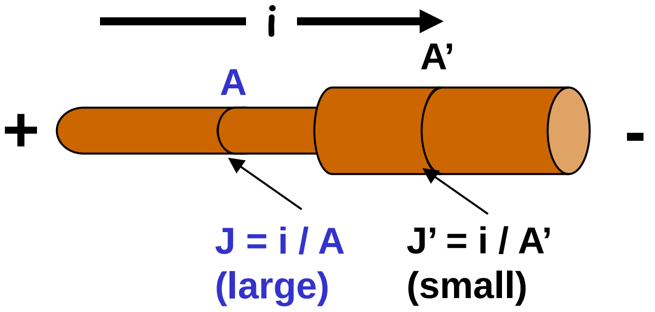
7-1: What is the current in the wire section marked *i*?

- A. 1 A.
- B. 2 A.
- C. 5 A.
- D. 7 A.
- E. Cannot determine from information given.



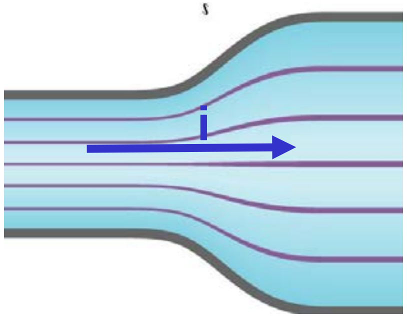
$$\boxed{\sum i_{in} = \sum i_{out}}$$

Current density J: Current / Unit Area (Vector)



Same current crosses larger or smaller Surfaces, current density J varies

High current density in this region



Small current density in this region

For uniform density $i = J A$ or $J = i/A$ units: Amperes/m^2

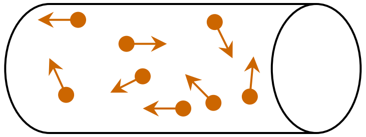
$$di \equiv \vec{J} \circ \hat{n} dA$$

$$\boxed{i = \int_{area} \vec{J} \circ d\vec{A}}$$

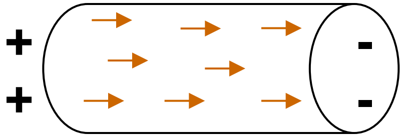
What makes current flow?

$$\boxed{J \propto E \qquad J = \sigma E}$$

E field in solid wire drives current.



APPLIED FIELD = 0
Random motion
flow left = flow right



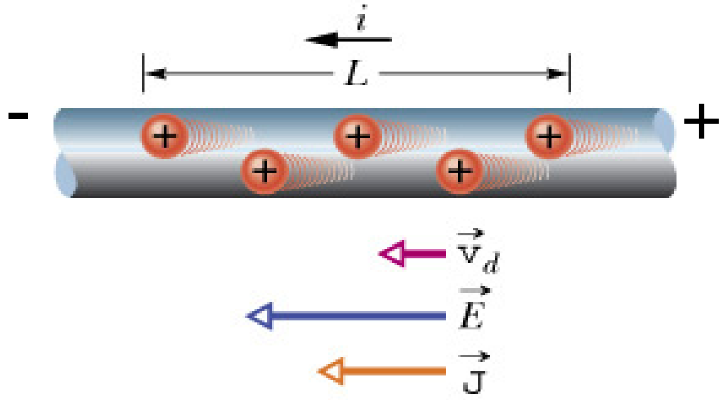
APPLIED FIELD NOT ZERO
Moving charges collide with fixed ions
and flow with **drift** velocity

Do charges in a current keep accelerating as they flow?

Electrons **collide** with ions, impurities, etc. causing resistance
Move at constant **drift speed v_D**:

- Thermal motions (random motions) have speed $v_{th} \approx 10^6 \text{ m/s} \quad (\frac{3}{2}k_{Boltz}T)$
- Drift speed is tiny compared with thermal motions.
- Drift speed in copper is $10^{-8} - 10^{-4}$ m/s.

$$\boxed{J \propto E \qquad J = \sigma E}$$



For E = 0: no current,
$v_D = 0$, J = 0, i = 0

For E not = 0
(battery voltage not 0):

$$n \equiv \text{ density of charge carriers} \qquad \text{Units : \# /volume}$$

$$nv_D = \text{\# of charge carriers crossing unit area per unit time}$$

$$\boxed{J = qnv_D = \text{net charge crossing area A per unit time}}$$

Note: for electrons, q & v_D are both reversed → J still to left
$|q| = e = 1.6 \times 10^{-19}$ C.

EXAMPLE: Calculate the current density J_{ions} for ions in a gas

Assume:

- Doubly charged positive ions
- Density $n = 2 \times 10^8$ ions/cm^3
- Ion drift speed $v_d = 10^5$ m/s

Find J_{ions} – the current density for the ions only (forget $J_{electrons}$)

$$J = qnv_D = 2 \times \underset{\text{coul/ion}}{1.6 \times 10^{-19}} \times \underset{\text{ions/cm}^3}{2 \times 10^{8}} \times \underset{\text{m/s}}{10^{5}} \times \underset{\text{cm}^3\text{/m}^3}{10^{6}}$$

$$\boxed{\therefore \quad J = 6.4 \quad A./m^2}$$

Increasing the Current

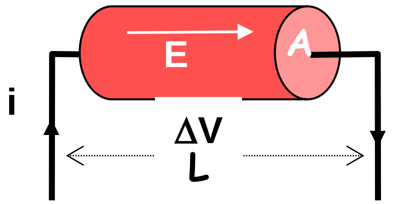
7-2: When you increase the current in a wire, what changes and what is constant?

A. The density of charge carriers stays the same, and the drift speed increases.
B. The drift speed stays the same, and the number of charge carriers increases.
C. The charge carried by each charge carrier increases.
D. The current density decreases.

$$\boxed{J \propto E \qquad J = \sigma E}$$

$$\boxed{J = qnv_D}$$

Resistance: Determines how much current flows through a device in response to a given potential difference.



$$\boxed{R \equiv \frac{\Delta V}{i} \qquad \text{units: } 1\text{ Ohm} \equiv 1\Omega = 1\,\text{volt/ampere}}$$

R depends on the material & geometry

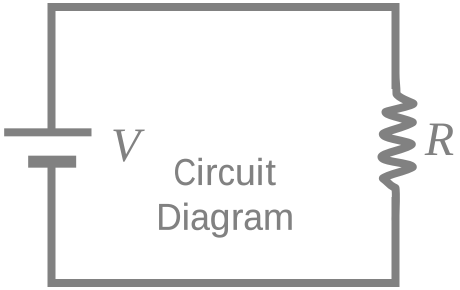
Note: C= Q/ΔV – inverse to R

Apply voltage to a conducting wire.

- Very large current so R is small.

Apply voltage to a poor conductor material like carbon

Tiny current so R is very large.



Resistivity "ρ" : Property of a material **itself**

(as is dielectric constant). Does not depend on dimensions

- The resistance of a device depends on resistivity ρ and also depends on shape
- For a given shape, different materials produce different currents for same ΔV
- Assume cylindrical resistors

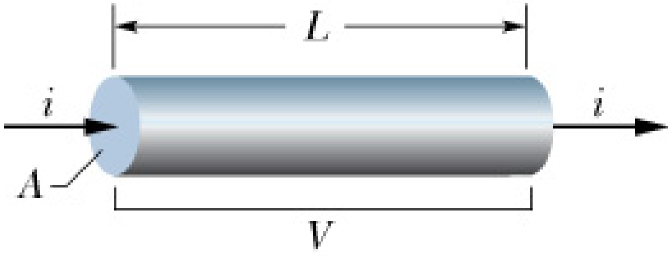
$$\boxed{R \equiv \text{resistance} = \frac{\rho L}{A}}$$

$$\boxed{\rho \equiv \text{resistivity} = \frac{RA}{L} \quad \text{for a resistor}}$$

$$\boxed{\text{resistivity units: Ohm-meters} \equiv \Omega.\text{m}}$$

For insulators: ρ → infinity

Calculating resistance, given the resistivity



resistivity

resistance

$$R = \frac{\rho L}{A}$$

proportional to length

inversely proportional to cross section area

EXAMPLE:

Find R for a 10 m long iron wire, 1 mm in diameter

$$R = \frac{\rho L}{A} = \frac{9.7\text{x}10^{-8}\ \Omega.\text{m x } 10\text{ m}}{\pi \text{ x } (10^{-3}/2)^2\ \text{m}^2} = 1.2\,\Omega$$

Find the potential difference across R if i = 10 A. (Amperes)

$$\Delta V = iR = 12\text{ V}$$

EXAMPLE:

Find resistivity of a wire with R = 50 mΩ, diameter d = 1 mm, length L = 2 m

$$\rho = \frac{RA}{L} = \frac{50\text{x}10^{-3}\,\Omega\text{ x } 10\text{ m}}{2\text{ m}} \text{ x } \pi \left(10^{-3}/2\right)^2 = 1.96\text{x}10^{-8}\ \Omega.\text{m}$$

Use a table to identify material. Not Cu or Al, possibly an alloy

Resistivity depends on temperature:

- Resistivity depends on temperature: Higher temperature → greater thermal motion → more collisions → higher resistance.

SOME SAMPLE RESISTIVITY VALUES

ρ in Ω.m @ 20° C.	
1.7×10^{-8}	copper
9.7×10^{-8}	iron
$2.5 \times 10^{+3}$	pure silicon

Reference Temperature

Simple model of resistivity: α = temperature coefficient

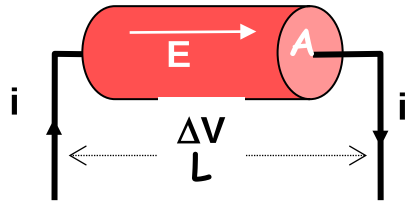
Change the temperature from reference T_0 to T

Coefficient α depends on the material

$$\rho = \rho_0(1 + \alpha(T - T_0))$$

$\alpha \equiv$ temperature coefficient

Conductivity is the reciprocal of resistivity



$$R = \frac{\rho L}{A}$$

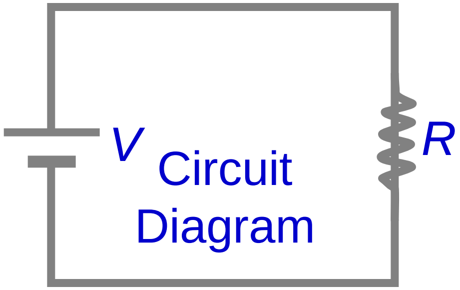
Definition:

$$\sigma \equiv \frac{1}{\rho} \quad \therefore \quad J = \sigma E \qquad \text{units: "mho"} \equiv (\Omega.m)^{-1}$$

$$\Delta V = EL = iR = JAR = J\rho L \quad \therefore \quad E/J = \rho$$

Current Through a Resistor

7-3: What is the current through the resistor in the following circuit, if V = 20 V and R = 100 Ω?



A. 20 mA.
B. 5 mA.
C. 0.2 A.
D. 200 A.
E. 5 A.

$$\boxed{\Delta V = iR}$$

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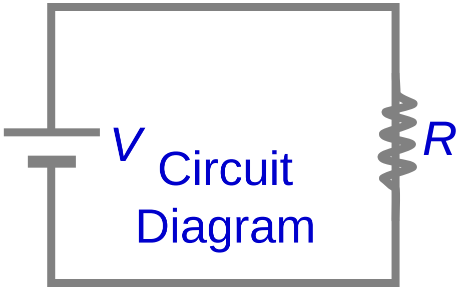
Current Through a Resistor

7-4: If the current is doubled, which of the following might also have changed?

- A. The voltage across the resistor doubles.
- B. The resistance of the resistor doubles.
- C. The voltage in the wire between the battery and the resistor doubles.
- D. The voltage across the resistor drops by a factor of 2.
- E. The resistance of the resistor drops by a factor of 2.

$$\boxed{\Delta V = iR}$$

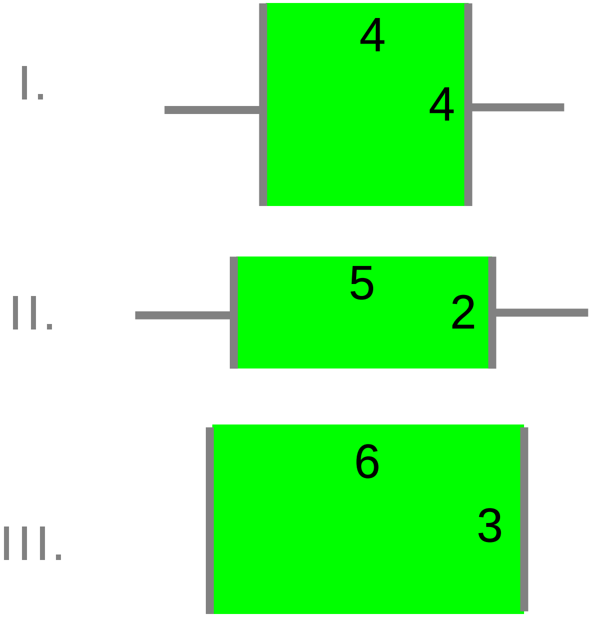


Resistivity of a Resistor

$$R = \frac{\rho L}{A}$$

7-5: Three resistors are made of the same material, with sizes in mm shown below. Rank them in order of total resistance, greatest first.

A. I, II, III.
B. I, III, II.
C. II, III, I.
D. II, I, III.
E. III, II, I.



Each has square cross-section

Ohm's Law and Ohmic materials (a special case)

Definitions of resistance:

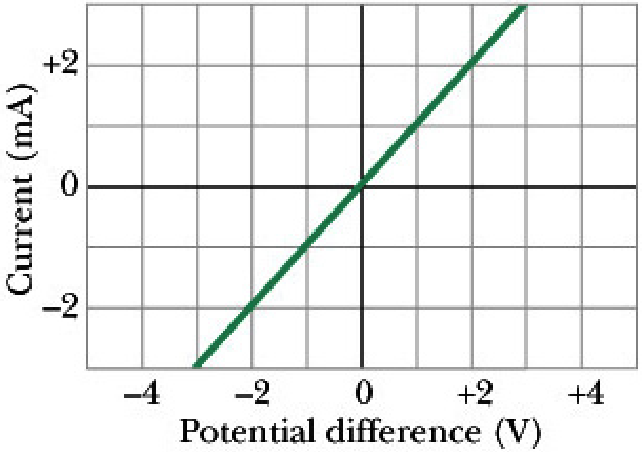
$R \equiv V / i$ but R *could* depend on applied V

$\sigma \equiv 1/\rho = J/E$ but ρ *could* depend on E

Definition of OHMIC conductors and devices:

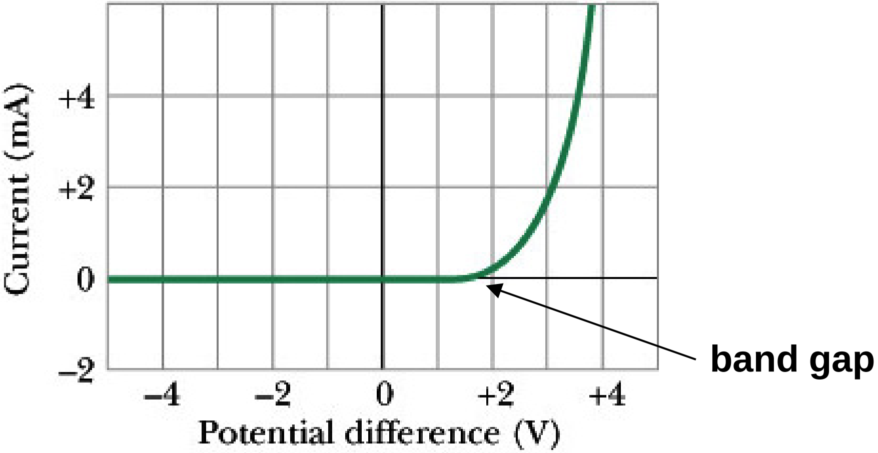
- Ratio of voltage drop to current is constant – it does not depend on applied voltage i.e., current is proportional to applied V
- Resistivity does not depend on magnitude or direction of applied voltage

Ohmic Materials
e.g., metals, carbon,…



constant slope = 1/R

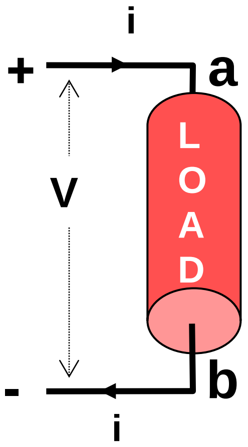
Non-Ohmic Materials
e.g., semiconductor devices



varying slope = 1/R

OHMIC CONDITION $\frac{1}{R} \equiv \frac{di}{dV}$ is CONSTANT

Power is dissipated in resistive circuits



- Apply voltage drop V across load
- Current flows through load which dissipates energy
- An EMF (e.g., a battery) does work, holding V and current i constant by expending potential energy

As charge dq flows from a to b it loses P.E. = dU

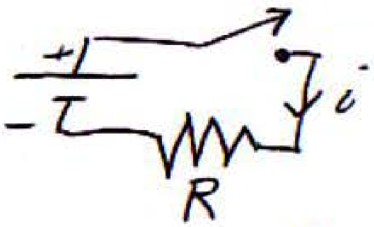
- potential is PE / unit charge
- charge = current x time

$$\Rightarrow \quad dU = V\,dq = Vi\,dt$$

$$\text{Power dissipation} = P \equiv \frac{dU}{dt} = Vi \quad \text{[Watts] for any load}$$

$$V = iR \quad \Rightarrow \quad P = i^2R = V^2/R \quad \text{resistors only}$$

EXAMPLE: Space heater: Find rate of converting electrical energy to heat



$V = 120$ Volts

$R = 14\,\Omega$

– FIND POWER $= V^2/R = 1$ KW

KW ≡ Kilowatt = 1000 WATTS

NOW, HOW MUCH DOES IT COST PER DAY TO RUN, IF 1 KWH = 12¢

COST = Power × time × Unit Energy Cost = 1 KW × 24 Hours × .12/KWH = \$2.88

EXAMPLE: A 47Ω resistor is rated for up to 10 watts what's the max voltage.

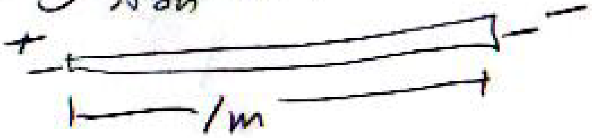
$P_{max} = V_{max}^2/R$ $V_{max} = \sqrt{P_{max} R} = 21.6$ Volts

EXAMPLE: A 100 watt light bulb is rated for 120 V. Find i in the light

$P = iV$ $i = P/V = \frac{100\ W}{120\ V} = .83\ A$

EXAMPLE: 10 Amps flows in an iron wire. 1m long and 1mm in diameter.

$\rho_{iron} = 9.7 \times 10^{-8}\ \Omega m$



a) How much power is dissipated

$P = i^2 R$ $R = \frac{\rho L}{A}$

$= \frac{9.7\times10^{-8} \times 1\,m}{\pi \times \frac{1}{4} \times 10^{-6} m^2} =$

$R = .123\Omega$

$P = 100 \times .123\Omega = 12.3$ Watts.

b) what voltage is across the wire.

$P = iV$ $V = P/i = 1.23$ Volts.

c) what should V be to dissipate 100 Watts?

$P = \frac{V^2}{R}$ $V = \sqrt{PR} = \sqrt{100 \times .123\Omega} = 3.5$ Volts

Ohmic and non-ohmic conductors

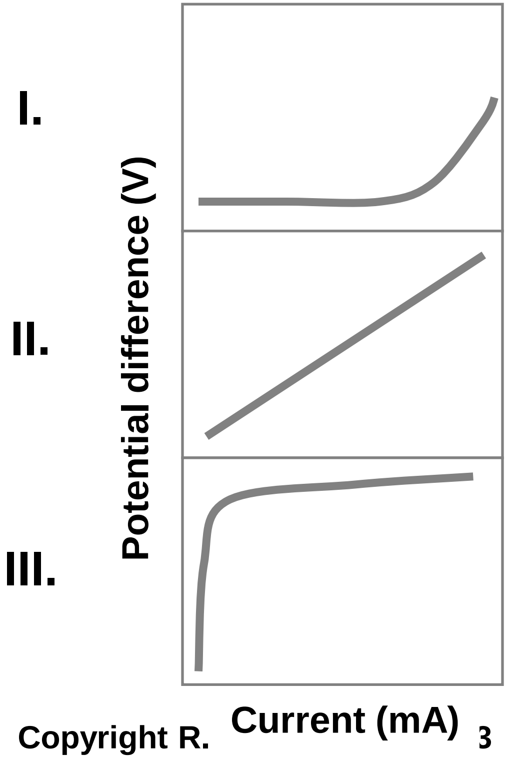
SUPERCONDUCTORS: At very low temperatures (~4 K) some conductors lose all resistance. Once you start current flowing, it will continue to flow "forever,"

- The current becomes enormous once the applied voltage exceeds a small value.

7-6: The three plots show voltage vs. current (so the slope is R) for three kinds of devices. Identify the devices in order appearing in charts I, II, III?

A. Resistor, superconductor, diode
B. Diode, superconductor, resistor
C. Resistor, diode, superconductor
D. Diode, resistor, superconductor
E. Superconductor, resistor, diode

$$R \equiv \frac{\Delta V}{\Delta i}$$



Circuit analysis with resistances and EMFs

GENERAL ANALYSIS METHOD: kirchhoff'S LAWS or RULES

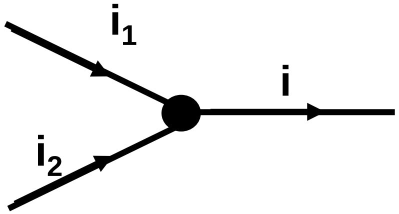
Junction Rule $\sum i_{in} = \sum i_{out}$ Charge conservation

Loop Rule $\sum \Delta V = 0$ (closed loop) Energy conservation

CIRCUIT ELEMENTS:

- **PASSIVE: RESISTANCE, CAPACITANCE, INDUCTANCE**
- **ACTIVE: EMF's (SOURCES OF POTENTIAL DIFFERENCE AND ENERGY)**

JUNCTIONS and BRANCHES



....etc....

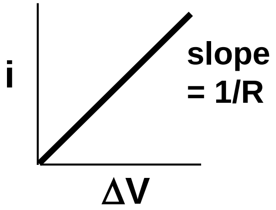
RESISTANCE:

$$\boxed{R \equiv \frac{\Delta V}{i}} \qquad R = \rho \frac{L}{A} \qquad \rho = \text{resistivity}$$

POWER:

$$P = \frac{dW}{dt} = -\frac{dU}{dt} = iV \qquad P = i^2R \ \text{(resistor)}$$

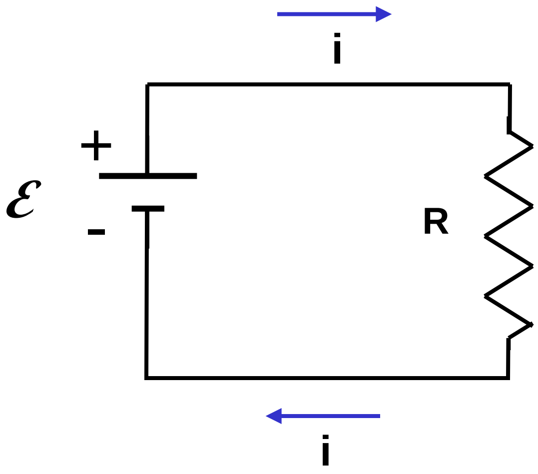
OHM's LAW:



R is independent of ΔV or i

EMFs “pump” charges to higher energy

- **EMFs move charges from low to high potential (potential energy).**
- **EMF’s (*electromotive force*) such as batteries supply energy:**
  - maintain constant potential at terminals
  - do work $\mathbf{dW = \mathcal{E} dq}$ on charges (source of the energy is usually chemical)
  - EMFs are “charge pumps”
- **Unit: volts (V). Symbol: script $\mathcal{E}$.**
- **Types of EMFs: batteries, electric generators, solar cells, fuel cells, etc.**
- **DC versus AC**



Current flows CW through circuit
from + to – outside of EMF
from – to + inside EMF

$$\mathcal{E} = \frac{\text{work done}}{\text{unit charge}} = \frac{dW}{dq}$$

Power supplied by EMF:

$$P = \text{power} = \frac{dW}{dt}$$

$$dW = \mathcal{E}\, dq = \mathcal{E}\, i\, dt = P\, dt$$

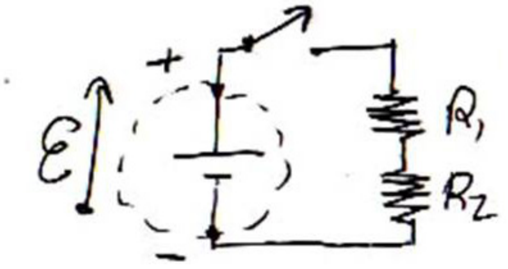
$$P_{emf} = \pm\, \mathcal{E} i = \vec{\mathcal{E}} \circ \vec{i}$$

Power dissipated by resistor:

$$P_R = i V = i^2 R = V^2 / R$$

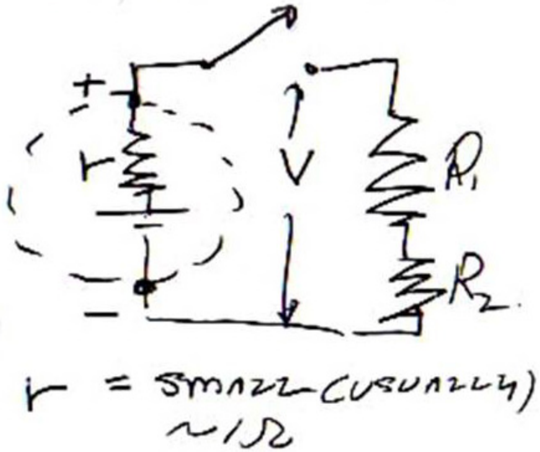
Ideal EMF device

- Zero internal battery resistance
- Open switch: EMF = $\mathcal{E}$
  no current, zero power
- Closed switch: EMF $\mathcal{E}$ is also applied across load circuit
- Current & power not zero



Real EMF device

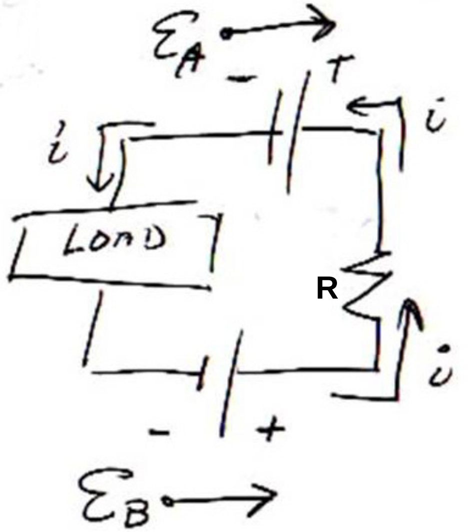
- Open switch: EMF still = $\mathcal{E}$
- r = internal EMF resistance in series, usually small ~ 1 Ω
- Closed switch:
  - V = $\mathcal{E}$ – ir across load, P_{ckt}= iV
  - Power dissipated in EMF
    P_{emf} = i($\mathcal{E}$-V) = i^2r



Multiple EMFs

Assume $\mathcal{E}_B$ > $\mathcal{E}_A$ (ideal EMF's)
Which way does current i flow?

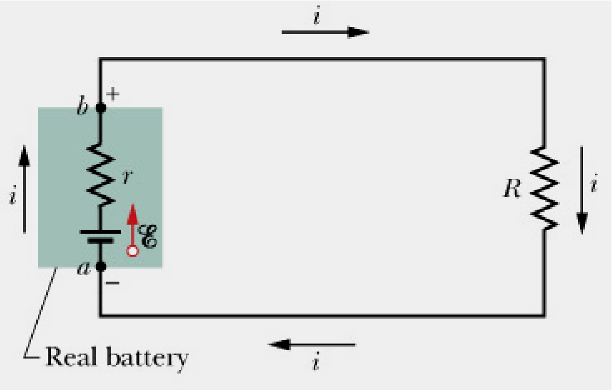
- **Apply kirchhoff Laws to find out**
- **Answer: From $\mathcal{E}_B$ to $\mathcal{E}_A$**
- $\mathcal{E}_B$ does work, loses energy
- $\mathcal{E}_A$ is charged up
- R converts PE to heat
- Load (motor, other) produces motion and/or heat



Generating Circuit Equations with the kirchhoff Loop Rule

- **The algebraic sum of voltage changes = zero around all closed loops through a circuit (including multi-loop)**
- **Assume either current direction. Expect minus signs when choice is wrong.**
- **Traverse circuit with or against assumed current direction**
- **Across resistances, voltage drop $\Delta V = - iR$ if following assumed current direction. Otherwise, voltage change is $+iR$.**
- **When crossing EMFs from – to +, $\Delta V = +\mathcal{E}$. Otherwise $\Delta V = -\mathcal{E}$**
- **Dot product $\underline{i}.\underline{\mathcal{E}}$ determines whether power is actually supplied or dissipated**

EXAMPLE: Single loop circuit with battery (internal resistance r)



Follow circuit from a to b to a, same direction as i

$$\mathcal{E} - ir - iR = 0 \quad \boxed{i = \frac{\mathcal{E}}{r + R}}$$

Power in External Ckt

$$P = iV = i(\mathcal{E} - ir)$$

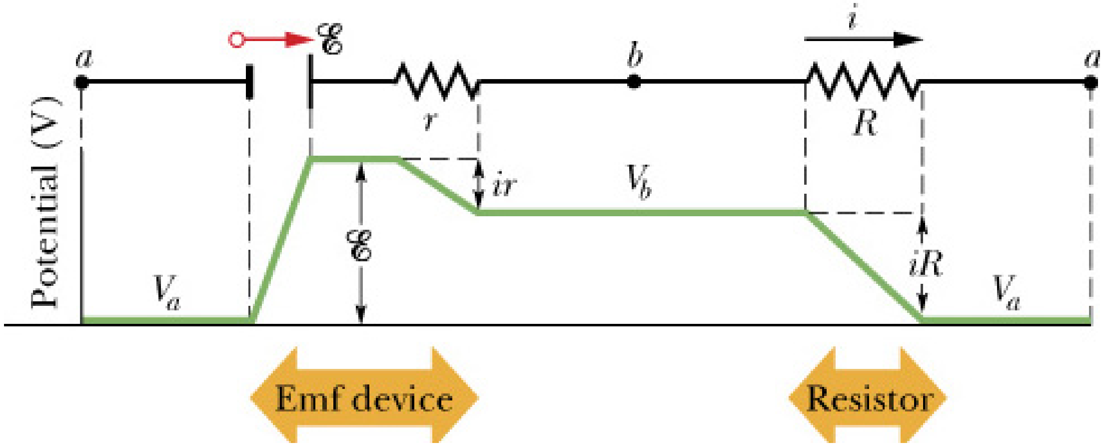
$$P = i\mathcal{E} - i^2 r$$

circuit dissipation

battery drain

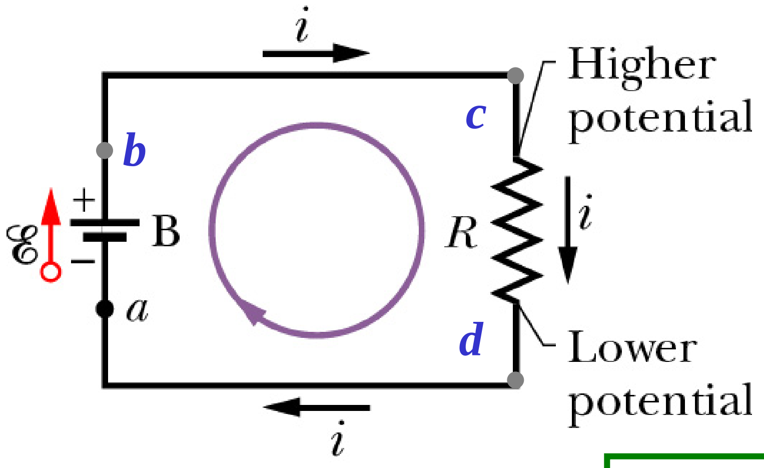
battery dissipation

Potential around the circuit



Example: CW or CCW around a single-loop circuit

Assume current direction as shown



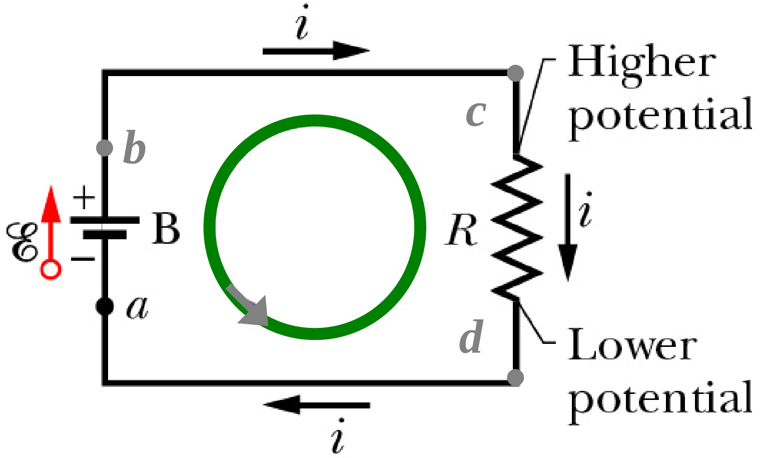
- **Traverse clockwise from a:**

$$\Delta V_{ba} = V_b - V_a = +\varepsilon$$
$$\Delta V_{cb} = 0$$
$$\Delta V_{dc} = V_d - V_c = -iR$$
$$\Delta V_{ad} = 0$$

$$\sum_{\text{closed loop}} \Delta V = 0 = \varepsilon + 0 - iR + 0$$

$$\boxed{\begin{gathered}\varepsilon - iR = 0 \\ i = \frac{\varepsilon}{R}\end{gathered}}$$

SAME RESULT



- **Traverse counterclockwise from a:**

$$\Delta V_{da} = 0$$
$$\Delta V_{cd} = V_c - V_d = +iR$$
$$\Delta V_{bc} = 0$$
$$\Delta V_{ab} = V_a - V_b = -\varepsilon$$

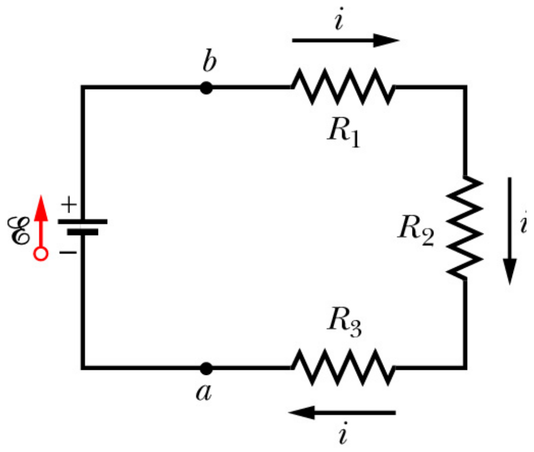
$$\sum_{\text{closed loop}} \Delta V = 0 = 0 + iR + 0 - \varepsilon$$

$$\boxed{\begin{gathered}iR - \varepsilon = 0 \\ i = \frac{\varepsilon}{R}\end{gathered}}$$

Equivalent resistance for resistors in series

Junction Rule: The current through all of the resistances in series (a single branch) is identical:

$$\mathbf{i = i_1 = i_2 = i_3}$$



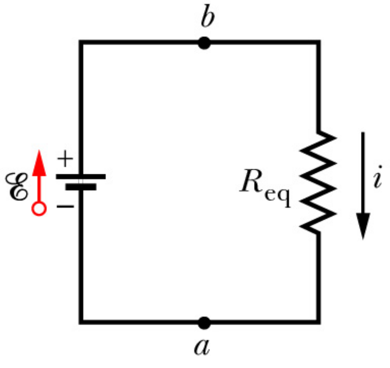
Loop Rule: The sum of the potential differences around a closed loop equals zero:

$$\varepsilon - iR_1 - iR_2 - iR_3 = 0 \quad \Longrightarrow \quad i = \frac{\varepsilon}{R_1 + R_2 + R_3}$$

The equivalent circuit replaces the series resistors with a single equivalent resistance:

same $\mathscr{E}$, same i as above

$$\varepsilon - iR_{eq} = 0 \quad \Longrightarrow \quad i = \frac{\varepsilon}{R_{eq}}$$



(b)

The equivalent resistance for a series combination is the sum of the individual resistances and is always greater than any one of them.

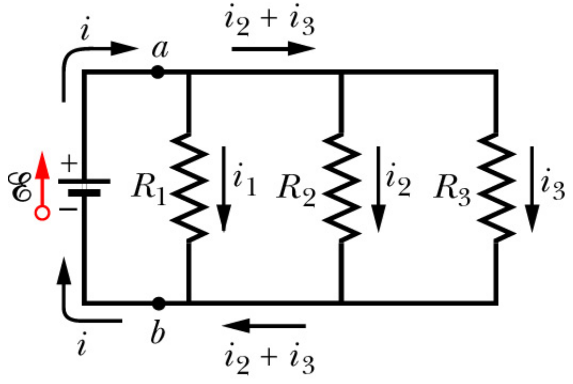
$$R_{eq} = R_1 + R_2 + R_3$$

$$\boxed{R_{eq} = \sum_{i=1}^{n} R_i}$$

inverse of series capacitance rule

Equivalent resistance for resistors in parallel

Loop Rule: The potential differences across each of the parallel branches are the same.



(a)

$$\mathcal{E} - i_1 R_1 = 0 \qquad \mathcal{E} - i_2 R_2 = 0 \qquad \mathcal{E} - i_3 R_3 = 0$$

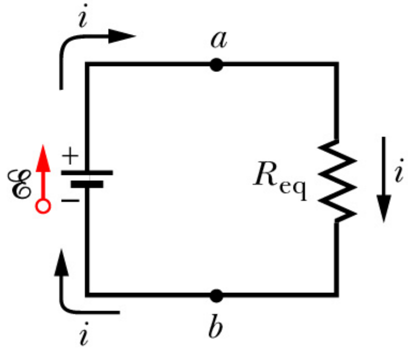
$$i_1 = \frac{\mathcal{E}}{R_1}, \quad i_2 = \frac{\mathcal{E}}{R_2}, \quad i_3 = \frac{\mathcal{E}}{R_3}$$

i not in equations

Junction Rule: The sum of the currents flowing in equals the sum of the currents flowing out. Combine equations for all the junctions at "a" & "b".

$$i = i_1 + i_2 + i_3 = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

The **equivalent circuit** replaces the series resistors with a single equivalent resistance:



(b)

same $\mathcal{E}$, same i as above

$$\varepsilon - iR_{eq} = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{R_{eq}}$$

The **reciprocal** of the equivalent resistance for a parallel combination is the **sum** of the individual **reciprocal** resistances and is always smaller than any one of them.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}}$$

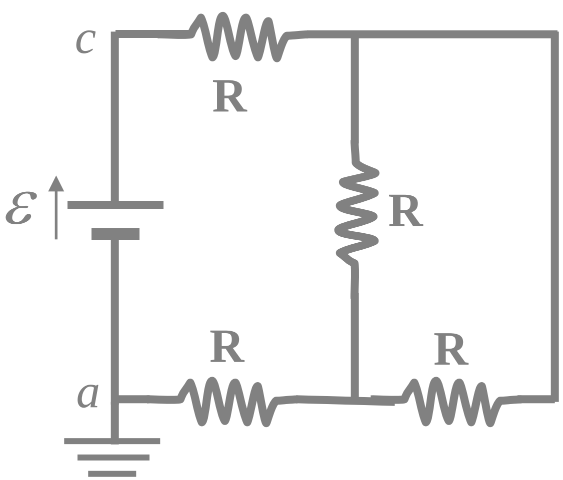
$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

inverse of parallel capacitance rule

Resistors in series and parallel

7-7: Four identical resistors are connected as shown in the figure. Find the equivalent resistance between points *a* and *c*.

- A. 4 R.
- B. 3 R.
- C. 2.5 R.
- D. 0.4 R.
- E. Cannot determine from information given.



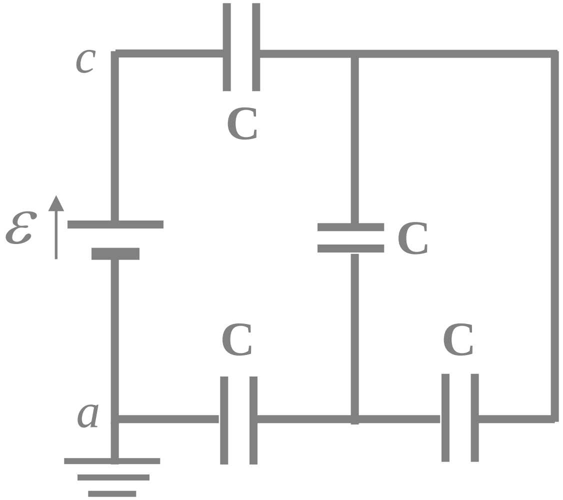
$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

$$R_{eq} = \sum_{i=1}^{n} R_i$$

Capacitors in series and parallel

7-8: Four identical capacitors are connected as shown in figure. Find the equivalent capacitance between points *a* and *c*.

A. 4 C.
B. 3 C.
C. 2.5 C.
D. 0.4 C.
E. Cannot determine from information given.

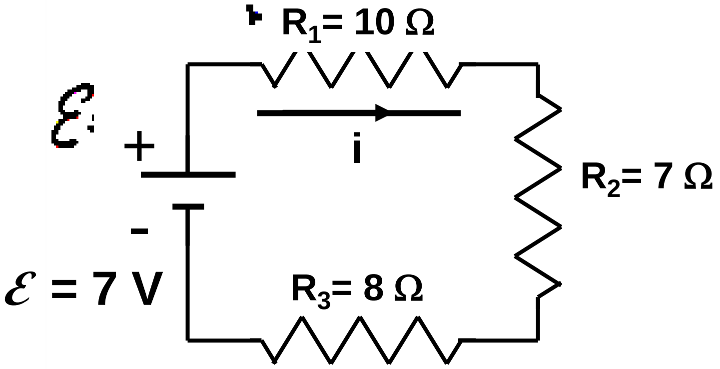


$$\boxed{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

$$\boxed{C_{eq} = \sum_{i=1}^{n} C_i}$$

EXAMPLE:

Find i, V_1, V_2, V_3, P_1, P_2, P_3



$$i = \frac{\mathcal{E}}{R_{eq}} = \frac{7}{10+7+8} = \frac{7}{25} = .28A$$

$$V_1 = iR_1 = .28 \times 10 = 2.80V$$

$$V_2 = iR_2 = .28 \times 7 = 1.96\ V$$

$$V_3 = iR_3 = .28 \times 8 = 2.24\ V$$

$$V = 7V.$$

$$P_1 = i^2R_1 = (.28)^2 \times 10 = .78W = iV = (.28) \times 2.8$$

$$P_2 = i^2R_2 = (.28)^2 \times 7 = .55W$$

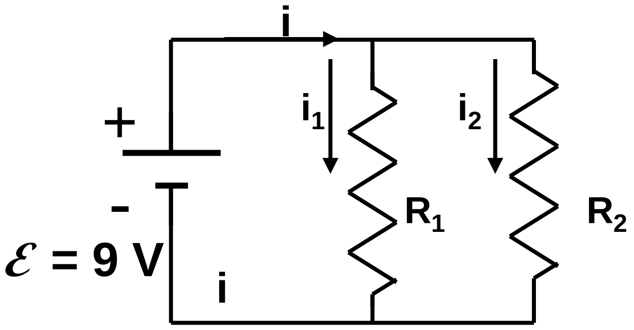
$$P_3 = i^2R_3 = (.28)^2 \times 8 = .627W$$

$$P = 1.96W = \underline{iV} = 1.96W \leftarrow \text{sum.}$$

EXAMPLE:

$R_1 = 4\Omega$, $R_2 = 11\Omega$.

Find currents and voltage drops



Loop Rule. $\mathcal{E} - i_1R_1 = 0$ $\mathcal{E} - i_2R_2 = 0$.

$\Rightarrow i_1R_1 = i_2R_2 = 9$ volts.

$i_1 = 9/4 = 2.25$ A. $i_2 = .818A$

Junction Rule: $i = i_1 + i_2 = 3.07A$.

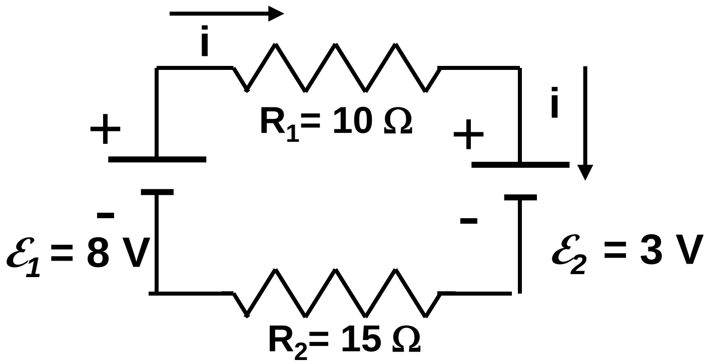
OR: Find $R_{eq} = \frac{4 \times 11}{4+11} = 2.93\Omega$.

$i = \frac{\mathcal{E}}{R_{eq}} = 3.07A$.

$P_2 = i_2^2 R_2 = 7.36W$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

EXAMPLE: MULTIPLE BATTERIES
SINGLE LOOP



Find $i, P_{\mathcal{E}_1}, P_{\mathcal{E}_2}, P_{R_1}, P_{R_2}$

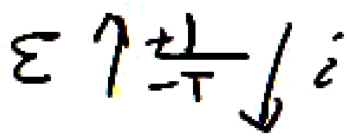
LOOP RULE:

$$\boxed{\mathcal{E}_1 - iR_1 - \mathcal{E}_2 - iR_2 = 0}$$

$$\mathcal{E}_1 - \mathcal{E}_2 = i(R_1 + R_2)$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{(R_1 + R_2)} = \frac{8-3}{10+15} = .2A.$$

A battery (EMF) absorbs power (charges up) when I is opposite to *E*



$$P_{emf} = \pm\ \mathcal{E}i = \vec{\mathcal{E}} \circ \vec{i}$$

$P_{\mathcal{E}_1} = \mathcal{E}_1 i = 8 \times .2) = 1.6W$ PROVIDED TO CIRCUIT.

$P_{\mathcal{E}_2} = -\mathcal{E}_2 i = -3 \times .6 = -.6W$ absorbed.

$P_1 = -i^2 R_1 = -(.2)^2 10 = -.4W$ dissipated

$P_2 = -i^2 R_2 = -(.2)^2 \times 15 = -.6W$ "

$\Big\}$ −1.6 Watts.

$$P_{\mathcal{E}_1} = P_{\mathcal{E}_2} + P_{R_1} + P_{R_2}.$$

EXAMPLE: Find the average current density J in a copper wire whose diameter is 1 mm carrying current of i = 1 ma.

$$J = \frac{i}{A} = \frac{10^{-3}\text{ amps}}{\pi \times (.5 \times 10^{-3}\text{ m})^2} = 1273 \text{ amps/m}^2$$

Suppose diameter is 2 mm instead. Find J':

$$J' = \frac{i}{A'} = \frac{J}{4} = 318 \text{ amps/m}^2$$

Current i is unchanged

Calculate the drift velocity for the 1 mm wire as above?

$$J = en_{Cu}v_d \quad \text{where} \quad n_{Cu} = \#\text{ conduction electrons/m}^3 \approx 8.49 \times 10^{28}$$

$$v_d = \frac{J}{en_{Cu}} = \frac{1273}{1.6\text{x}10^{-19}\text{x}8.49\text{x}10^{28}} = \boxed{9.37\text{x}10^{-8}\text{m/s} \quad \text{About 3 m/year !!}}$$

So why do electrical signals on wires seem to travel at the speed of light (300,000 km/s)?

Calculating n for copper: One conduction electron per atom

$$n_{Cu} = 1 \text{ electron / atom} \times \frac{8.96 \text{ gm/cm}^3}{63.5 \text{ gm/mole}} \times 6.02 \times 10^{23} \text{ atoms/mole} \times 10^6 \text{ cm}^3/\text{m}^3$$

$$= 8.49 \times 10^{28} \text{ electrons/m}^3$$