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A very simple and inexpensive apparatus for detecting superconducting transitions via magnetic screening

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We report a very simple and inexpensive technique for detecting the screening of the B field by high temperature superconductors via the change in the inductance of a coil containing the sample. While related to previously reported techniques for measurement magnetic susceptibility via changes in inductance, the apparatus we report does not require lock-in amplification and is suitable for general physics labs.

I. INTRODUCTION

Lab exercises involving the detection of superconductivity and determination of the transition temperature T_c from measurement of the resistance of a sample are common. Inexpensive kits with samples of $YBa_2Cu_3O_{7-\Delta}$ prepared for four-lead resistance measurements are available commercially[1] and are often used in general physics labs.

Detection of superconductivity via observation of the screening of the magnetic induction B by a superconductor is less common in undergraduate labs, but two recent articles in the American Journal of Physics[2,3] describe apparatus suitable for such experiments and provide detailed analyses of inductive measurement of magnetic susceptibility. In this paper we report an extremely simple and inexpensive alternative method for detecting a superconducting transition via inductive measurement of magnetic susceptibility. Our apparatus requires only a polycrystalline sample of $YBa_2Cu_3O_{7-\Delta}$ and a thermocouple which can both be obtained commercially,[1] a coil potted in epoxy, a standard student signal generator, two standard $3\frac{1}{2}$ digit digital multimeters (DMMs), one DMM with 0.1-mV dc resolution for reading the thermocouple, and a Styrofoam cup filled with sand. The apparatus is sufficiently simple and inexpensive to allow construction of a copy for each lab station in an instructional lab.

II. THE EXPERIMENT

The apparatus is shown in Fig. 1. A standard signal generator[4] generates a sinusoidal signal at about 1 kHz. The frequency is not critical, but multiples of 60 Hz should be avoided. The signal generator is connected to an inductor containing a cylindrical sample of $YBa_2Cu_3O_{7-\Delta}$ as indicated in Fig. 1. One of the DMMs is used to measure the rms current I in the circuit and another is used to measure the rms voltage V_L across the inductor which is given by

$$V_L = I\sqrt{R^2+\omega^2L^2}$$

or

$$\omega L = \sqrt{(V_L/I)^2 - R^2}, \tag{1}$$

where R is the resistance of the coil and L is its inductance. Suitable coils have an impedance of less than 10 Ω at 78 K, and some signal generators can be damaged by driving a low impedance load. To prevent damage to the signal generator, we have used a 100 Ω resistor in series with the coil. The construction of the coil is discussed in Sec. III below.

If the coil contains a paramagnetic sample of susceptibility χ and permeability $\mu=\mu_0(1+\chi)$, the ac flux through the coil will increase, resulting in an increase in the inductance of the coil. Likewise, the presence of a diamagnetic sample will decrease the inductance of the coil. In particular, the flux of the B field through a coil immersed in a medium of permeability μ is enhanced by a factor of $(1+\chi)$ compared to an identical coil in a vacuum, and thus the inductance of the coil in the medium is

$$L = L_0(1+\chi) \quad \text{or} \quad \chi = L/L_0 - 1, \tag{2}$$

where L_0 is the inductance of the coil in a vacuum. This result holds for any coil, provided the sample occupies all

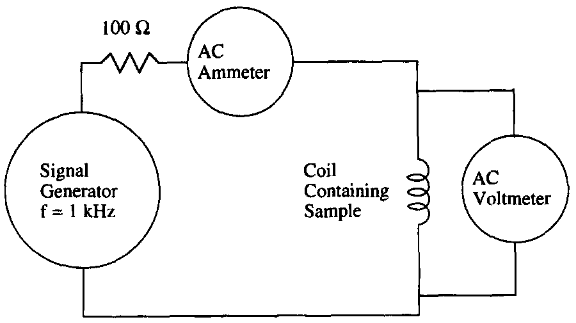

Fig. 1. A schematic diagram of the apparatus. The coil, which contains the sample, is buried in sand in a Styrofoam cup and a thermocouple attached to the sample is used to monitor the sample temperature.

space in which the coil produces a field. If the sample does not fill all space in which the coil produces a field, geometrical corrections (of order unity) will appear in the equation for χ.

An ideal superconducting sample screens the B field completely in B fields weaker than the lower critical field. This is perfect diamagnetism and the corresponding susceptibility is $\chi=-1$. In the normal state the susceptibility of a superconductor is small and the presence of the sample does not significantly affect the inductance of the coil. Thus the transition from the normal to the superconducting state results in a large change in the inductance of the coil. Our experiment consists of measuring as a function of temperature the rms[5] current through the coil, the rms voltage across the coil, and the resistance of the coil. Equation (1) is then used to find the inductance of the coil, and the superconducting transition is indicated by a sudden change in the inductance.

A. Temperature control

We have used a very simple technique for changing the sample temperature slowly enough to allow data collection. The coil containing the sample is buried in sand (rinsed to remove dust) which partially fills a Styrofoam cup. Following standard safety precautions, liquid nitrogen is added slowly to the cup until the sand and superconductor are cooled to below 80 K, and the apparatus is then allowed to warm slowly on a lab bench. By adjusting the size of the cup and the amount of sand, one can vary the rate at which the sample warms. Students record the current, the voltage across the inductor, and the voltage across the thermocouple mounted on the sample as the apparatus warms up. Students typically require between 2 and 3 h to complete the experiment using a 16-oz Styrofoam cup filled with sand.

Proper operation of the thermocouple requires that the junction be thermally anchored to the sample. It is also essential that the room temperature connections to the thermocouple remain at room temperature. Cold vapors escaping from the apparatus can cool these connections, resulting in erratic and incorrect temperature readings. One can see this effect by warming the room temperature connections between two fingers. This results in a substantially different temperature reading. We have found that the cold vapors flow mostly near the surface of the lab bench on which the apparatus sits. Placing the connections to the thermocouple as far as possible from the apparatus and at least a few inches above the lab bench greatly reduces errors in temperature measurement.

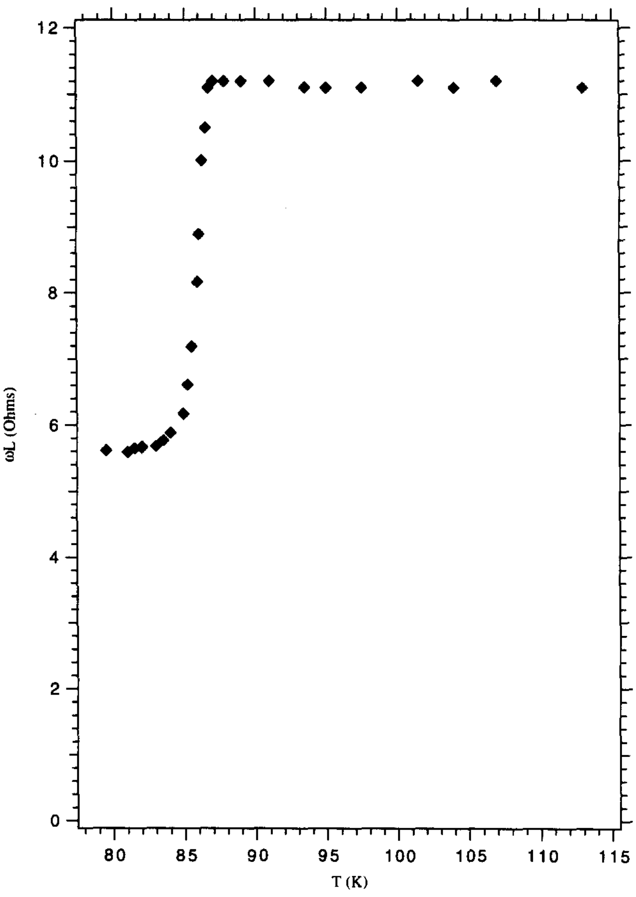

Fig. 2. The inductance of the coil versus temperature. This is typical data collected by a student and shows the superconducting transition clearly.

B. Resistance of the coil

The resistance of the coil is strongly temperature dependent, and students are asked to determine the resistance of the coil as a function of temperature using one of the DMMs as an ohmmeter. The sample is cooled off as above and allowed to warm gradually on the lab bench while students record the resistance of the coil and the thermocouple voltage. It is this temperature-dependent coil resistance which is used in determining the inductance of the coil from Eq. (1). It is also possible to determine the resistance of the coil by replacing the signal generator with a dc power supply and recording the dc voltage across the coil and the dc current as functions of temperature. The use of this method, immediately following measurement of the impedance using the signal generator, reinforces Lenz's law.

C. Results

A typical set of data is presented in Fig. 2. Note that the superconducting transition is very clear in the plot of inductance versus temperature.

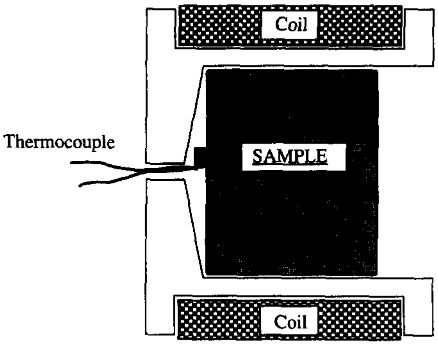

Fig. 3. A cross section of the coil mounted on the coil form and containing the sample and thermocouple. The dimensions of our coil are given in the text.

D. Calibration of the apparatus

The geometrical factors which modify Eq. (2) are difficult to calculate for experimentally convenient geometries, and thus standard practice in inductive susceptibility measurements involves defining a calibration constant α for a susceptometer as follows:

$$\chi=\alpha(L/L_0)-1. \tag{3}$$

The calibration constant is usually determined by using a sample with well-known magnetic susceptibility (e.g., a paramagnetic salt). Of course the calibration constant depends on the shape of the sample, and thus any samples to be studied with the inductive susceptometer must have the same shape as the calibration sample. Moreover, measurement of the small susceptibility of most materials via changes in the inductance of a coil typically requires sensitive detection techniques employing lock-in amplification. The apparatus described here is orders of magnitude less sensitive than inductive susceptometers based on lock-in amplification and cannot be used to measure the susceptibility of paramagnetic materials normally used to calibrate an inductive susceptometer. It would be possible to determine the calibration constant for this apparatus using a superconducting sample with a well-known screening factor, but calibration of the apparatus is unnecessary for observation of the transition, and simple observation of the transition is appropriate for general physics courses. Moreover, nonidealities in available samples of $YBa_2Cu_3O_{7-\Delta}$ make quantitative determination of the susceptibility difficult. In particular, polycrystalline samples of $YBa_2Cu_3O_{7-\Delta}$ contain grains of material with relatively large lower critical fields which undergo a superconducting transition at above 90 K, but material between the grains has smaller lower critical fields[3] and not all of this material will become superconducting in this apparatus. Moreover, the distribution of lower critical fields results in a dependence of the fraction of the sample which screens the B field on the strength of the B field applied by the coil, and samples from the same production lot vary significantly.

We studied a small portion of our sample in a dc SQUID magnetometer at the University of California–San Diego. We found that about 50% of the sample is screened at 80 K in a field of 1 gauss and application of stronger fields (a few gauss) reduces the screening further. The effect can be seen in the inductive apparatus described here by increasing the amplitude of the ac current through the coil which results in a decrease in the magnitude of the drop in the inductance at the critical temperature and a broadening of the transition. As expected, dc SQUID magnetometer measurement of the screening at lower temperatures (5 K) and weak field (1 gauss) indicated improved screening. Given the sample variations and dependence of the screening on the amplitude of the field even for the small fields used in this experiment, we have not sought to calibrate our apparatus.

III. DESIGN AND CONSTRUCTION OF THE COIL

We have used a coil wound of 400 turns of number 34 enamel coated copper magnet wire. This may seem like an inconveniently large number of turns, but the inductance of a coil of a given shape with N turns is proportional to N^2 and the resistance of the coil is proportional to N. If the number of turns is too small, the voltage across the inductor depends very little on the inductance (and thus on the susceptibility of the sample) and is determined mostly by the resistance of the coil. This makes clear observation of the transition difficult. One can increase ωL in comparison to R by increasing the frequency, but typical student DMMs do not measure ac voltage and ac current accurately at frequencies much higher than 1 kHz.

A coil can be wound in-house as follows. A coil form is cut from a plastic rod as shown in Fig. 3. The coil form is mounted in a lathe or another rotating apparatus (low speed) equipped with a counter to count the number of rotations. The magnet wire is fastened to one end of the coil form and is moved slowly across the coil form as the apparatus rotates. After one layer of turns is laid down, the coil is coated with 5-min epoxy. It is helpful to rub the epoxy into the coil with a finger. The second layer turns is then laid down before the epoxy on the first layer hardens and is covered with epoxy. The process is repeated until about 400 turns have been laid down. Thin magnet wire is quite delicate and is prone to break where it emerges from the epoxy. Strain relief is provided by soldering thicker wires to the thin coil wire and then looping the thicker wire once around the coil. A final layer of epoxy is applied to hold the thick wire in place, and the resulting coil is fairly sturdy. Our coil has an average diameter of 1.4 cm and a length of 1.4 cm. Our sample consists of three disks of $YBa_2Cu_3O_{7-\Delta}$ with a diameter of 1.2 cm and a total length of 0.6 cm. Of course it is possible to design the apparatus with a different shape. For example, the sample can be made longer so that it extends beyond the ends of the coil. This increases the change in inductance associated with the transition, but it has the disadvantage of decreasing the homogeneity of the applied field. An inhomogeneous applied field artificially broadens the transition since the transition temperature for screening is strongly field dependent.

The rms magnitude of the B field inside the coil is approximately

$$B=\mu_0 nI, \tag{5}$$

where n is the number of turns per unit length of the coil. For our coil, with 400 turns in a length of 1.4 cm, the rms B field is about 2 gauss when the rms current is 5 mA. Larger rms B fields will significantly decrease the fraction of the sample which screens the B field.

The thermocouple is inserted into the coil form with the wires protruding from the hole as shown in Fig. 2. The hole is then sealed with 5-min epoxy, locking the thermocouple in place. The superconducting disks are inserted into the coil form and pressed tightly together and tightly against the thermocouple. We have applied Apiezon N grease[6] to the ends of the pellets before inserting them into the coil form to improve thermal contact between the pellets and to the thermocouple. This helps to keep the sample temperature uniform. When the thermocouple is in place and the epoxy has hardened, the coil is ready for use. The plastic coil form and the epoxy will gradually deteriorate with repeated thermal cycling and it will be necessary to eventually replace the coil.

IV. CONCLUSION

We have described an apparatus for detecting superconducting transitions in $YBa_2Cu_3O_{7-\Delta}$ via screening of the B field in high temperature superconductors. The simplicity and low cost of the apparatus makes it suitable for use in almost any physics department and in both upper division and lower division courses.

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We would like to thank Colorado Superconductor Inc. for providing samples of $YBa_2Cu_3O_{7-\Delta}$ and thermocouples. We would also like to thank Seamus Lagan, Ward Beyermann, and Doug MacLaughlin for useful discussions. We also acknowledge the W. M. Keck foundation which provided the SQUID magnetometer we used at the University of California—San Diego.

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[1] Superconducting samples in various forms and thermocouples with calibration data can be obtained from Colorado Superconductor Inc., Fort Collins, Colorado. Moreover, Colorado Superconductor is marketing a kit based on the design presented here.

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[4] We used a BK Precision 3011B Function Generator, but almost any function generator capable of producing a 1-kHz signal with an amplitude of about 1-Volt rms into 100 Ω will work.

[5] The operation of DMMs as ammeters and voltmeters should be checked at the operating frequency (1 kHz in our apparatus) by comparison to a scope.

[6] Apiezon N Grease is available from Lakeshore Cryotronics Inc., Westerville, Ohio.

Velocity measurements through magnetic induction

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A simple experiment based on magnetic induction is presented to measure the velocity of a magnet flying through air using a signal induced by a magnet when it passes through a coil. The results show that some properties of this signal depend only on the geometry of the coil and this fact can easily be applied to obtain the velocity of the magnet. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

Experiments dealing with velocity measurement that are normally carried out in a standard physics laboratory are based on methods that make use of two or three basic devices. First, there is the bodies in motion experiment using an air track. In a normal situation, high velocities cannot be used due to the length of the air track. A second device, which is sometimes complementary to the first one, consists of photoelectric gates which are placed at a predefined distance, and both of them are connected to a counter. There are some disadvantages in using this type of device, except in some particular cases.[1] The velocity obtained is an average velocity with subsequent errors.[2] When the experiment is not restricted to a body moving on a track, i.e., a body flying in air, there will be alignment problems with the gates, or photoelectric gates must be used with a wide beam of light to avoid such problems. In addition, this type of photoelectric gate is very expensive and is not available in all laboratories. A third device commonly employed to measure velocity is a ballistic pendulum.[3] With this device an indirect measure is used, because the velocity is determined by using the vertical displacement of a collector system.

A very simple experiment based on magnetic induction is presented here. This experiment can be easily carried out in a secondary school laboratory by means of a device similar to the one traditionally used for the ballistic pendulum. This device usually consists of a launcher system based on compressed air and a box containing plasticine, which is used as a collector system, plus a digital oscilloscope or any kind of standard data acquisition system. This is an ideal situation