Physics 234 Exam # 1, 2/13/02

Name:____________________

Section A. Circle one of the answers (4 points each).

1. In Fig. 1 a weight of mass m is attached to a spring with spring constant k. The frequency ω of free oscillations will increase if the incline angle increases from 40° to 70°.

True False

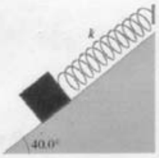


Fig. 1.

2. The displacement of the block away from zero position of the spring (relaxed state) increases with increasing angle

True False

3. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion:

$\frac{d^2x}{dt^2} = -\omega^2 x$

(a) $a = 0.5x$, (b) $a = 400x^2$, (c) $a = -20x$, (d) $a = -3x^2$?

4. The following four waves are sent along strings with the same linear densities (x is in meters and t is in seconds). Which wave has the highest velocity?

(a) $y_1 = (3 \text{ mm}) \sin(x - 3t)$, (b) $y_2 = (6 \text{ mm}) \sin(2x - t)$, (c) $y_3 = (1 \text{ mm}) \sin(4x - t)$, and (d) $y_4 = (2 \text{ mm}) \sin(x - 2t)$.

$v = 3$ $v = 1/2$ $v = 1/4$ $v = 2$

$y = y_0 \sin(kx - \omega t)$ $v = \omega/k$

5. Two waves with the same amplitude and wavelength interfere to produce a standing wave. Which equation represents the standing wave?

(a) $y'(x, t) = 4 \sin(5x - 4t)$, (b) $y'(x, t) = 4 \sin(5x) \cos(4t)$, (c) $y'(x, t) = 4 \sin(5x + 4t)$

6. For a particular instrument four of the six harmonic frequencies below 1000 Hz are: 300, 600, 750, and 900 Hz. What is the fundamental frequency?

(a) 200 Hz, (b) 150 Hz, (c) 300 Hz, (d) 900 Hz

Section B- Longer Problems

7. (20 Pts.) A block of mass m = 0. 25 kg is connected by a spring (with constant k = 40 N/m) to a vertical support. Assume that the surface on which the block slides is frictionless. Let the motion of the block start a t=0 by displacing the block 10 cm from its equilibrium position and giving it a velocity in the +x direction of 40 cm/s. Compute the following:

(a) the period

$$T = 1/f = \frac{1}{2.01} = 0.498\ s$$

(b) the frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\ 12.65\ rads/sec = 2.01\ Hz$$

(c) the angular frequency

$$\omega = \sqrt{k/m} = \sqrt{\frac{40\ N/m}{0.25\ kg}} = \sqrt{160 s^{-2}} = 12.65\ rads/s$$

(d) the total energy

$$E = const = \frac{1}{2} m v^2 + \frac{1}{2} k x_0^2 = \frac{1}{2}(0.25 kg)(0.4 m/s)^2 + \frac{1}{2}\, 40\, (0.10)^2 = 0.22\ J$$

(e) the amplitude of the oscillations

at $x = x_m$ – total energy is potential energy
– $V = 0$

$$0.22 J = \frac{1}{2} k x_m^2 \qquad x_m = \sqrt{\frac{2(0.22 J)}{40 N/m}} = 0.105 m$$

(f) the phase angle

$$X(t) = X_m \cos(\omega t + \phi)$$
$$X(0) = X_m \cos(\phi) \qquad \phi = \cos^{-1}\left(\frac{X(0)}{X_m}\right) = \cos^{-1}\left(\frac{0.10}{0.11}\right) = 24.6^\circ$$

(g) the maximum velocity

$$V_m = \omega X_m = (12.65\ rads/s)(0.11 m) = 1.4\ m/s$$

(h) the maximum acceleration

$$a_m = +\omega^2 x_m = (12.65\ \text{rads/sec})^2 * (0.11\ \text{m})$$
$$= 17.6\ \text{m/s}^2$$

(i) the position, velocity and acceleration at $t = \pi/8$

$x(t) = x_m \cos(\omega t + \phi) \longrightarrow x(\pi/8) = 0.11 \cos(12.65(\pi/8) + 24.6)$
$= 0.097\ \text{m}$

$v(t) = -x_m \omega \sin(\omega t + \phi)$ $\quad v(\pi/8) = -(0.11)(12.65) \sin(12.65 \frac{\pi}{8} + 24.6)$
$= 0.69\ \text{m/s}$

$a(t) = -x_m \omega^2 \cos(\omega t + \phi)$ $\quad a = -(12.65)^2 (0.097) = 15.5\ \text{m/s}^2$

8. (10 Pts.) The period of a physical pendulum is T_1 when on the surface of the earth where the free fall acceleration is g. If the pendulum is placed on a rocket accelerating upward with acceleration a. What will be the new period of the pendulum in terms of g and T_1?

$$T_1 = 2\pi \sqrt{\frac{I}{mgh}}$$

$\vec{a} = \vec{g} - \vec{a}_{rocket}$ relative to rocket frame

$a = g + a_{rocket}$ choose down as positive

$$T_2 = 2\pi \sqrt{\frac{I}{m(g+a)h}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g+a}} \qquad T_2 = T_1 \sqrt{\frac{g}{g+a}}$$

9. (8 Pts.) An electrical transmission line is strung across a valley between two poles. The distance between the poles is 200 m and the mass of the line is 40 kg. If you strike the line on one pole the wave created travels to the other pole and is reflected back. What is the tension if the wave take 10 sec to travel to the second pole and back?

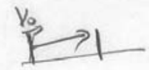
$$v = \sqrt{\frac{T}{\mu}} \qquad v^2 = \frac{T}{\mu}$$

$$T = v^2 \mu$$

$$v = \frac{400\text{m}}{10\ \text{sec}} = 40\ \text{m/s}$$

$$\mu = \frac{m}{L} = \frac{40\text{kg}}{200\text{m}} = 0.2\ \text{kg/m}$$

$$T = (40\ \text{m/s})^2\ 0.2\ \text{kg/m} = 320\ \text{N}$$



10. (8 Pts) Standing waves are produced by the superposition of two waves of the form (lengths in m):

$y_1 = 15\sin(5x - 3\pi t + \pi/3)$

and

$y_2 = 15\sin(5x + 3\pi t)$.

What is the amplitude of the motion at x = 21 m.

$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\alpha = A+B$
$\beta = A-B$

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$y' \quad y_1 + y_2 = (15)(2)\sin(5x + \pi/6)\cos(3\pi t + \pi/6)$

$y' = [30\sin(5x + \pi/6)]\cos(3\pi t + \pi/6)$

↑ amplitude of standing wave ↑ time dependent part

Amplitude = $30\sin(5(21) + \pi/6) = 28.9$ m

11. (14 pts.) A stone is dropped into a well. The sound of the splash is heard 3.00 s later. What is the depth of the well?

$V_{sound} = 343$ m/s
$g = 9.81$ m/s^2

$T = t_1 + t_2$ ⇒ free fall $h = \frac{1}{2}g t_1^2$

↑ free fall; time for sound to travel back up

sound travel $h = V t_2$ ↑ sound velocity

$T = \sqrt{\frac{2h}{g}} + \frac{h}{V}$

$\sqrt{\frac{2h}{g}} = T - \frac{h}{V} \Rightarrow \frac{2h}{g} = \left(T - \frac{h}{V}\right)^2 = T^2 + \frac{h^2}{V^2} + \frac{2hT}{V}$

$V^2 2h/g = T^2V^2 + h^2 - 2VhT$

$0 = h^2 + h(2VT + 2V^2/g) + T^2V^2$

↓

$$0 = h^2 + h\left(2(343\,m/s)(3\,sec) + 2(343)^2/9.81\right) + 3^2(343)^2$$

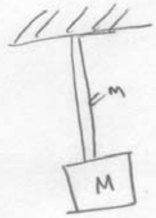
$$0 = h^2 - 26043.5h + 1058841$$

$$h = 40.72\ m$$

$$h = \frac{+26043.5 \pm \sqrt{(26043.5)^2 - 4(1058841)}}{2}$$

NJ

12. (16 pts.) A cord of mass m and length L hang from the ceiling. A mass M is then attached to the cord. (a) What is the tension in the cord as a function of the distance from the lowest point on the cord? What is the velocity of a wave at the midpoint of the cord? (c) Write down an expression for time required for a wave to travel from the bottom of the cord to the top?



(a) $T = Mg + g\frac{y}{L}m$

$T(y) = g(M + m\frac{y}{L})$

(b) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{g(M + m y/L)}{m/L}}$

$= \sqrt{g\frac{(ML + my)}{m}}$

10

$V(L/2) = \sqrt{\frac{g(M + m/2)L}{m}}$

(c) $\frac{dy}{dt} = V$

$\frac{dy}{V(y)} = dt$

$\int_0^T dt = \int_0^L \frac{dy}{V(y)} = \int_0^L \frac{dy}{\sqrt{\frac{g(ML + my)}{m}}}$