# Physics 121 - Electricity and Magnetism

# Lecture 08 - Multi-Loop and RC Circuits

## Y&F Chapter 26 Sect. 2 - 5

- **Kirchhoff's Rules**
- **Multi-Loop Circuit Examples**
- **RC Circuits**
  - **Charging a Capacitor**
  - **Discharging a Capacitor**
- **Discharging Solution of the RC Circuit Differential Equation**
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- **Examples**
- **Charging Solution of the RC Circuit Differential Equation**
- **Features of the Solution**
- **Examples**
- **Summary**

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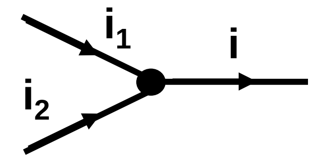
# Kirchhoff's Rules:




- Branch/Junction Rule (charge conservation):
  The current through all series elements in a branch is the same. At any junction:

$$\sum i_{in} = \sum i_{out}$$

- Loop Rule (energy conservation):
  The net change in potential difference is zero for any closed path around a circuit:

$$\sum \Delta V = 0$$

## Generating Circuit Equations with the Kirchoff Loop Rule

- **The algebraic sum of voltage changes = zero around all complete loops through a circuit (including multi-loop).**
- **OK to assume either current direction. Expect minus signs when choice is wrong.**
- **OK to traverse circuit with or against assumed current direction**
- **Across resistances, voltage drop DV = - iR if following assumed current direction. Otherwise, set $\Delta V = +iR$.**
- **When crossing EMFs from – to +, DV = +$\mathcal{E}$. Otherwise DV= -$\mathcal{E}$**
- **Dot product $\underline{i}.\underline{\mathcal{E}}$ determines whether power is actually supplied or dissipated in EMFs**

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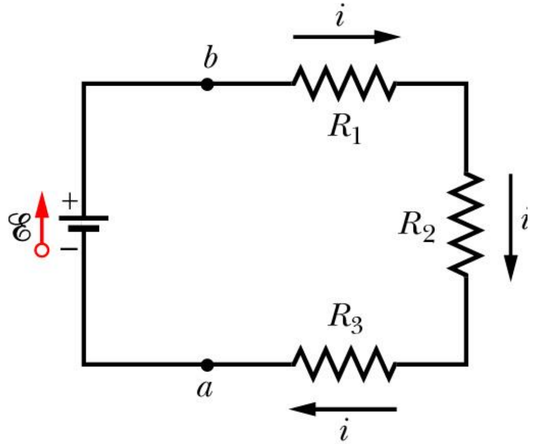
# Equivalent resistance for resistors in series

Junction Rule: The current through all of the resistances in series (a single branch) is identical:

$$i = i_1 = i_2 = i_3$$



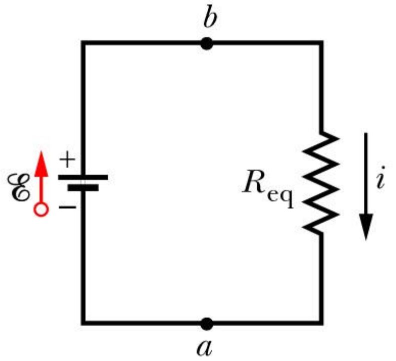

Loop Rule: The sum of the potential differences around a closed loop equals zero:

$$\varepsilon - iR_1 - iR_2 - iR_3 = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{R_1 + R_2 + R_3}$$

The **equivalent circuit** replaces the series resistors with a single equivalent resistance:

same $\mathcal{E}$, same i as above

$$\varepsilon - iR_{eq} = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{R_{eq}}$$



(b)

The equivalent resistance for a series combination is the **sum** of the individual resistances and is always greater than any one of them.

$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = \sum_{i=1}^{n} R_i$$

inverse of series capacitance rule

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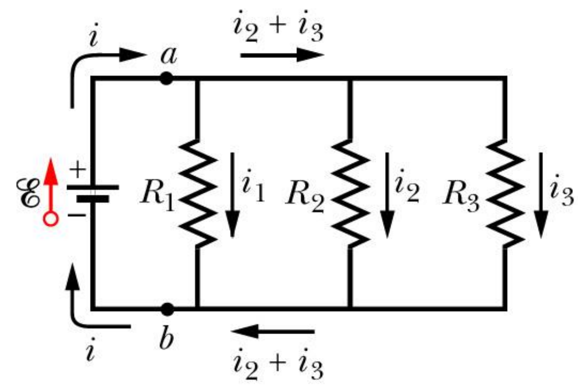
# Equivalent resistance for resistors in parallel

**Loop Rule: The potential differences across each of the parallel branches are the same.**

$$\mathcal{E} - i_1 R_1 = 0 \qquad \mathcal{E} - i_2 R_2 = 0 \qquad \mathcal{E} - i_3 R_3 = 0$$

$$i_1 = \frac{\mathcal{E}}{R_1}, \quad i_2 = \frac{\mathcal{E}}{R_2}, \quad i_3 = \frac{\mathcal{E}}{R_3}$$

i not in equations




(a)

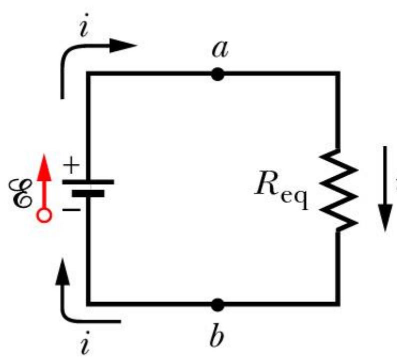
**Junction Rule: The sum of the currents flowing in equals the sum of the currents flowing out. Combine equations for all the junctions at "a" & "b".**

$$i = i_1 + i_2 + i_3 = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

The **equivalent circuit** replaces the series resistors with a single equivalent resistance:

same $\mathcal{E}$, same i as above

$$\varepsilon - iR_{eq} = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{R_{eq}}$$




(b)

The **reciprocal** of the equivalent resistance for a parallel combination is the **sum** of the individual **reciprocal** resistances and is always smaller than any one of them.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

*inverse of parallel capacitance rule*

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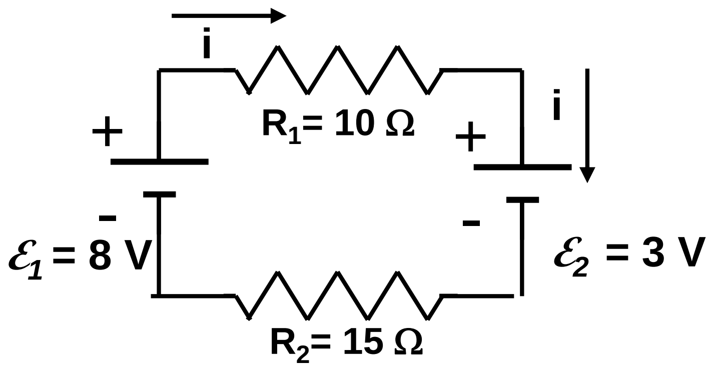
## EXAMPLE: MULTIPLE BATTERIES SINGLE LOOP




Find $i$, $P_{\mathcal{E}_1}$, $P_{\mathcal{E}_2}$, $P_{R_1}$, $P_{R_2}$

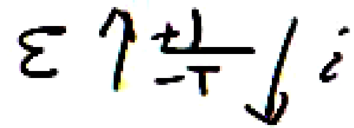
Loop Rule:

$$\mathcal{E}_1 - iR_1 - \mathcal{E}_2 - iR_2 = 0$$

$$\mathcal{E}_1 - \mathcal{E}_2 = i(R_1 + R_2)$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{(R_1 + R_2)} = \frac{8-3}{10+15} = .2A.$$

**A battery (EMF) absorbs power (charges up) when I is opposite to *E***



$$P_{emf} = \pm\, \mathcal{E}\mathbf{i} = \vec{\mathcal{E}} \circ \vec{\mathbf{i}}$$

$P_{\mathcal{E}_1} = \mathcal{E}_1 i = 8 \times .2) = 1.6W$ Provided to circuit.

$P_{\mathcal{E}_2} = \mathcal{E}_2 i = -3 \times .6 = -.6W$ absorbed.

$P_1 = -i^2 R_1 = -(.2)^2 10 = -.4W$ dissipated

$P_2 = -i^2 R_2 = -(.2)^2 \times 15 = -.6W$ "

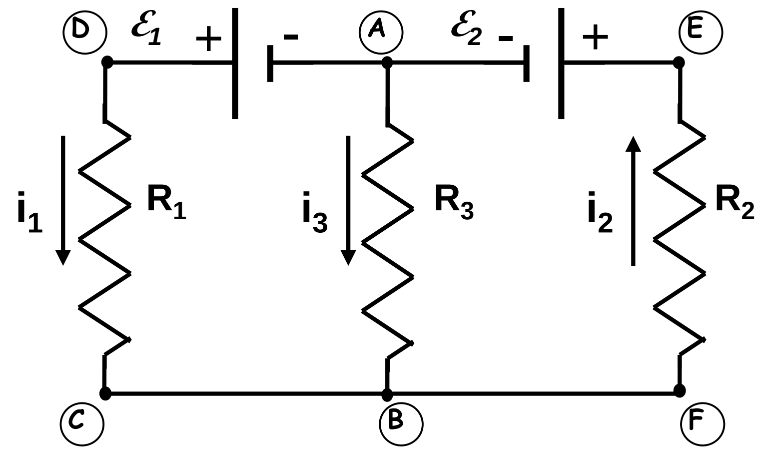
} $-1.6$ Watts.

$$P_{\mathcal{E}_1} = P_{\mathcal{E}_2} + P_{R_1} + P_{R_2}.$$

# Example: Multi-loop circuit with 2 EMFs

Given all resistances and EMFs in circuit:

- Find currents ($i_1$, $i_2$, $i_3$), then potential drops and power dissipated by resistors
- 3 unknowns (currents) imply 3 **independent** equations needed




## Apply Procedure:

- Identify branches & junctions. Name all currents (3) and other variables.
- Same current flows through all elements in any series branch.
- Assume arbitrary current directions; negative result means opposite direction.
- Find junctions, write Junction Rule equations for all.

$$\sum i_{in} = \sum i_{out}$$

- Same equation at junctions A and B (not independent).
- Junction Rule yields only 1 of 3 equations needed
- Are points C, D, E, F junctions? (No)

$$i_2 = i_1 + i_3 \quad (1)$$

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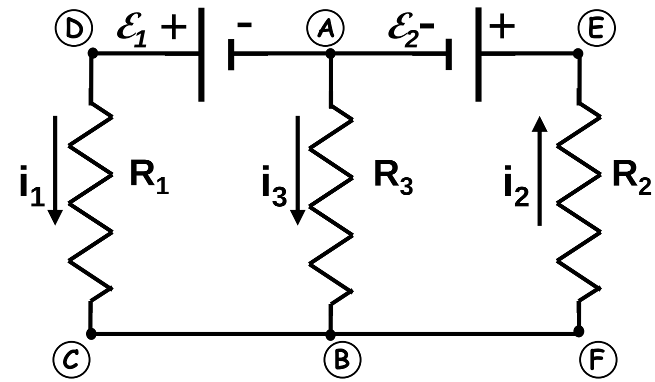
# Procedure, continued:




- Apply Loop Rule as often as needed to find equations that include all the unknowns (3).
- Traversal direction is arbitrary.
- IR's are voltage drops when following the assumed current direction: use **- iR**
- IR's are steps up when going against assumed current
- EMF's are positive when traversed from – to + side
- EMF's are negative when traversed from + to - sides

$$\sum \Delta V = 0$$

**Loop equations for the example circuit:**

ADCBA - CCW $\quad \mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$

ADCBFEA - CCW $\quad \mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 = 0$

ABFEA - CCW $\quad -i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$

- Only 2 of these three are independent
- Now have 3 equations in 3 unknowns

**Solution:** (after a lot of algebra)

**Define:**

$$[\;] = R_1 R_2 + R_2 R_3 + R_1 R_3$$

$$i_1 = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_1 R_3 - \mathcal{E}_2 R_3}{[\;]}$$

$$i_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2 R_3 - \mathcal{E}_2 R_1}{[\;]}$$

$$i_3 = \frac{-\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2}{[\;]}$$

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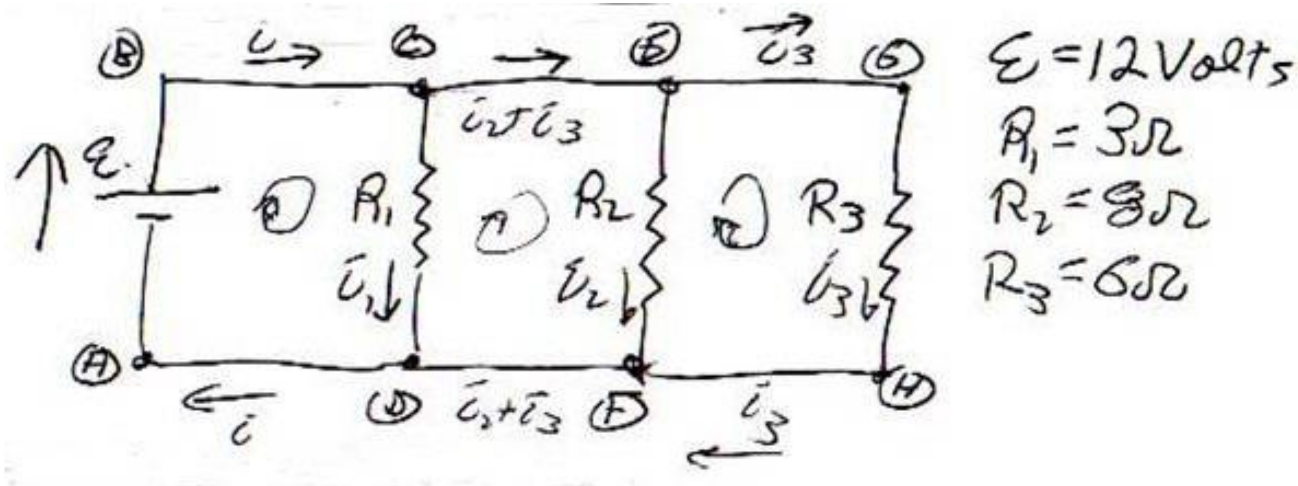
# Example: find currents, voltages, power

## 6 BRANCHES → 6 CURRENTS.

**•JUNCTION RULE:**

Branches C,E,G are the same point, as are D, F, H. 4 currents left. Remaining 2 junction equations are dependent 1 junction equation

$$i = i_1 + i_2 + i_3$$




**LOOP RULE:**

ABCDA - CW $\quad \mathcal{E} - i_1 R_1 = 0 \;\Rightarrow\; \mathcal{E} = i_1 R_1 \Rightarrow i_1 = \mathcal{E}/R_1 = 12/3 = 4.0\text{ A.}$

CEFDC - CW $\quad -i_2 R_2 + i_1 R_1 = 0 \;\Rightarrow\; i_2 = i_1 R_1 / R_2 = 4\text{x}3/8 = 1.5\text{ A.}$

EGHFE - CW $\quad -i_3 R_3 + i_2 R_2 = 0 \;\Rightarrow\; i_3 = i_2 R_2 / R_3 = 1.5\text{x}8/6 = 2.0\text{ A.}$

**CHECK:** $\quad i = i_1 + i_2 + i_3 = 4.0 + 1.5 + 2.0 = 7.5\text{ A.}$ $\qquad R_{eq} = 1.6\,\Omega$

$\mathcal{E}$ should $= V_{R1} = i_1 R_1 = 4.0\text{x}3.0 = 12.0$ Volts

**POWER:**

$P_{R1} = i_1^2 R_1 = 48.0$ Watts

$P_{R2} = i_2^2 R_2 = 18.0$ Watts

$P_{R3} = i_3^2 R_3 = 24.0$ Watts

$P_{\mathcal{E}} = \vec{\mathcal{E}} \circ \bar{i} = 90.0$ Watts $= P_{R1} + P_{R2} + P_{R3}$

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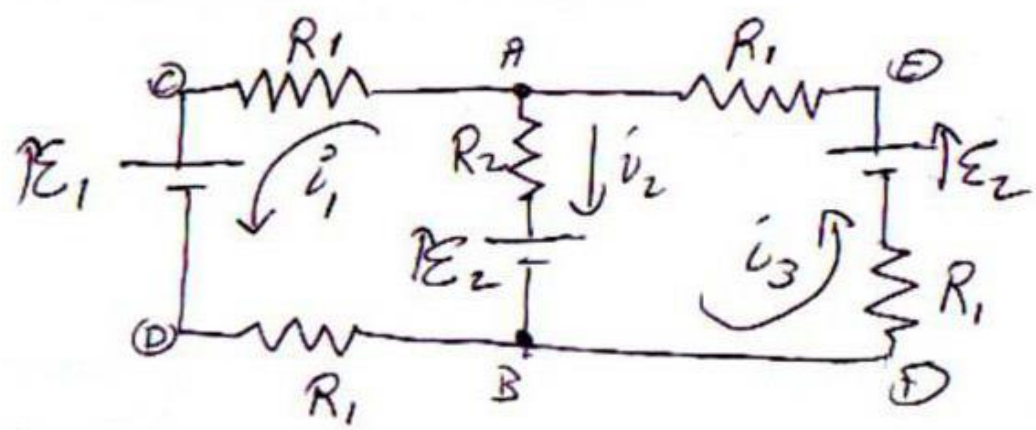
# Multiple EMF Example: find currents, voltages, power




$R_1 = 2$ W　　$R_2 = 4$ W

$\mathcal{E}_1 = 3$ V　　$\mathcal{E}_2 = 6$ V

MULTIPLE
EMF
CIRCUIT

USE THE
SAME RULES

JUNCTION RULE at A & B: $i_3 = i_1 + i_2$

LOOP ACDBA:

$$-i_1R_1 - \mathcal{E}_1 - i_1R_1 + \mathcal{E}_2 + i_2R_2 = 0 \;\Rightarrow\; i_2 = i_1 - 3/4$$

LOOP BFEAB:

$$-i_3R_1 + \mathcal{E}_2 - i_3R_1 - i_2R_2 - \mathcal{E}_2 = 0 \;\Rightarrow\; i_3 = -i_2$$

USE JUNCTION EQUATION:

$$i_3 = -i_2 = i_1 + i_2 \;\Rightarrow\; i_1 = -2i_2$$

For power use:

$$V = i_iR_i \qquad P_i = i_i^2R_i$$

EVALUATE NUMERICALLY: $i_1 = 1/2, \quad i_2 = -1/4, \quad i_3 = +1/4$

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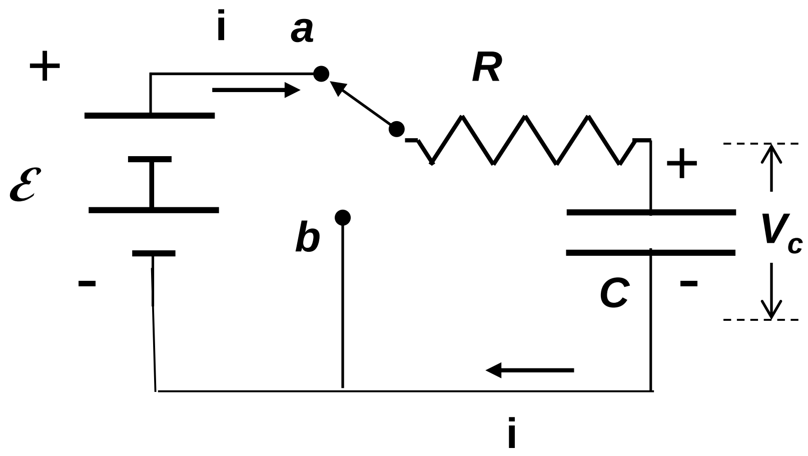
# RC Circuits: Time dependance




**Can constant current flow through a capacitor indefinitely?**

- Given Capacitance + Resistance + EMF
- Loop Rule + Junction Rule
- Find Q, i, V, U for capacitor as functions of time

**First charge C (switch to "a") then discharge (switch to "b")**

**Charging: Switch to "a". Loop equation:**

$$\mathcal{E} - iR - V_c = 0$$

- Assume current i through R is clockwise
- Expect largest current at t = 0,
- Expect zero current as t → infinity
- $V_{cap} \rightarrow \mathcal{E} = V_{inf}$ as t → infinity
- Energy stored in C, plus some dissipated in R

**Discharging: Switch to "b". no $\mathcal{EMF}$, Loop equation:**

$$- iR - V_c = 0$$

- Energy stored in C now dissipated in R
- Arbitrarily assume current is still CW
- $V_{cap} = \mathcal{E}$ at t =0, but it must die away
- $Q_0$= full charge = $CV_{inf} = C\mathcal{E}$
- Result: i through R is actually CCW

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# RC Circuit: solution for discharging

Loop Equation is : $iR + V_c = 0$

Substitute : $i(t) = \frac{dQ}{dt}$ $V_c(t) = \frac{Q(t)}{C}$

Circuit Equation:

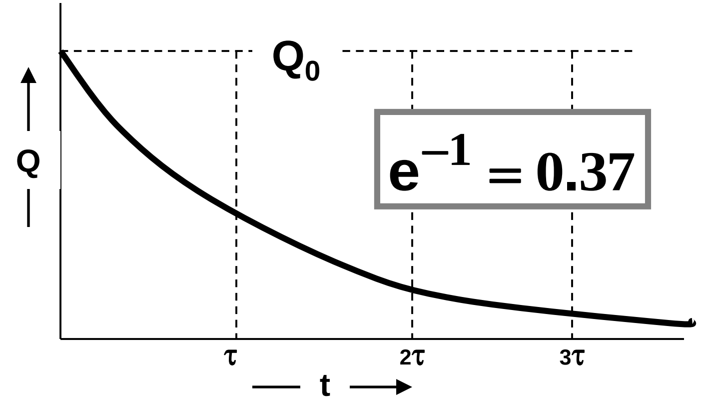
$$\frac{dQ}{dt} = -\frac{Q(t)}{RC}$$

First order differential equation, form is Q' = -kQ → Exponential solution

Charge decays exponentially:

- t/RC is dimensionless

$$Q(t) = Q_0 e^{-t/RC}$$




**RC = τ = the TIME CONSTANT**

Q falls to 1/e of original value

Voltage across C also decays exponentially:

$$Q_0 = C\mathcal{E}$$
$$Q(t) = CV_c(t)$$

$$V_c(t) = \mathcal{E}\, e^{-t/RC}$$

Current also decays exponentially:

$$i(t) \equiv \frac{dQ}{dt} = i_0 e^{-t/RC}$$

$$i_0 \equiv \frac{\mathcal{E}}{R} = \frac{Q_0}{RC}$$

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# Solving for discharging phase by direct integration

$$\frac{dQ}{dt} = -\frac{Q(t)}{RC}$$ RC is constant

Initial conditions ("boundary conditions")
$Q(t) = Q_0$ at $t = 0$ where $Q_0 = C\mathcal{E}$

$$\frac{dQ}{Q} = -\frac{dt}{RC} \Rightarrow \int_{Q_0}^{Q} \frac{dQ'}{Q'} = -\frac{1}{RC}\int_0^t dt' \Rightarrow \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

exponentiate both sides of above right $e^{\ln(x)} = x$

$$e^{\ln\left(\frac{Q}{Q_0}\right)} = \frac{Q}{Q_0} = e^{-\frac{t}{RC}} \Rightarrow \boxed{Q(t) = Q_0 e^{-t/RC}}$$ exponential decay

RC = time constant = time for Q to fall to 1/e of its initial value

$$RC \equiv \tau$$

$$e^{-1} = \frac{1}{e} = \frac{1}{2.71828} \approx .37$$

| Time | t | 2t | 3t | 4t | 5t |
|---|---|---|---|---|---|
| Value | $e^{-1}$ | $e^{-2}$ | $e^{-3}$ | $e^{-4}$ | $e^{-5}$ |
| % left | 36.8 | 13.5 | 5.0 | 1.8 | 0.67 |

After 3-5 time constants the action is over

# Units for RC

**8-1: We defined $\tau$ = RC, which of the choices best conveys the physical units for the decay constant $\tau$ ?**

$$[\tau] = [RC] = [(V/i)(Q/V)] = [Q/Q/t] = [t]$$

**A.** $\Omega \cdot F$ **(ohm·farad)**
**B.** **C/A** **(coulomb per ampere)**
**C.** $\Omega \cdot C/V$ **(ohm·coulomb per volt)**
**D.** **V·F/A** **(volt·farad per ampere)**
**E.** **s** **(second)**

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# Examples: discharging capacitor C through resistor R

## a) When has the charge fallen to half of it's initial value $Q_0$?

set: $Q(t) = \frac{1}{2}Q_0 = Q_0 e^{-t/\tau}$ $\Rightarrow$ $\frac{1}{2} = e^{-t/\tau}$ (solve for t - depends only on $\tau$)

take log: $\ln(\tfrac{1}{2}) = -t/\tau$ $\quad \ln(1) = 0 \quad \ln(a/b) = \ln(a) - \ln(b)$

$-\ln(2) = -t/\tau \quad \ln(2) = 0.69$ $\Rightarrow$ $\therefore\ t = 0.69\,\tau$

## b) When has the stored energy fallen to half of its original value?

recall: $U(t) = \frac{Q^2}{2C}$ and $Q(t) = Q_0 e^{-t/RC}$

at any time t: $U(t) = U_0\, e^{-2t/RC}$ $\quad$ at t = 0: $U(t=0) \equiv U_0 = \frac{Q_0^2}{2C}$

set: $U(t) = \frac{U_0}{2} = U_0\, e^{-2t/\tau}$

take log: $\ln(\tfrac{1}{2}) = -2t/\tau$ $\Rightarrow$ $\therefore\ t = 0.69\,\tau/2 = 0.35\,\tau$

## c) How does the power delivered to C vary with time?

power: $P \equiv \frac{dU}{dt} = U_0 \frac{d}{dt}[e^{-2t/\tau}] = U_0[\frac{-2}{\tau}]e^{-2t/\tau} = \frac{-2}{2}\frac{Q_0}{C}\frac{Q_0}{RC}e^{-2t/\tau}$

recall: $\frac{Q_0}{RC} \equiv i_0 \qquad \frac{Q_0}{C} \equiv \mathcal{E}$

C supplies rather than absorbs power
Drop minus sign

power supplied by C: $P = i_0 e^{-t/\tau} \times \mathcal{E} e^{-t/\tau} = i(t) \times V(t)$

$\equiv P_0 e^{-2t/\tau}$

$\Rightarrow$ $P(t) = i(t)\,V(t)$

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# RC Circuit: solution for charging

**Loop Equation is :** $\mathcal{E} - iR - V_c = 0$

**Substitute :** $i(t) = \dfrac{dQ}{dt} \qquad V_c(t) = \dfrac{Q(t)}{C}$

→

**Circuit Equation:**

$$\frac{dQ}{dt} = -\frac{Q(t)}{RC} + \frac{\mathcal{E}}{R}$$

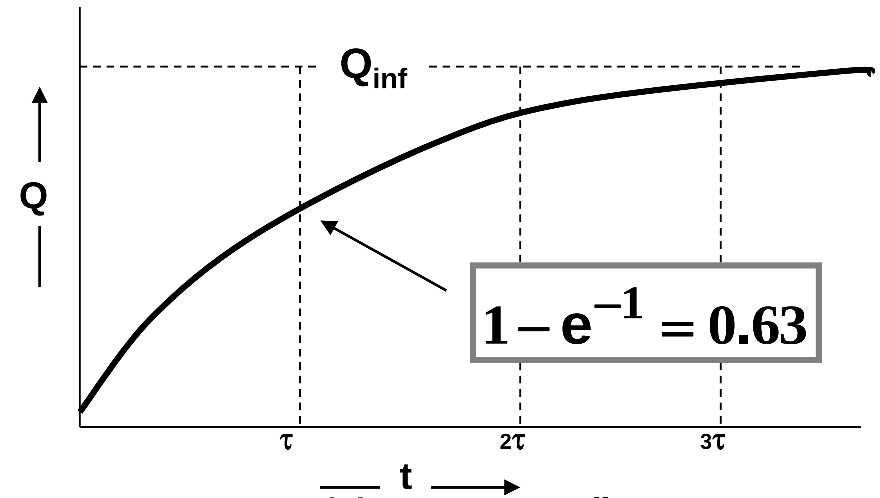
- First order differential equation again: form is Q' = - kQ + constant
- Same as discharge equation, but $i_0 = \mathcal{E} / R$ is on right side
- At t = 0: Q = 0 & $i = i_0$. Large current flows (C acts like a wire)
- As t → infinity: Current → 0 (C acts like an open circuit)

  $Q \rightarrow Q_{inf} = C\mathcal{E}$ = same as $Q_0$ for discharge

*Solution: Charge starts from zero, grows as a saturating exponential.*

$$Q(t) = Q_{inf}\left(1 - e^{-t/RC}\right)$$

- RC = τ = TIME CONSTANT describes time dependance again
- Q(t) → 0 as t → 0
- Q(t) → $Q_{inf}$ as t → infinity

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# RC Circuit: solution for charging, continued

## Voltage across C while charging:

$$Q = CV_c \quad \text{and} \quad Q_{inf} = C\mathcal{E} \quad \Rightarrow \quad V_c(t) = \mathcal{E}\,(1 - e^{-t/RC})$$

Voltage across C also starts from zero and saturates exponentially

## Current in the charging circuit:

$$i(t) \equiv \frac{dQ(t)}{dt} = Q_{inf}\frac{d}{dt}\left(1 - e^{-t/RC}\right) = Q_{inf}\frac{1}{RC}e^{-t/RC} \quad \Rightarrow \quad i(t) = i_0 e^{-t/RC}$$

$$i_0 \equiv \frac{\mathcal{E}}{R} = \frac{Q_{inf}}{RC}$$

Current decays exponentially just as in discharging case

Growing potential $V_c$ on C blocks current completely at t = infinity

At t=0 C acts like a wire. At t=infinity C acts like a broken wire

## Voltage drop $V_R$ across the resistor:

$$V_R(t) = i(t)R = i_0 R e^{-t/RC} \quad \Rightarrow \quad V_R(t) = \mathcal{E}\, e^{-t/RC}$$

Voltage across R decays exponentially, reaches 0 as t→ infinity

## Form factor: 1 – exp( - t / τ )

| Factor | .63 | .865 | .95 | .982 | .993 | .998 |
|---|---|---|---|---|---|---|
| Time | τ | 2τ | 3τ | 4τ | 5τ | 6τ |

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After 3-5 time constants the action is over

# RC circuit – multiple resistors

**8-2: Consider the circuit shown, The battery has no internal resistance. The capacitor has zero charge.**

**Just after the switch is closed, what is the current through the battery?**

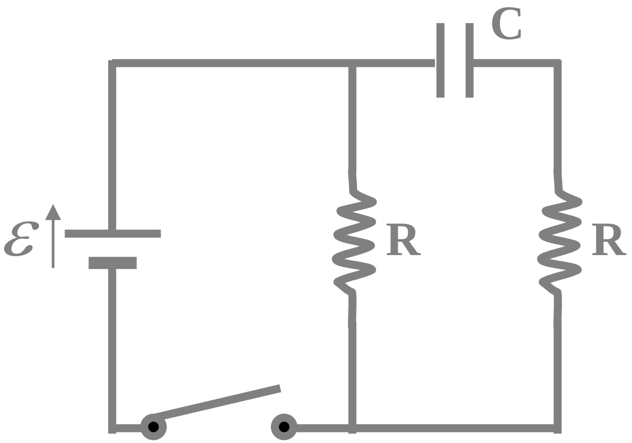
**A. 0.**

**B. ε/2R.**

**C. 2ε/R.**

**D. ε/R.**

**E. impossible to determine**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# RC circuit – multiple resistors

**8-3: Consider the circuit shown. The battery has no internal resistance. After the switch has been closed for a very long time, what is the current through the battery?**

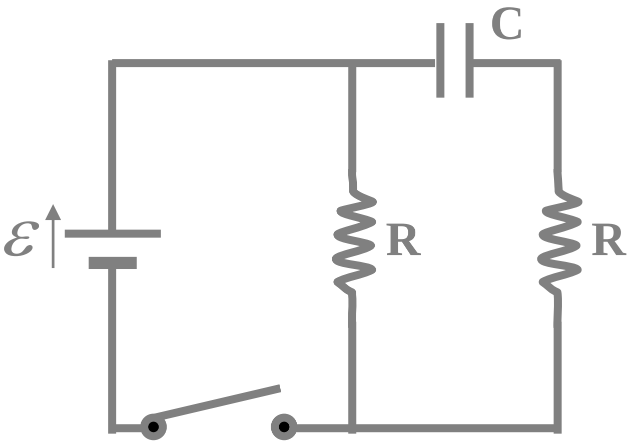
**A. 0.**

**B. ε/2R.**

**C. 2ε/R.**

**D. ε/R.**

**E. impossible to determine**

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**Discharging Example: A 2 mF capacitor is charged and then connected in series with a resistance R. The original potential across it drops to ¼ of it's starting value in 2 seconds. What is the value of the resistance?**

**Use:** $V_c(t) = V_0\, e^{-t/RC}$

**Set:** $\frac{V_c(t)}{V_0} = \frac{1}{4} = e^{-t/RC}$

**Take natural log of both sides:**

$$\ln(1) - \ln(4) = \ln[e^{-2/RC}] = \frac{-2}{RC}$$

$$\ln(4) = 1.39 \qquad \ln(1) = 0 \qquad \ln[e^{x}] = x$$

$$1.39\ RC = 2 \quad \Rightarrow \quad R = \frac{2}{1.39}\frac{1}{2\text{x}10^{-6}}$$

$$R = 0.72\ \text{M}\Omega$$

**Define: $1\ \text{M}\Omega = 10^6\ \Omega$**

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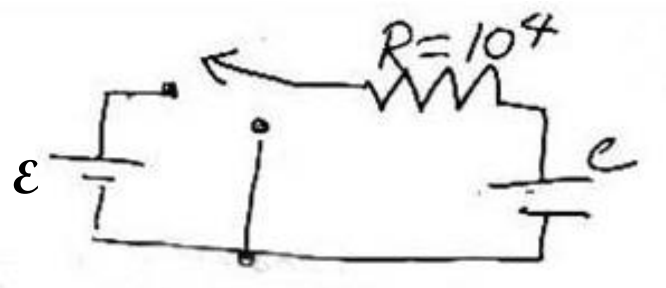
# Example: Discharging




C = 500 mF    R = 10 KW    $V_0 = \mathcal{E} = 12$ V

Capacitor C is charged for a long time, then discharged.

## a) Find current at t = 0

$$i(t) \equiv \frac{dQ}{dt} = i_0 e^{-t/RC} \qquad i_0 \equiv \frac{\mathcal{E}}{R} = \frac{Q_0}{RC} \quad \Rightarrow \quad i(t=0) = \frac{\mathcal{E}}{R} e^0 = \frac{12}{10^4} = 1.2\text{ mA}$$

## b) When does $V_{Cap}$ (voltage on C) reach 1 Volt?

$$V_{cap}(t) = \mathcal{E}\, e^{-t/RC} \qquad RC = 10^4 \times 5 \times 10^2 \times 10^{-6} = 5\text{ sec} \qquad V_0 = \mathcal{E} = 12\text{ Volts}$$

$$\frac{V_{cap}}{V_0} = \frac{1}{12} = e^{-t/5} \qquad -\ln(12) = -t/5 \quad \Rightarrow \quad \boxed{t = 5\ln(12) = 12.4\text{ sec}}$$

## c) Find the current in the resistor at that time

$$i(t) \equiv \frac{dQ}{dt} = i_0 e^{-t/RC} \quad \Rightarrow \quad i(t = 12.4\text{ sec}) = 1.2\text{ mA} \times e^{-12.4/5}$$

$$\boxed{i(t = 12.4\text{ sec}) = 0.1\text{ mA}}$$

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## Charging Example: How many time constants does it take for an initially uncharged capacitor in an RC circuit to become 99% charged?

**Use:** $Q(t) = Q_\infty\left(1 - e^{-t/\tau}\right)$ $\tau \equiv RC = \text{time constant}$

**Require:** $\dfrac{Q(t)}{Q_\infty} = 0.99 = 1 - e^{-t/\tau}$ ⇨ $0.01 = e^{-t/\tau}$

**Take natural log of both sides:**

$$\ln(0.01) = -4.61 = -t/\tau$$

$$\therefore\ t/\tau = 4.61 = \#\ \text{of time constants}$$

**Did not need specific values of RC**

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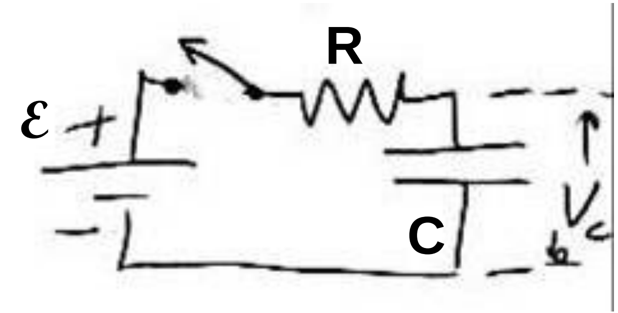
**Example: Charging a 100 μF capacitor in series with a 10,000 Ω resistor, using EMF $\mathcal{E}$ = 5 V.**




a) How long after voltage is applied does $V_{cap}(t)$ reach 4 volts?

$$V_c(t) = \mathcal{E}\,(1 - e^{-t/RC}) \qquad RC = 10^4 \times 100 \times 10^{-6} = 1.0 \text{ sec}$$

$$\frac{V_c(t)}{\mathcal{E}} = \frac{4}{5} = 0.8 = 1 - e^{-t/RC} \qquad \therefore \quad e^{-t/RC} = 0.2$$

Take natural log of both sides:

$$\ln(0.2) = -1.61 = \ln[e^{-t/RC}] = \frac{-t}{RC} = -t$$

**t = 1.61 sec**

b) What's the current through R at t = 2 sec?

$$i(t) = i_0 e^{-t/RC} \qquad i_0 \equiv \frac{\mathcal{E}}{R}$$

$$i(t=2) = i_0 e^{-2.0/1.0} = \frac{\mathcal{E}}{R} e^{-2.0/1.0} = \frac{5}{10^4}(0.37)^2 = 6.77 \times 10^{-5}$$

**$i(t=2) = 6.8\ \mu A.$**

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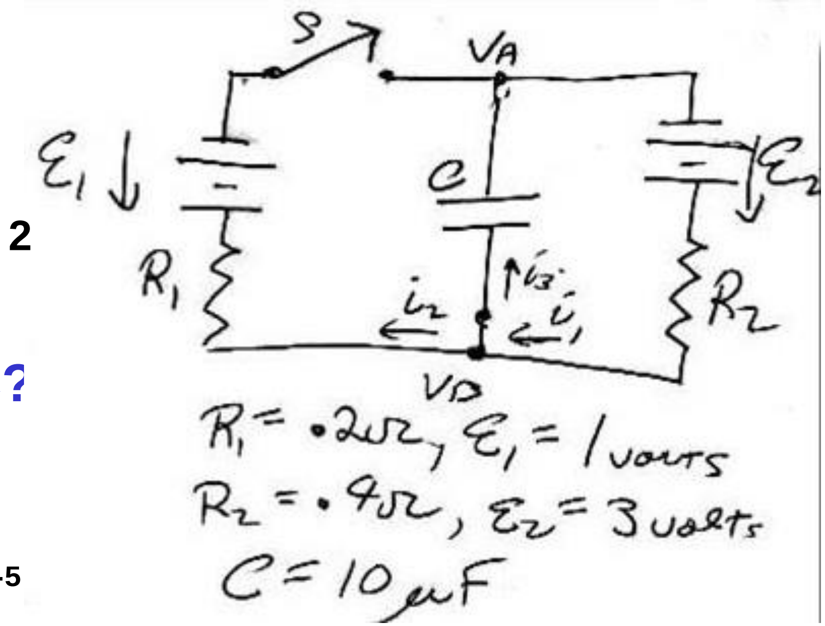
# Example: Multiple loops and EMFs

- Switch S is initially open for a long time.
- Capacitor C charges to potential of battery 2
- S is then closed for a long time




## What is the CHANGE in charge on C?

### First: $\mathcal{E}_2$ charges $C$ to have:

$$V_c = \mathcal{E}_2 = 3 \text{ volts with current } i_1 = 0$$

$$Q_0 = \text{final charge for first phase} = C\mathcal{E}_2 = 3.0 \times 10^{-5}$$

$$\boxed{Q_0 = \text{inital charge for second phase} = 30\ \mu C}$$

### Second: Close switch for a long time

At equilibrium, current $i_3$ though capacitor → zero

Find outer loop current $i = i_1 = 1_2$ using loop rule

$$\mathcal{E}_2 - iR_2 - iR_1 - \mathcal{E}_1 = 0$$
$$3 - i(0.4 + 0.2) - 1 = 0$$

→ $\boxed{i = 2.0/0.6 = 3.33 \text{ A.}}$

Now find Voltage across C, same as voltage across right hand branch

$$V_b - V_a = \mathcal{E}_2 - iR_2 = 3 - 3.33 \times 0.4 = 1.67 \text{ v}$$

Final charge on C:

$$Q_{final} = C(V_b - V_a) = 10 \times 10^{-6} \times 1.67$$

$\boxed{Q_{final} = 16.7\ \mu C}$ → $\boxed{Q_{final} - Q_0 = -13.3\ \mu C}$

# Lecture 8A Chapter 27 - Circuits, Part 1

EMF: ELECTROMOTIVE FORCE. SOURCE OF POWER + POTENTIAL DIFFERENCE


- IDEAL EMF: $r=0$, $V=\mathcal{E}$
- REAL EMF: $V=\mathcal{E}-ir$

$$\mathcal{E}=\frac{dW}{dq}$$

$P$ = Power Supplied $= i\mathcal{E}$

Power Dissipated $= i\mathcal{E}-V$

BRANCH RULE: SAME CURRENT IN ALL SERIES ELEMENTS. N BRANCHES $\Rightarrow$ N CURRENTS

JUNCTION RULE: $\sum_{in} i_s = \sum_{out} i_s$ AT EACH  $i=\frac{dq}{dt}$

LOOP RULE: • $\sum \Delta V's = 0$ AROUND EVERY CLOSED LOOP

• POTENTIAL DIFFERENCE BETWEEN A PAIR OF POINTS IS THE SAME FOR EVERY PATH.

USE RULES TO GET N EQUATIONS IN N UNKNOWNS

- NAME + IDENTIFY CURRENTS. ASSUME DIRECTIONS
- USE JUNCTION RULE. EQUATIONS NOT ALL INDEPENDENT
- USE LOOP RULE - TRAVERSE EACH ONE
  - FOR R's: $\Delta V=-iR$ WHEN FOLLOWING ASSUMED CURRENT
    $\Delta V=+iR$ " GOING OPPOSITE TO " "
  - FOR EMF's: $\Delta V=+\mathcal{E}$ WHEN TRAVERSING FROM (−) TO (+)
    $\Delta V=-\mathcal{E}$ " " " (+) TO (−)
    IGNORE CURRENT
- SOLVE RESULTING SYSTEM OF EQUATIONS FOR ALL CURRENTS
- CALCULATE POWER, OTHER QUANTITIES NEEDED.

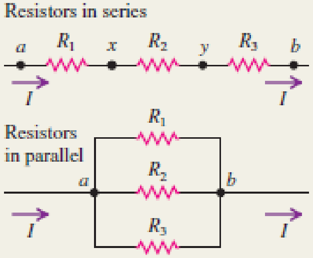
# Summary: Lecture 8B Chapter 27 – RC Circuits, Part 2

CHAPTER 26 SUMMARY

**Resistors in series and parallel:** When several resistors $R_1, R_2, R_3, \ldots$ are connected in series, the equivalent resistance $R_{eq}$ is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of the equivalent resistance $R_{eq}$ is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2.)

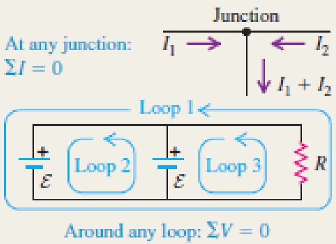
$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad (26.1)$$
(resistors in series)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad (26.2)$$
(resistors in parallel)

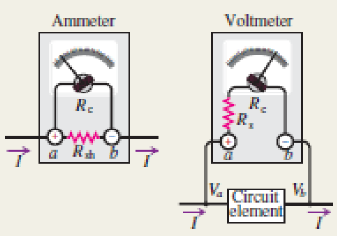



**Kirchhoff's rules:** Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules. (See Examples 26.3–26.7.)

$$\sum I = 0 \quad \text{(junction rule)} \quad (26.5)$$
$$\sum V = 0 \quad \text{(loop rule)} \quad (26.6)$$




**Electrical measuring instruments:** In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil. Such an instrument is called an ammeter. If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage. The instrument is then called a voltmeter. A good ammeter has very low resistance; a good voltmeter has very high resistance. (See Examples 26.8–26.11.)

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**R-C circuits:** When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13.)

**Capacitor charging:**

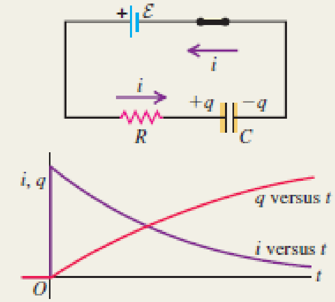
$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \quad (26.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.13)$$

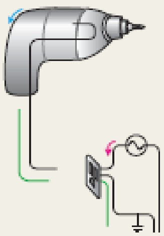
**Capacitor discharging:**

$$q = Q_0 e^{-t/RC} \quad (26.16)$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.17)$$




**Household wiring:** In household wiring systems, the various electrical devices are connected in parallel across the power line, which consists of a pair of conductors, one "hot" and the other "neutral." An additional "ground" wire is included for safety. The maximum permissible current in a circuit is determined by the size of the wires and the maximum temperature they can tolerate. Protection against excessive current and the resulting fire hazard is provided by fuses or circuit breakers. (See Example 26.14.)



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